

METHODOLOGIES OF
HEALTH INSURANCE
RESERVING

Christine Wiehe

110 Pages

August 2005

This thesis describes the calculation of reserves in health insurance in the United States as it is done by deterministic methods nowadays and improvements by stochastic methods.

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The exact calculation of the reserves of an insurance company which has to be shown in the company's annual statement is an important matter, not only for health insurance companies. An appropriate prediction of the future liabilities of a health insurance company, especially the liabilities for reported or not yet reported claims, is even harder, since no one can really predict, how often a physician will be visited or how much the total costs of a claim will be. Another problem is that the access to the claim related data is limited for health insurances. Therefore the actuary has only knowledge of historical claim behavior of similar claims, i.e. the factor time depends on the knowledge of previous claim development.

This thesis describes shortly the health insurance system of the U.S. and the different kinds of reserves for health insurance. Afterwards the calculation of health claim reserves will be discussed in more detail. The most common used methods are the chain-ladder method and the completion factor method. The disadvantages of these methods are that they do not provide a measure of variability or diagnostic

tests. Therefore statistical methods can improve the prediction of the reserves. These improvements are mainly based on statistical methods, a regression approach with independent variable time and dependent variable claim amount. The advantages and the disadvantages of these methods for the claim reserving process are discussed, too. Two different approaches are discussed, whether hybrids between the distribution-free and regression models or whether purely statistical methods are more appropriate to be used. This thesis provides an overview over both ideas.

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A Thesis Submitted in Partial
Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

Department of Mathematics

ILLINOIS STATE UNIVERSITY

2005

THESIS APPROVED:

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ACKNOWLEDGEMENTS

I would like to thank the members of my Thesis committee for their help during my work on this thesis.

Especially I would like to thank Krzysztof Ostaszewski for his guidance over the year and his ambition to teach us the American Insurance System. He offered me the great opportunity of working in the Health Insurance Project which gave me the possibility to understand the system even better.

I would like to thank Hans-Joachim Zwiesler for his help before and during the exchange program with the Illinois State University.

Furthermore I would like to thank Solveig Bosse and Thomas Kamberger for proof-reading my thesis.

I would like to thank Julie and Allison Riddle for proof-reading my thesis and the whole family Riddle for being a great host family during my time in Normal.

Also I would like to thank Dieter Kiesenbauer for his support throughout the year, whenever I needed him, and for proof-reading my thesis.

Finally I would like to thank my parents, Annegret and Hartmut Wiehe, for their support during my studies of mathematics in Germany and abroad.

C.W.

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CHAPTER I

INTRODUCTION

In the annual statement of any insurance company, reserves build a main part. In health insurance business they describe the liabilities of the insurance company to the insured individual, the policyholders of a group insurance contract or the providers of health care in the case of managed care organizations. Therefore they are somehow "customers' money" and need to be calculated accurately. A careful calculation is required and is strongly supervised by regulators to make sure that the reserves are estimated in an appropriate way and that the insurance company remains solvent.

But how can one calculate payments in the future which are not known today? This question can be solved by several calculation approaches. Some are like the ones used in the life insurance business, some are similar to the reserving methods for property/casualty insurance. Nevertheless, reserving methods for health insurance are of their own interest, since an exact calculation is not easy and nobody can predict how many health care claims will occur and how much they will cost. Due to always new development of medical technologies these can rise new levels, since costs for medical procedures will increase. This may cause a

problem for the estimation of policy reserves which describe most of the time a long-term character.

Another problem occurs for the estimation of reserves for a specific claim, the so-called claim reserves. Health insurances are not allowed to get many claim describing information which would be helpful for the claim amount evaluation. Different to property/casualty insurance for which the actuary has knowledge about the factors influencing the ultimate claim (e.g. the type of a car, the vehicle number, etc.) the health insurance company knows only the time as influencing factor. This is given by the date the claim incurred (i.e. the moment an insured uses a medical service), the date when the insurance company gets knowledge about the claim and the date when a payment for the given claim is made. Thus, models describing the claim development process should be mainly based on the factor time. These models will be an integral part of this thesis which relies mainly on procedures for claim reserve calculating.

