From Numerals to Intervals and Back: How Young Children Think About Measuring.

Jenni McCool

Craig Cullen

Jeff Barrett

Illinois State University

Doug Clements

University at Buffalo (SUNY)

Measurement Learning Trajectory

- Learning Trajectory
  - An educational goal
  - Children's increasingly sophisticated reasoning
  - Tasks at each successive level of reasoning

Levels of Thinking

- Pre Quantity Recognizer

- Length Quantity Recognizer

- Direct Comparer

- Indirect Comparer

- **End to End Accumulation**

- **Unit Repeating and Relating**

- **Consistent Length Measuring**

- Conceptual Ruler Measuring

- Integrated Conceptual Path Measuring

- Coordinated, Integrated Abstract Measures with Derived Units

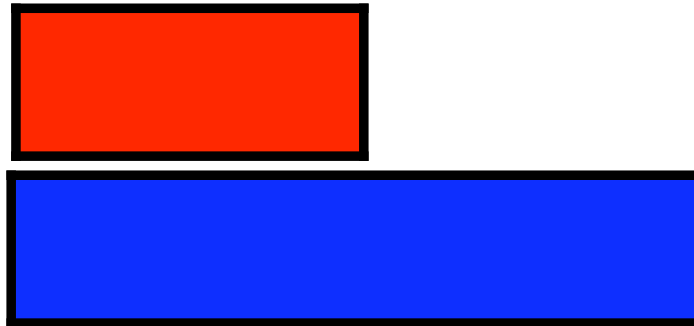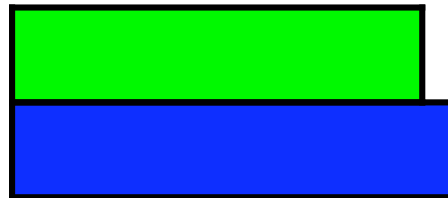# Length Measurement Trajectory Levels

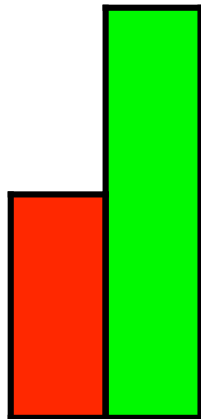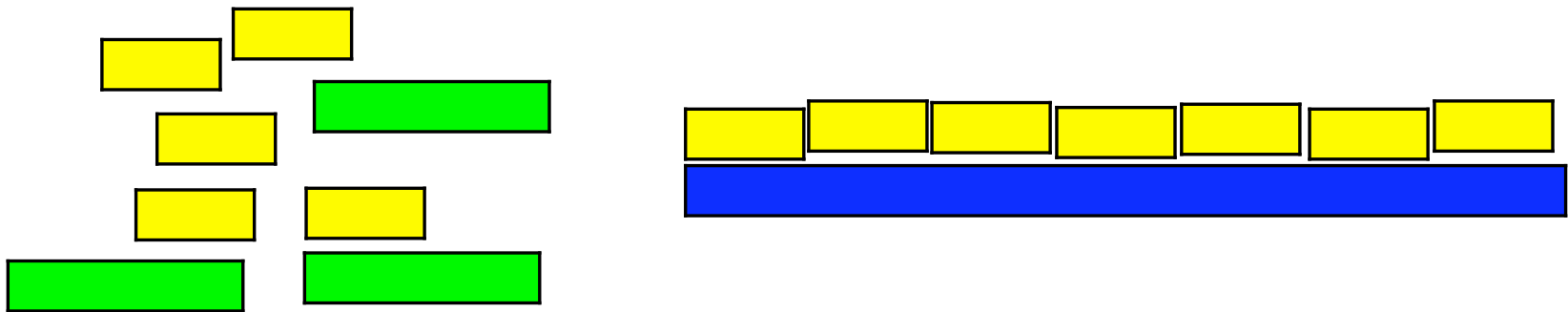**Pre-Quantity Recognizer Length**

