

# Children's Abstraction of Iterative Units to Measure Linear Space: A Trajectory

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# Overview of Presentation

- 1. What is the value of a Learning Trajectory?
- 2. Describe our Learning Trajectory for Length (handout)
- 3. Our study on Linear Measurement Development

# What is the value of a Learning Trajectory (LT)?

- A tool for assessment, for improving Learning Standards, and for differentiating instruction
- Learning trajectories incorporate developmental progressions into a larger framework (Clements & Sarama, 2004; 2007; also Smith, Wiser, Anderson & Krajcik, 2006 )
  - An educational goal (concepts of length measurement);
  - A developmental account of thinking and strategies related to the goal;
  - A sequence of instructional or assessment tasks coordinated with the developmental account (cf. *Key Developmental Understandings*, Simon, 2007)

# Our Learning Trajectory for Length (handout)

- Serial Ordering
- End to End Accumulation
- Unit Repeating and Relating
- Consistent Length Measuring
- Conceptual Ruler Measuring
- Integrated Conceptual Path Measuring
- Coordinated, Integrated Abstract Measures with Derived Units

# An illustration of part of a LT: A student at the UR Level

- Modal Age 7: Length Unit Repeater and Relater (level)
- Hypothetical mental actions on objects:  
iterates a unit to find a length by counting  
Requires supportive figural image or actions
- Tasks: e.g., Use a broken ruler to find length

# Our study: Research Questions

- How do students develop coherent, integrated strategies for measurement from Pre-Kindergarten through Grade 5?
- In what ways might a Learning Trajectory for Length Measurement support formative instruction and assessment?

# Our study: Method of Inquiry

- Using a Teaching Experiment (Steffe & Thompson, 2000) to predict and check models of student learning and development through designed cycles of formative instruction.
- Two classes of children
- Beginning in Grade 2 spanning 2007-08 and on into 2008-09...
- Focus Cases: 8 children using a stratified sample
- Background Cases: 8 more children
- Clinical Interviews and Classroom Lessons

# Growth Chart for 8 Focus Students

<i>Child's name</i>	Initial assessment	Follow-up	TE 1: April 2008	TE 2 May	TE 3 May	TE 4 Fall 2008
<i>Sara</i>	ILC	ILC	EE	EE	EE	URR
<i>Anselm</i>	EE	URR	URR	URR	URR	URR
<i>David</i>	EE	URR	(LM)	URR	URR	LM
<i>Abby</i>	URR	URR	URR	(LM)	LM	LM
<i>Owen</i>	URR	URR	URR	URR	(LM)	LM
<i>Ryan</i>	URR	URR	LM	LM	LM	LM
<i>Drew</i>	URR	URR	LM	LM	(LM)	LM
<i>Arielle</i>	URR	URR	LM	LM	LM	(CM)



# Situating 8 Focus Students within the Trajectory

- End to End up into Unit Repeater (student Sara) (May 08 to Oct 08)
- Unit Repeater and Relater (student Anselm) (Jan 08 to Oct 08)
- Unit Repeater and Relater up into Length Measurer (six other students) (earliest: Feb 08, latest: May 08)

# Finding: LT emphasis on Units supported procedural work

- Focus on unit concept in the Learning Trajectory supported procedural development
- Anselm (strips he labeled 4, 3 and 1)

4



3

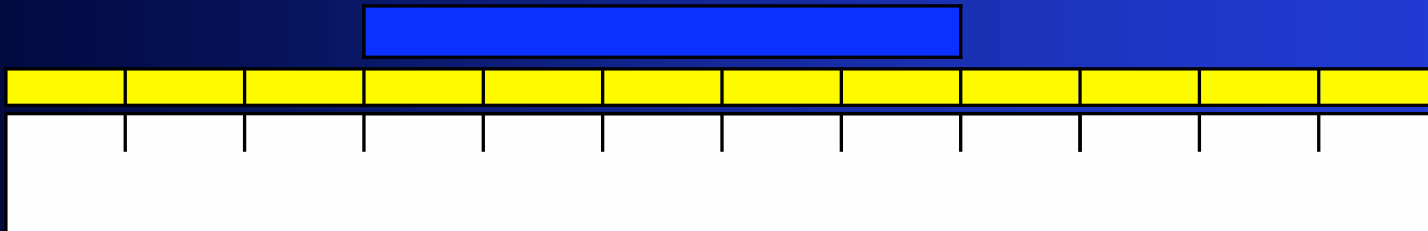


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# Finding: LT supported integrating number line and measures

- Abby: For 3 sessions since Jan 08 she has struggled to coordinate point counting and interval counting. In May 08, a breakthrough:



# Finding: The LT met criteria for a theory of levels

- **Four Criteria** (Clements & Battista, 1992):
  - The levels provide adequate distinctions over time (KDU, Simon, 2006)
  - The sequence is invariant and stepwise
  - Each level is integrative of prior levels
  - Each level is coherent
- **Analysis of our Growth Chart**
  - Our accounts of 8 students show distinctions, growth along steps. This finding generally confirms the stepwise progress with short term regressions but long term stability.

# Finding: Prompting Growth

- False dichotomy between counting intervals and counting tick marks (endpoints) needs to be resolved:
  - Decreasing sequence of segments (3, 2 and 1 unit in length) used to prompt ratio and unit image. The ordinal sequence contrasts with cardinality.
  - Juxtaposing interval strip with line of tick marks promoted unit integration.
  - Find length of a strip that obscures a mid-portion of an extended ruler (80 inches) to restore use of number labels at unit endpoints.

# Research Issues to pursue

- More work to assess children's grasp of continuous and discrete quantity
- Tasks requiring coordination of ordinal and cardinal values on measures support comparative reasoning and ratio concept
- Prompting children to use close transitional forms of rulers bridging from interval sets to tick mark sequences prompted more abstracted unit concepts (units of units)

# Thank you!

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