This thesis will describe the methods of Health Insurance Reserving in the United States as they are used today but emerging methods will be analyzed, too. Since the American health insurance system differs from most of the health insurance systems in the world, a short description of it will be given in chapter 2. The main types of health insurance reserves in the United States will be described in chapter 3. Chapter 4 discusses the most common methods which are used for the estimation of claim reserves of health insurance companies today. Normally, health

claim reserves are calculated with the same methods used in the property/casualty business. These reserving methods are approved by federal and state regulators. However, they have disadvantages, too. Although these methods are known for property/casualty claim reserving, they are more problematic for health insurance companies, since for these not so many data are known for each claim. Therefore new approaches need to be considered. These mainly statistical methods will be discussed in chapter 5.

CHAPTER II

HEALTH INSURANCE IN THE US

Health insurance in the United States is provided - as stated by O'Grady (1988) - by three different sources: as social insurance provided by federal or state institutions, as group insurance or as individual insurance. Reserves are only important in the two latter branches, since due to the sharing of costs and expenses in the social insurance no reserve-calculation is needed for this kind of health insurance. For that reason this thesis confines itself to describing only the private and the group insurance sector.

Private Insurance

"Private" or "individual" health insurance is the term used for an insurance contract which provides medical coverage for only one individual and in special cases for family members of that individual. Normally, the policy is issued to the covered individual. Before the individual enters into the insurance contract he has to prove insurability, i.e. he has to provide the insurance company with information about himself, his medical condition and other circumstances which may influence his health. This is done to protect the insurer from adverse selection.

Private health insurance is offered as full health insurance, i.e. the individual

insurance is the only source of health insurance, or as supplemental health insurance to existing health insurance contracts which are either offered by the government or as group insurance. Since - according to Bluhm (2003) most of the employees in the United States have health insurance via group insurance by their employers, the need for full individual health insurance is given only to a small percentage of the population of the U.S. Black and Skipper (2000) describe the typical groups of individuals who buy individual health insurance. These are students who are not insured in their parents health insurance, self-employed people, employees whose employers do not offer group health insurance plans, unemployed who are not eligible for governmental plans or part-time, temporary and contract workers who are not eligible for group insurance plans offered by their employer.

Individual health insurance is offered in so-called health insurance plans. According to Black and Skipper (2000) individual health insurance can be distinguished into three different categories of insurance plans:

1. Comprehensive Medical Insurance Plans which cover most medical costs
2. Long-term care insurance which pays for nursing care needed in old age
3. Disability Income Insurance which provides coverage for loss of income due to sickness or injury

Each category is covered under a separate policy, because each of these plans has different characteristics which will be described in the following.

Comprehensive Medical Insurance

Comprehensive medical insurance policies cover the expenses which occur due to medical care in case of illness or by preventive medical procedures. These expenses originate from in- and outpatient hospital services, visiting physicians or other medical institutions who do medical diagnostics, and from the purchase of prescription drugs.

This kind of insurance is offered to individuals by traditional life insurers who also provide health insurance or by health service providers like Blue Cross / Blue Shield insurers or health maintenance organizations (HMO). The two latter will be described in the section "Group Insurance", because they are the main sellers of group insurance in the United States. Normally, the policy for a comprehensive medical insurance plan is issued for one year, but it is guaranteed renewable. That means the insurance company cannot cancel the policy or change the benefits, if the insured pays his premiums on time, but it is allowed to change the premiums. This change of premiums is subject to strong regulation and cannot be done for a single policy, but only for a group of insured persons in the same class.

To reduce his premiums the insured can choose to pay medical expenses up to a predetermined limit on his own, the so-called deductible. This amount is limited most of the time by an "out-of-pocket" amount per year (see Black and Skipper (2000), p. 138). As soon as this amount is reached by the insured, the insurance company pays for all further medical expenses even if the costs would normally be

covered by the deductible payment of the insured.