Length Quantity Recognizer
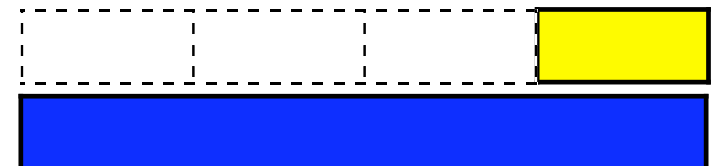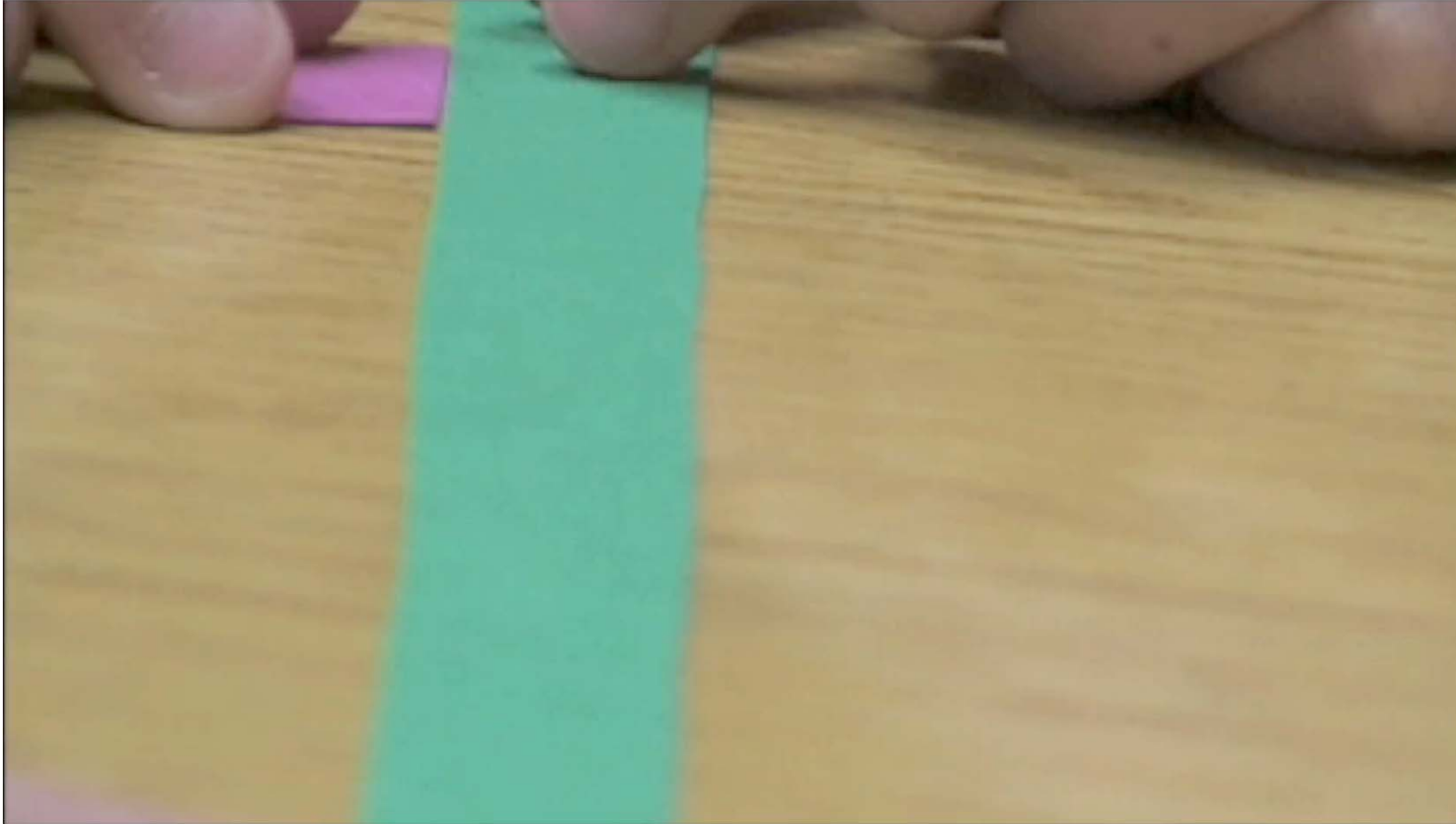

"These are both short."