Some supplemental coverage is provided under the term of comprehensive medical insurance, too. These policies will not be covered in detail, but a short overview of the kinds of policies will be given (see Black and Skipper (2000)):

1. Hospital confinement indemnity policies which pay a fixed amount per day of hospital treatment
2. Government supplemental policies which are supplemental to governmental coverage such as Medicare; for example they cover deductibles and coinsurance the insured has to pay under Medicare and / or pay for the expenses in periods when the protection by Medicare has been ruled out
3. Specified disease policies which cover the costs incurred by one disease specified by the insurance contract such as cancer

Long-Term Care Insurance

Since due to medical development and a better environmental climate in the countries of the European Union, Canada and the United States, the number of elderly people is increasing in these countries. Many elderly people cannot take care of themselves - according to Black and Skipper (2000) (p.141) these are approximately 40% of the elderly in the United States - and have no dependents who can, as in former times when the family took care of them. Consequently, there is a need of nursing care. This kind of care is very expensive because of time

and care intensity. Therefore an insurance which covers the costs of nursing care is necessary to eliminate the loss of property during old age. This kind of insurance is given by the so-called long-term care insurance.

Long-term care insurance covers the costs for medical, personal and social needs for people who are not able to perform the basic activities of daily living like eating or dressing because of accident, illness or frailty. These services are provided as "Nursing Home Care" or as "Community Care" (Black and Skipper (2000)). If the insured receives "Nursing Home Care", he lives in a nursing home where he gets all the help for daily life activities he needs. "Community Care" is provided for elderly persons who live at their home and need assistance only in the activities of daily living. This assistance is provided by nurses who visit the elderly persons at home.

The need for individual long-term care insurance is given since - according to Black and Skipper (2000) - governmental programs like Medicare or Medicaid cover only a small percentage of the population, because these programs are designed to help elderly and poor people. Normally, they cover only a small percentage of the costs of nursing care services such that an individual long-term care insurance makes it possible that the insured individual receives a better treatment. Benefits for long-term care insurance are paid in a by the policy specified daily amount over a specified period in time. The length of payment and the benefit amount depend on the wishes of the insured and are defined in the insurance policy.

The premiums for a long-term care policy are determined by factors as age of the insured, sex of the insured, medical conditions and medical history, the amount and duration of benefit payment provided as well as the existence of an elimination period for the benefit payment (see Black and Skipper (2000), p. 146).

Disability Income Insurance

The two health insurance plans which have been introduced so far cover the expenses for medical services in case of illness and the costs for nursing care if necessary. However, there are questions arising in one's mind: "What happens if I am not able to work any longer due to an accident or an illness? How can I pay for the needs of daily life in such a case when I do not have any regular income?" Then a disability income insurance is helpful.

Disability income insurance provides monthly benefits in case of loss of income due to a disability related to an accident or an illness. As a comprehensive medical insurance it is either a full or a supplemental insurance. Since it is not clear when somebody is disabled - Is it if he cannot work in his former profession or is it if he cannot work in any job? -, the term of disability has to be specified in the insurance contract. Two different types are distinguished. The "own occupation" definition states that a disability is given if the insured is not able to "perform the principal duties of his/her occupation" (O'Grady (1988), p. 37). The "any gainful occupation" definition of disability provides the payment of a benefit only if the

insured is not able to perform any gainful occupation for which he has the skills because of education, training or experience.

The amount of premium which has to be paid by the insured depends on three major factors (see Black and Skipper (2000), p. 153). First, the premium depends on the benefit amount the insurance company has to pay in case of a disability which is based on the actual monetary loss the insured will suffer in case of disability. The benefit will be paid on a monthly basis. Secondly, the premium is determined by the length of the benefit payment period after the disability has occurred. Typical periods of benefit payment are short-term, i.e. two to five years, until a specified age like the normal retirement age at 65 and sometimes continuously for the whole life time. A third factor which influences the amount of premium is the duration of an elimination period. This is the period during which, after the incidence of the disability, no benefits are paid. This elimination period can last between 30 days and one year.

Group Insurance

Most of the health insurance contracts in the United States are group insurance contracts. Therefore it is important to have a closer look at this kind of insurance. A group insurance does not cover only one individual and his dependents but a whole group of persons under one single insurance contract. This

differs from other forms of insurance, because normally a person buys insurance coverage for himself, for his family and other dependents. In this section a short overview over the main characteristics of group insurance will be given and advantages and disadvantages of this insurance type are shown.

As for individual health insurance policies, medical group insurance plans cover the main medical benefits as hospital and surgical expense benefits, but they can also cover - referring to Black and Skipper (2000) - extended care services like nursing home care, community care and hospice care expenses. The benefits can be limited in the amount or the duration of payment. The major health insurance group plans are major medical expenses plans, basic hospital/surgical plans and supplemental plans to the basic coverage. Group insurance plans cover the loss of income because of short-term or long-term disability, too.