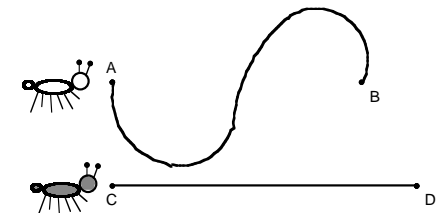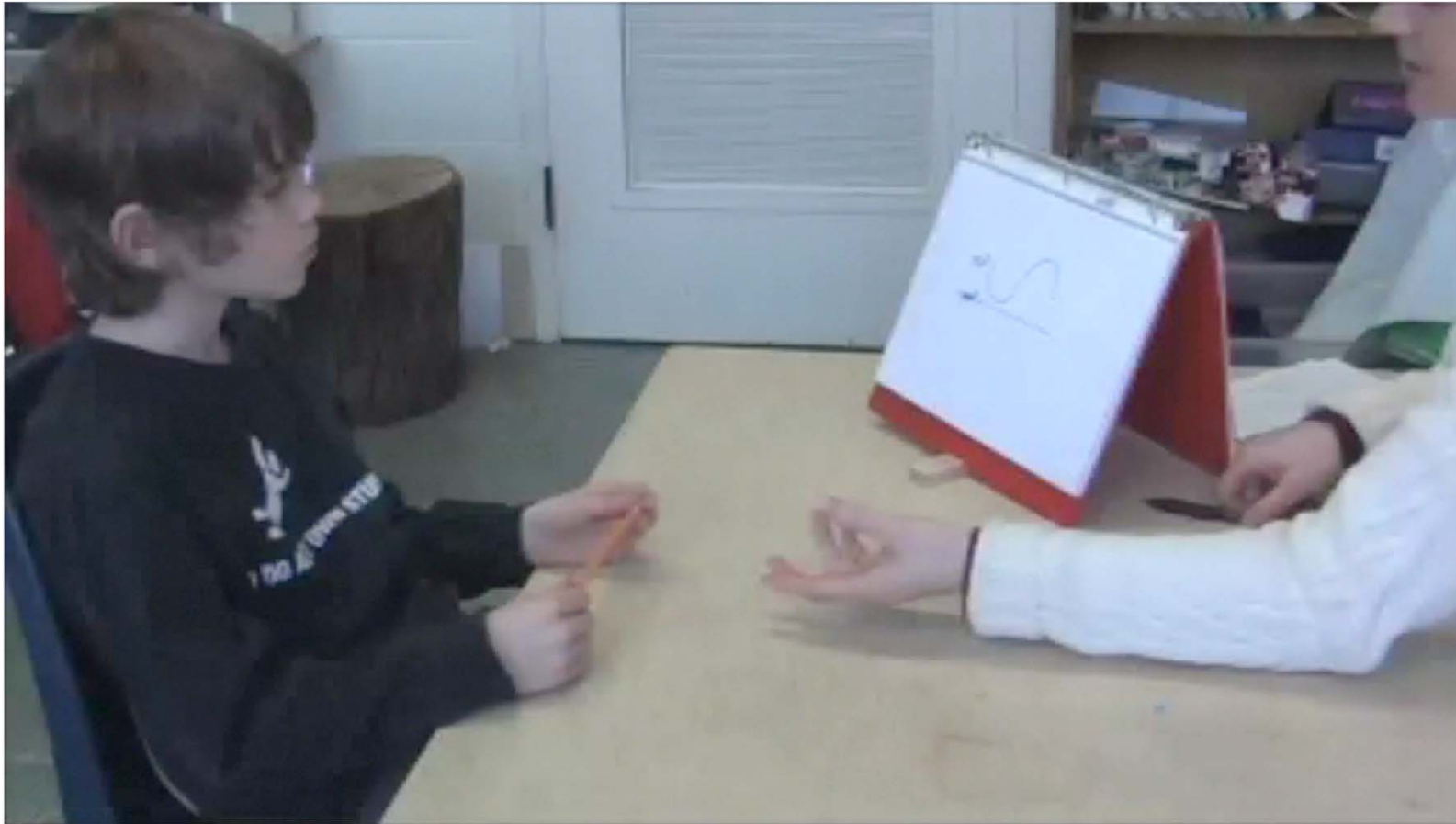- Direct Comparer

Indirect Comparer

End-to-End

- Unit Repeater and Relater

- Consistent Length Measurer (Example 1)

- Consistent Length Measurer (Example 2)

- "3 yards, 15 inches"

Length Measurement Trajectory

| Level	Thinking & Actions	Tasks
End-to-End	Expects that length is quantifiable as a composition of shorter lengths. Compares an end-to-end train of countable objects to the linear extent of an object.	Fewer objects than needed to fill the space. Exaggerated mistakes for gapping and overlapping.
Unit Repeater & Relater	Is able to iterate a unit along an object to find length.	Contrasting a tick mark tool and an interval tool. Diminishing line segments. Broken ruler tasks.
Length Measurer	Can compose and partition length units. Can think of the length of a bent path as the sum of its parts. Mentally iterates a unit and sub units (internalized ruler).	Predict with mental iterations and check with a tool.

Growth Chart for 8 Focus Students

| Child's name	Initial assessment	Follow-up	TE 1: April 2008	TE 2 May	TE 3 May	TE 4 Fall 2008
Sara	ILC	ILC	EE	EE	EE	URR
Anselm	EE	URR	URR	URR	URR	URR
David	EE	URR	(LM)	URR	URR	LM
Abby	URR	URR	URR	(LM)	LM	LM
Owen	URR	URR	URR	URR	(LM)	LM
Ryan	URR	URR	LM	LM	LM	LM
Drew	URR	URR	LM	LM	(LM)	LM
Arielle	URR	URR	LM	LM	LM	(CM)

The Big Story

- Identification of units of length
- Coordinating intervals with units of length
- Coordinating numerals to length

The Broken Ruler:
A Revealing Task
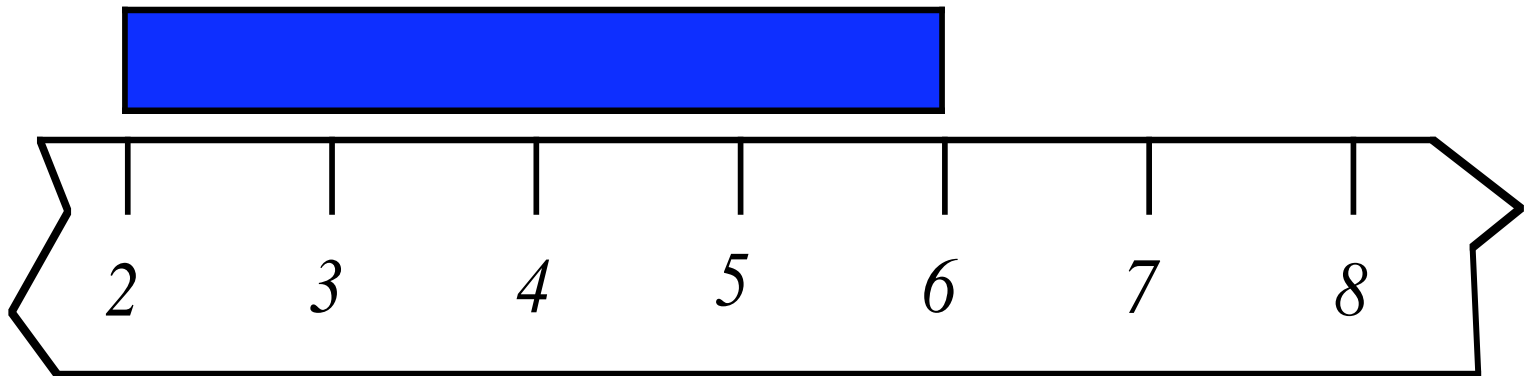
Where is the unit?
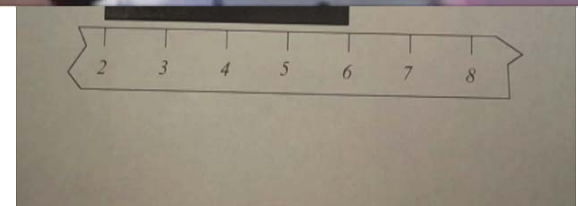
The unit identified

Where is the unit?

The unit identified

Make a record

Where is the unit?

Transition???