Important Facts about Group Insurance

In most group insurance contracts the participants - those are the covered individuals under the contract - do not have to provide a proof of insurability as it is common in individual health insurance contracts which is supposed to protect the insurer against adverse selection. Therefore some different regulations for group insurance have been made (see Black and Skipper (2000)).

In most of the group insurance contracts, the group has to be formed for another reason than to get insurance, for example as employees of a single employer

or of multiple employers. Possible buyers of group insurance contracts will be explained in the section "Buyer of Group Insurance". Also, the group for which group insurance is bought should not be closed, i.e. entries into the group and leaving the group should be possible on a steady basis. These regulations allow to make predictions about morbidity and mortality of the group and to make an insurance coverage possible without high risks for the insurance company.

Furthermore the group insurance contract should cover the majority of group members or all group members. For non-contributory plans a one hundred percent participation is prescribed, because the policyowner pays the whole premium. In contributory plans in which the participants are required to pay a part of the premium for their benefits on their own a participation of 75 percent (Black and Skipper (2000), p. 451) is needed. This regulation is supposed to protect the insurer from only highly risky persons taking part in the group insurance. Additionally, the policyowner should pay at least a part of the premium to give incentives to join the group.

Another very important regulation is a prescribed basic coverage for each individual. This protects the insurer against the fact that healthy and therefore less risky participants buy only low coverage whereas sick and therefore risky participants buy high insurance coverage. Together with the basic coverage the insured participant under the group insurance contract has the possibility to choose benefits which fits his needs perfectly. In some group insurance plans the

participant can choose between the amounts of benefits (s)he will receive in case of illness. These are only amounts which are beyond the basic coverage. Another type of plan in which the participant has the ability to choose the "best coverage for himself" is the so-called cafeteria plan (Black and Skipper (2000), p.450). In this type of plan the participant can choose from different benefits those which fits best to her/him.

For small groups a proof of insurability of all participants is necessary to protect the insurer against adverse selection. That is done because the probabilistic structure of small groups is not as significant as the one for larger groups. Therefore a small group cannot compensate high risks as larger groups can.

Group Insurance Pricing

The group insurance pricing is done most of the times like the pricing of individual insurance. Nevertheless, there is one big difference. Especially for groups with a large number of participants, the pricing can be based on the experience of the group. This type of pricing is the so-called experience rating. According to Black and Skipper (2000) the rates for experience-rating are dependent on the type of contract, for employer-based group insurance plans the type of industry, the geographic region where the group is based and the composition of group members, e.g. age, sex and income level of the participants.

For smaller groups for which the calculation of premiums and the prediction about future claims are more complicated, uniform rates are used.

Buyer of Group Insurance

Since group insurance should be sold to groups of individuals which belong together for another reason than to obtain insurance, a definition for groups eligible for group insurance is needed. The main types of groups which are eligible to buy group insurance coverage in the United States are defined by the National Association of Insurance Commissioners (NAIC) Model Group Insurance Bill. Nevertheless each state can point out other eligible groups.

By the NAIC Model Group Insurance Bill the following groups are eligible to buy group insurance coverage (see Black and Skipper (2000), p. 454):

1. Employees of a single employer

The employees of a single employer and sometimes their dependents are covered by the group insurance contract. The employer or a trust act as the policyowner.

2. Multiple employer trust

These trusts give small employers the ability to buy group insurance for their employees. The trust will act as the policyowner.

3. Government employee groups

4. Labor Unions

Labor Unions can deliver group insurance coverage to their members. The union itself will act as the policyowner.

5. Associations and organizations

Associations or some organizations can offer group insurance protection for their members. Examples for possible organizations and associations are professional, trade or alumni organizations and also associations founded by another reason than obtaining insurance like the American Automobile Association, etc.

6. Creditor groups

They can sell group insurance to their debtors but most sell only life and disability insurance and not health insurance.

Additionally, several states allow group insurance coverage for groups not defined in the NAIC Model Group Insurance Bill .