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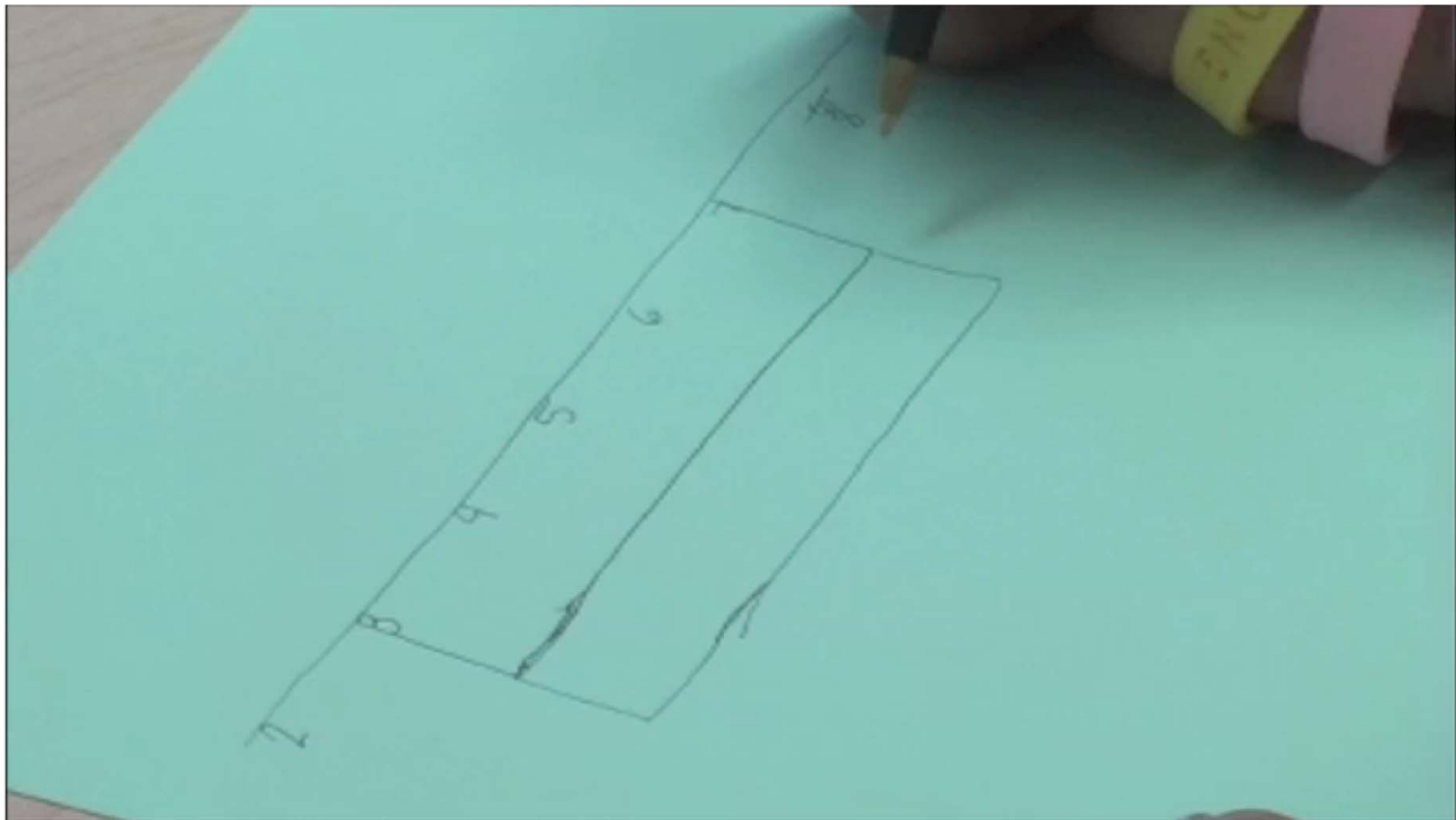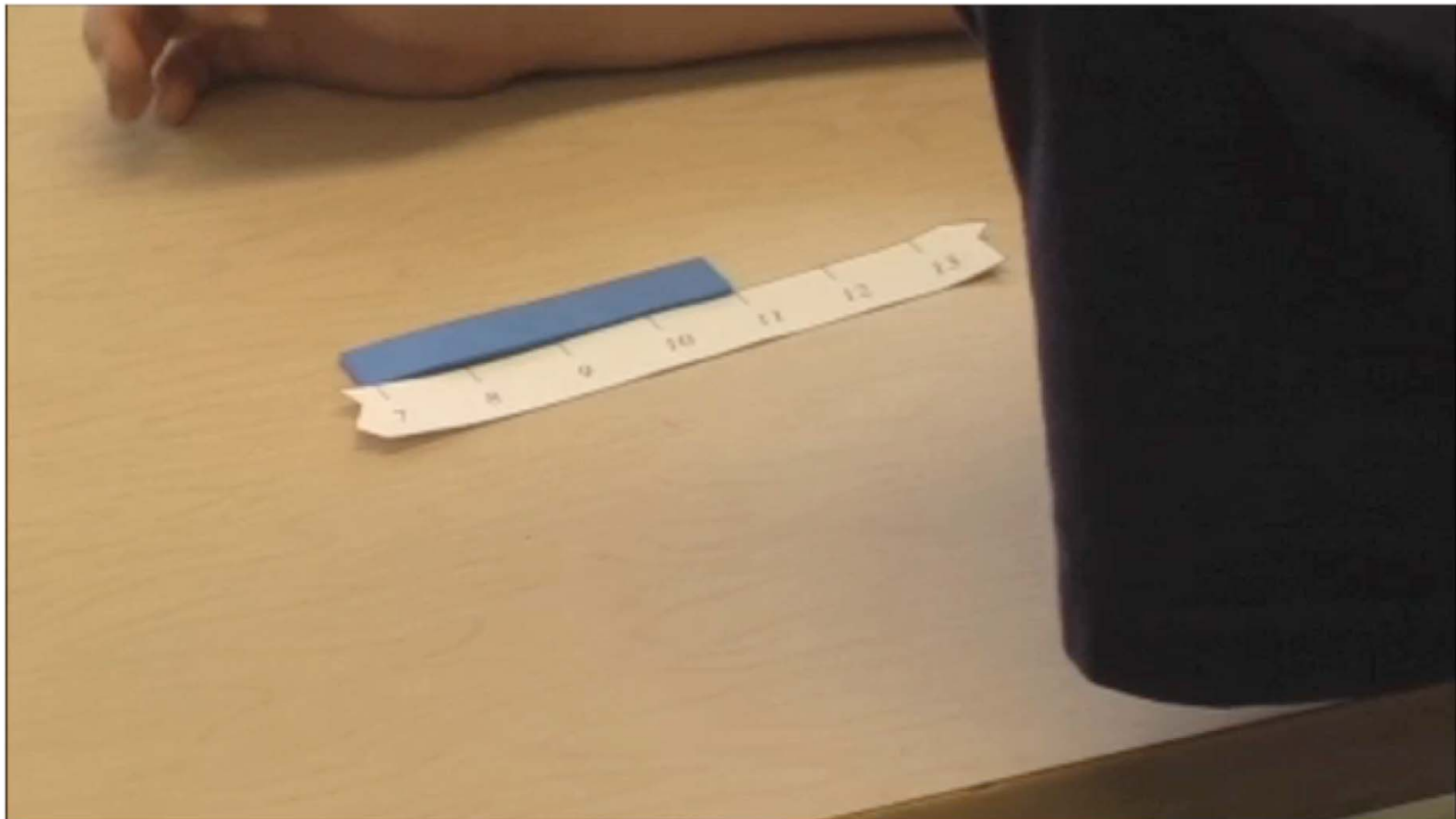