Seller of Group Insurance

Group Health Insurance can be purchased from several different health insurance providers. In the following the main providers of group health insurance (Bluhm (2003)) will be described.

Similar to individual health insurance, group insurance is sold by insurance companies. They offer the full range of health care plans like indemnity plans,

Point-Of-Service (POS) plans, in which the access to a special network of providers is controlled by a primary care physician, dental, disability and long-term care plans. Also they provide Administrative Services Only (ASO) for self-insured plans. Since most insurance companies have access to the insurance market in almost every state of the United States and since they offer the full range of health insurance coverage, they have an advantage over other forms of group insurance. This advantage is given by the fact that a buyer of group insurance can buy all kinds of products he wants to offer his members at one company and does not have to switch between different sellers of group insurance. Another advantage is that the insurance companies can use their knowledge gained in other states to offer group insurance in specific, not so widely spread occupational fields.

Other providers of group health insurance are Health Care Service Corporations like Blue Cross / Blue Shield. The "Blues" (Black and Skipper (2000)) are non-profit organizations which provide health insurance coverage on a prepayment basis. Often they are granted a special tax treatment by federal or state law.

Blue Cross plans offer hospital expense plans with member hospitals and reimburse the hospital directly for the offered service.

Blue Shield plans offer surgical and medical expense benefits for treatment by a physician similar to the benefits commercial plans provide. These are prepaid plans as well.

A third provider for group health insurance are the so-called Managed Care organizations like the Health Maintenance Organizations (HMOs) and the provider based health insurance like Preferred Provider Organizations (PPOs) and Exclusive Provider Organizations (EPOs). These organizations do not only provide health insurance, but also deliver the health care services.

Health Maintenance Organizations are growing providers of managed care health insurance. These are "legally organized entities" (Black and Skipper (2000), p. 494) which most of the time do not only provide the financing of health care services via insurance, but also provide the health care services to their members on a prepaid fixed fee. The participating hospitals are sometimes owned by the HMO and the physicians are employees of the HMO on a fixed salary. Some plans are on a prepaid basis so that members of the HMO can use one or more hospitals on a service type basis and they can visit a special group of physicians for other services.

The emphasis of HMO based plans is in the preventive medicine and in early treatment of diseases, because those two factors help to lower the costs for health care. Therefore routine medical examinations are made on a regular basis. In case of an injury or sickness HMOs also provide the adequate hospital and medical care by their member providers. The members of a HMO plan are allowed to visit a non-participating hospital or physician only in case of emergency.

Another type of Managed Care plans is a provider-based plan. Both, Preferred Provider Organizations and Exclusive Provider Organizations are formed

by a group of health care providers which offer health care services to their members to a reduced fee on a fee-for-service basis (see Black and Skipper (2000), p.496). While PPOs allow their members to choose between health care providers but get the lower fees only at preferred providers, EPOs limit their members to the EPO providers.

The last form of providing group health insurance is self-insurance. The employer, labor union or trust pays the premiums into a fund which is used to pay for their members or employees. Most of the times the self-insured does not do the administrative work on his own. He hands it over to the so-called Administrative Service Only Providers which are for example insurance companies.

Comparison of Individual and Group Insurance

Based on Black and Skipper (2000) the main advantages and disadvantages of group insurance over individual insurance will be pointed out in this section.

As we have seen before group insurance plans are most of the time plans which are sponsored partially or even paid completely by a third party. Therefore they make it possible that persons who cannot afford to buy individual health insurance can get at least a basic insurance coverage. Also the concept of group insurance allows that uninsurable persons from the point of view of individual health insurance can get insurance coverage. The reason for this is that a proof of

insurability is not required in most of the group contracts, because the risk is spread among the group participants. Additionally, group insurance for larger groups is less costly than individual insurance for all participants. This is due to the fact that it is distributed among many persons at the same time instead of several individual insurances. Another reason that group insurance is not so expensive is the fact that the commissioners' and the administrative costs are decreased by the issuing and administration of only one huge contract instead of several small ones. The costs for medical examinations is also minimized, because no proof of insurability is needed for group insurance most of the time.

One big and severe disadvantage of group insurance, especially for employer-based plans, is the temporary coverage. The insured person is only insured as long as the participant is employed by the employer. Another disadvantage is that no individual can have an overview about her/his own needs. This might have consequences if (s)he has to buy her/his own insurance coverage due to a change of the employer or of unemployment.

CHAPTER III

RESERVING IN HEALTH INSURANCE

As in life insurance, reserving plays an important role in the health insurance business. The insurance company has to calculate the amount of future obligations at the end of every accounting period. Although nobody can tell the exact amounts, the adequate calculation is important to make sure that the insurance company remains solvent in the future. This chapter discusses the main types of reserves and liabilities a health insurance company has to set up for their existing contracts.

According to Gamage et al. (2005), the following three types of reserves can be classified for health insurance. These will be covered in more detail in this chapter:

1. Policy reserves:

These include the amount that is needed to pay future contract obligations as well as reserves for unearned premiums

2. Claim reserves:

These should cover those amounts which are needed to pay claims which are still in the development process

3. Expense liabilities:

These include those amounts the health insurer needs to pay for loss settlement of expenses and taxes in periods prior to the statement date

As in life insurance health insurance reserving is liable to strong regulation which is governed by the NAIC Model Minimum Reserve Standards for Individual and Group Health Insurance (see American Academy of Actuaries (1998)).

Policy Reserves

The policy reserves can be distinguished into two different categories, the reserves for long-term health care insurance and the reserves for unearned premiums. A description of both kinds of policy reserves is given in the following sections. In addition to reserves for long-term health insurance and premium reserves, sometimes the necessity of further reserve calculation exists. This depends on different effects in the claim developing process as well as on contract specific assumptions. Therefore the actuary has to consider if additional reserves like premium deficiency reserves or outcome-based contractual reserves have to be calculated. A short description of these reserves will be given, too.

Reserves for Long-Term Health Insurance

Defined by the NAIC Health Reserve Guidance Manual (see Bluhm (2003), p. 414) as "a reserve set up when a portion of the premium collected in early years is

meant to help pay for higher claim costs arising in the later years”, reserves for long-term health insurance are required only for those policies which have a long-term character and which are priced on a level periodical premium basis. Examples of these kinds of policies are individual disability policies, long-term care policies and hospital benefit policies. They should cover the increasing costs of aging and therefore they need to be saved in earlier years.

Referring to Gamage et al. (2005), the reserves for long-term care insurance are mainly calculated as life insurance reserves, because they cover long-term insurance. Therefore all calculating methods which are known from life insurance can be used to obtain the reserves. Using the equivalence principle and a prospective model, the net reserves at a given time t are calculated as the difference of the expected present value of future benefits and the expected present value of future valuation net premiums under the condition that the policy exists at time t . In difference to life policies the amounts of benefits for a health insurance policy are not necessarily fixed, because the benefits are influenced by higher medical costs for the use of new technologies. The influence and the duration of special illness are not known. Therefore the expected present value of future benefits should include this fact.

The basic principles for the statutory valuation of long-term reserves are the designated morbidity tables, the assumption of a maximum interest rate at the date of issue of the given policy and the designated persistency rate tables. The

persistence rate (see Black and Skipper (2000), p. 805) is given for a group of policies and it is defined as the ratio of the number of policies in force at the premium-due date to the number of policies in force at the preceding statement date. The consideration of the basic principles is important to fulfill the minimum reserve requirements demanded by the NAIC.

Furthermore, full-preliminary term (FPT) is allowed only in the first two policy years of a long-term health policy. However, the recommended time of full-preliminary term is one year as Black and Skipper (2000), p. 820, point out. Full-preliminary term is used, as in life insurance, to make it possible that the insurance company has more funds available in the first policy years to cover the high acquisition costs of an insurance contract.

The idea of full-preliminary term is to calculate the first year (or the first two years if one uses two-year full-preliminary term) of a policy as a one-year (two-year) term insurance. Therefore the reserve at the end of year one (or in the case of two-year preliminary term at the end of the second policy year) is zero. For the calculation of reserves in later policy years one uses the net-level premium of the same kind of policy for a beginning age one (two) year(s) higher than the beginning policy age. As described before this method lowers the reserve in the first policy year - in the special case of one-year full-preliminary term it will be zero - and it is amortized over the following policy years by calculating with higher net premiums as they were if we had calculated the same policy for someone one year older.