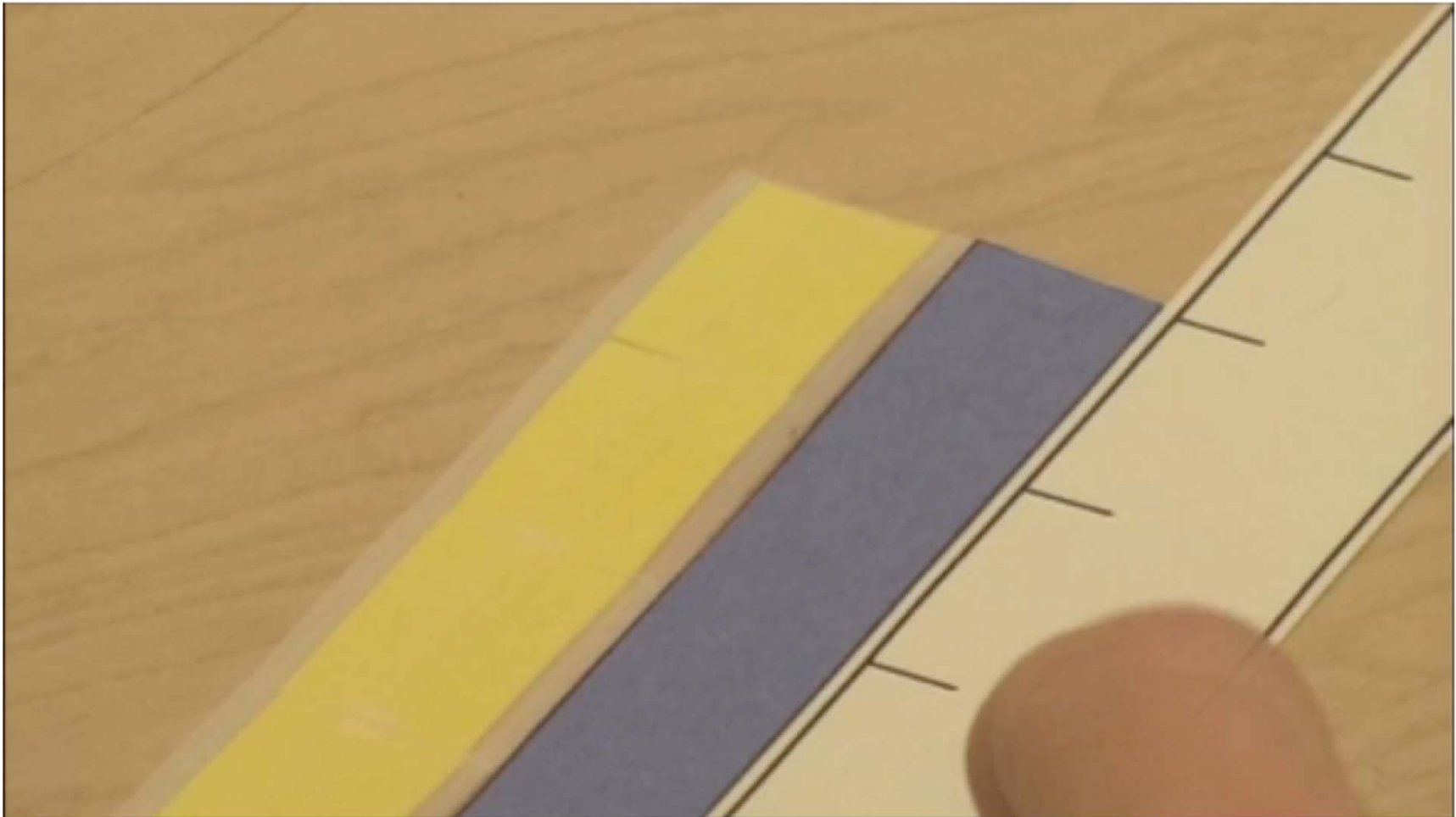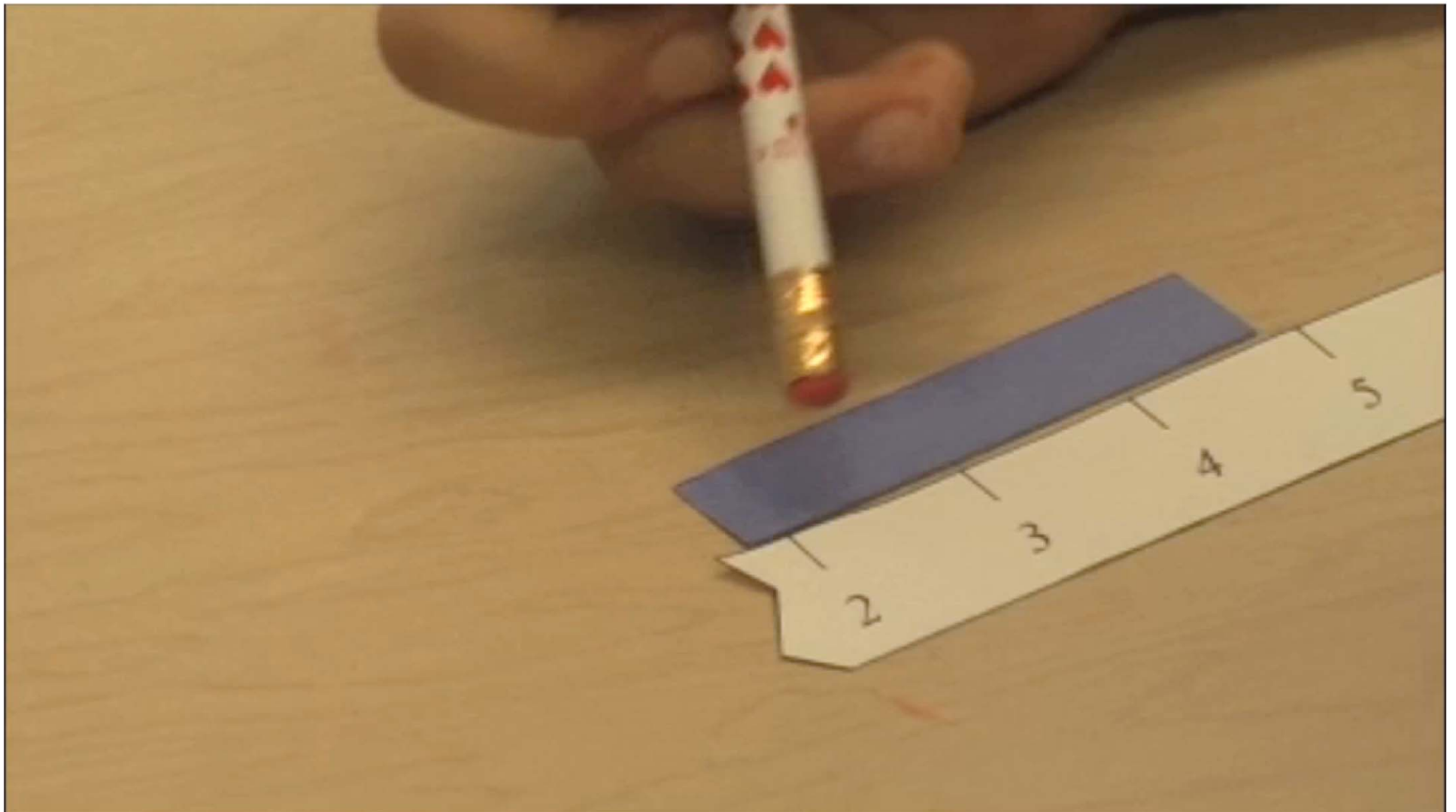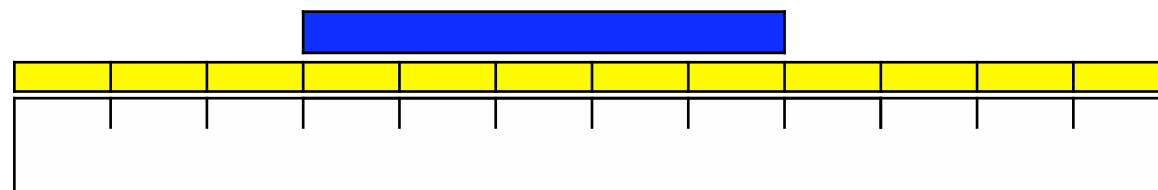