According to Gamage et al. (2005), the basic valuation methods for General Accepted Accounting Principles (GAAP) are based on the realistic expectations of the insurer for morbidity, mortality, persistency and interest rates. The reserves are calculated on a net level premium basis.

Unearned Premium Reserves

Most insurance contracts have payments which are not monthly. Instead they get premium payments only once a year at the policy's anniversary which in most of the cases is not the statement date. Therefore the insurer has to guarantee that the fraction of the premium which pays for the time period after the statement date does not count for the previous statement period, because if the contract is canceled this unearned premium amount is due to the insured. Because of this the insurer has to calculate a reserve paying respect this concept of unearned premiums. The amount of the unearned premium reserve has to be the "pro-rata portion of the full gross-premium from the statement date to the end of the period for which premium have been paid on the policy" (Black and Skipper (2000), p. 821), i.e. it has to be calculated in the same way the gross-premiums are paid. In the majority of cases insurance companies use approximation methods, especially the method of assuming that the premium payment is uniformly distributed throughout the year. This assumption implies that on average one half of the total premiums are unearned at the statement date.

This unearned premium reserve covers a reserve allowance for the unearned gross premium expense, i.e. it is a reserve for payment of expenses incurred later than the statement date and a reserve for benefit payments after the statement date out of premiums already received.

In addition to the unearned premium reserve there are other kinds of reserves which shows already earned premiums which are not due before the statement date. These "premium reserves" (O'Grady (1988)) are:

1. Reserves for advanced premiums, i.e. reserves for premiums which have already been collected, but which pay for a beginning period after the statement date
2. Reserves for uncollected premiums, i.e. reserves for premiums which are already due, but which will not have been collected until the statement date
3. Reserves for deferred premiums, i.e. reserves for the uncollected portion of the net level premium if the insurer uses the mean policy reserving method

Premium Deficiency Reserves

Premium deficiency reserves need to be calculated if the expected claim payments or cost for claims as of the valuation date exceed the calculated and stated premium (Lloyd (2000)). These higher costs for the insurance company were not known at issue of the contract and therefore not included into the premium calculation. Since premiums cannot be changed during the policy year, the

insurance company has to expect a loss. This has to be stated in the annual statement to make aware of the deficiency.

In general a premium deficiency reserve is calculated by combining a number of several similar policies to one group. Since grouping could cause the effect that deficiencies for a subset of the group would be offset by other policies in the group, the grouping is strongly regulated. In the Statement of Statutory Accounting Principles, # 54 (see Bluhm (2003), p. 405), it is stated that "contracts shall be grouped in a manner consistent with how policies are marketed, serviced and measured. A liability shall be recognized for each grouping where a premium deficiency is indicated." This implies that for example vision and dental policies are considered as two groups as well as individual and group insurance contracts build two groups for the calculation of deficiency reserves. The calculation period for a deficiency reserve should be the period from the valuation date until the time the deficiency is no longer existent, because premium adjustment could be conducted because of renewal of the policy.

In general, premium deficiency reserves can be calculated as described by Lloyd (2000) as the sum of the present value of future benefits and the present value of future expenses less the sum of present value of future premiums, the current reserve for long-term health insurance (if existent), the current claim reserve and the current unearned premium reserve.

Outcome-Based Contractual Reserves

Outcome-based contractual reserves (Lloyd (2000)), which occur especially for group insurance contracts, are not reserves established for the protection of insured individuals. They show the liabilities of the insurance company to an employer or health care provider which are defined in the insurance contract. One can distinguish between employer-based contractual liabilities and provider liabilities.

Employer-based contractual liabilities have to be calculated if the insurance contract defines some kind of risk-sharing with the employer who is the policyholder in this case. If a favorable claim experience is made, the insurer will build a liability which is used in future times to reduce future losses or future rate increases because of poor claim experience.

Since Managed Care Organizations become more and more popular in the United States, provider liabilities become increasingly important. These liabilities are set up for further payments which are not directly connected to a special claim to the health care provider after the valuation date. According to Lloyd (2000), this could be a further payment because of a stop-loss arrangement between health care provider and insurance company, a bonus payment or capitations made to the provider at the end of the policy period based on the experience outcome. In addition, the insurance company has to build a liability for the case that the provider becomes insolvent. In most of these cases the insurance company has to provide fee-for-service protection for the insured.