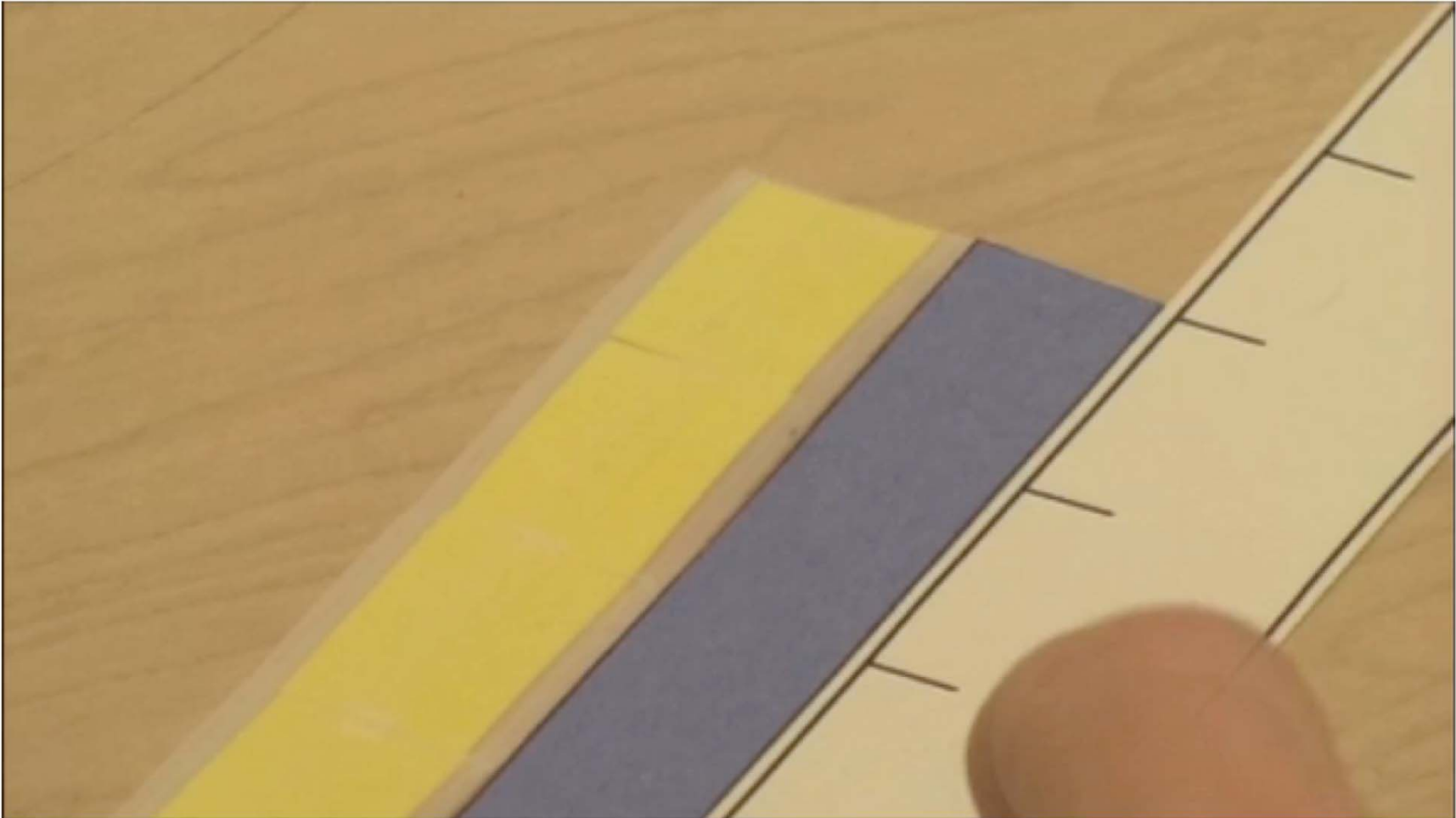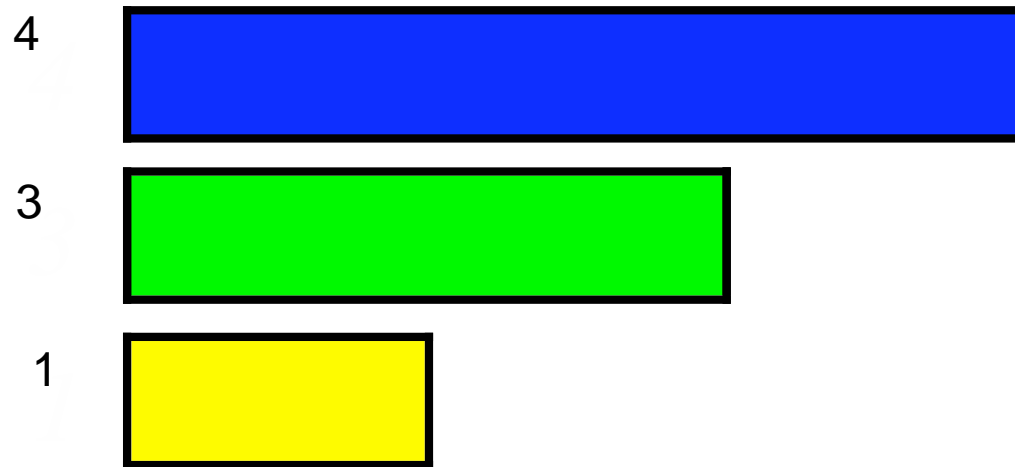# Length Measurement Trajectory

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Diminishing Segments

- Anselm (strips he labeled 4, 3 and 1)

4

3

1

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- Abby: For 3 sessions since Jan 08 she has struggled to coordinate point counting and interval counting. In May 08, a breakthrough:
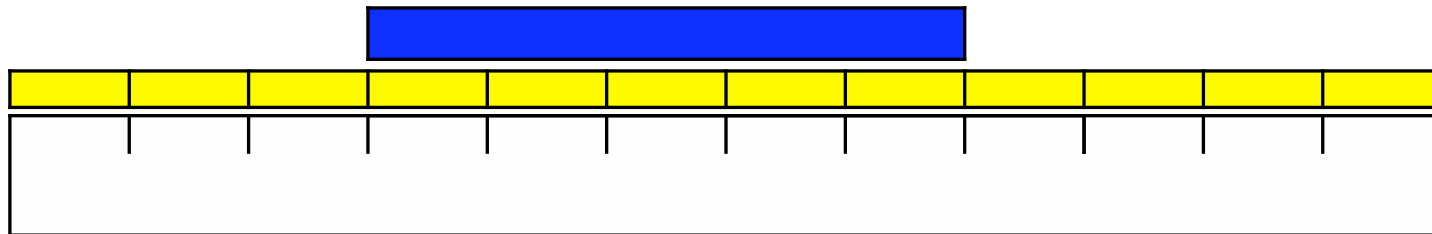
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Summary

- False dichotomy between counting intervals and counting tick marks (endpoints) needs to be resolved:
  - Decreasing sequence of segments (3, 2 and 1 unit in length) used to prompt ratio and unit image. The ordinal sequence contrasts with cardinality.
  - Juxtaposing interval strip with line of tick marks promoted unit integration.
  - Find length of a strip that obscures a mid-portion of an extended ruler (80 inches) to restore use of number labels at unit endpoints.