Expense Reserves

The expense reserve is estimated from previous data when the expenses for incurred claims prior to the statement date are significantly high (Gamage et al. (2005)). There are two main calculation methods, either the costs are estimates per claim or the expense reserve is calculated as a percentage of all liabilities for incurred claims prior to the statement date. A third method of estimation of expense reserves would be a combination of the two previous ones. Other expenses are normally included (as described above) in the unpaid premium reserve. However, especially in the first years, it could be that an extra amount for those expenses has to be included in the expense reserve to cover the higher commission costs and other expenses of the first year of a policy.

Claim Reserves

Claim reserves are defined by the NAIC Health Reserve Guidance Manual (see Bluhm (2003), p. 413) as "a measurement of a reporting entity's contractual obligation to pay benefits as of specific date". Therefore, claim reserves are the reserves which should cover the expenses for those claims that are incurred in the previous statement periods, but which are not (fully) paid until the statement date. Different than the policy reserves they are only for a special claim and not for the policy at whole. Like the policy reserves one can distinguish between different

kinds of claim reserves: The reserve for due but unpaid claims, the reserve for "amounts not yet due on claims" and the reserves for claims in settlement.

The reserve for due but unpaid claims (Black and Skipper (2000), p.821) is known very precisely, because the amounts which have to be paid in the future are almost known. They are not yet paid as a result of timing lags in the claim development process, i.e. the claim is already processed, but the final payment has not yet been made. Therefore the insurer has a precise knowledge about the amounts due in the future and does not need special calculation methods. The amount for this kind of reserve is either the average of historical data or shown for each claim if there are only few such claims.

The reserve for "amounts not yet due on claims" (Black and Skipper (2000), p.821) shows the expected value for future payments which are not certain. This occurs especially on disability claims when the insurer is not sure if the payment has to be made or if the insured recovers. As for other claim reserves one can distinguish between reported and unreported claims until the statement date, where the second kind can be a little more difficult to calculate. The estimation of this type of reserves depends on the character of the claim. The age of the claimant, the duration of his disability and the remaining years of benefit payment have to be considered for the amount needed for disability annuities payable in the future, because the chance of recovering depends on these factors. Therefore those reserves need to be calculated for each claim on itself. Instead hospital, surgical

and medical reserves of this type can be considered as a group, because they are not dependent on individual factors as the age, etc. The calculation of these reserves has to be done very accurately.

The last part of the claim reserves are presented by the reserves for claims in settlement (Gamage et al. (2005)). These are the reserves which are still in the development process, because the exact amount of the payments is not known at the valuation date or the insurer has not even knowledge about the incurred claim. The latter reserve is called "incurred but not reported"-reserve or short the IBNR-reserve. For both cases one can not give a very precise amount for future payments as for the two examples before. Because of the uncertainty of future payments for already incurred claims, the claim reserves need to be estimated accurately by former data. This estimation is done almost always with reserving methods which are also used in the property/casualty business, e.g. the chain-ladder-method. These methods are described in more detail in chapter 4.

However, these methods - according to Gamage et al. (2005) - are not very appropriate for health insurers, because compared to property/casualty insurers there are more types of claims. That depends on the greater number of medical methodologies and the big variety of diseases, but even more important is the restricted access to health insurance data. Nevertheless, it is used for the reserve calculation today and because of the lack of data these reserves are calculated not for every claim on its own, but as an average amount per month and per member.

CHAPTER IV

CALCULATION OF CLAIM RESERVES

Claim reserves form a main part of the reserves in the annual statement of a health insurance company. Compared to other types of reserves like policy reserves or expense reserves, their calculation is especially for IBNR-reserves not very easy because of the lack of data (Gamage et al. (2005)). Nevertheless, since claim reserves are "customers' money" - as most of the reserves - their calculation has to be done in an accurate way. In this chapter, the main methods used by actuaries nowadays for the calculation of claim reserves of health insurance companies will be discussed, especially the methods for calculation of IBNR-reserves. Although these methods are approved by the regulators, they have some disadvantages. This disadvantages will be pointed out in this chapter, too.

Terminology

For the calculation of claim reserves, but also for the process of ratemaking, the insurance company needs knowledge of the expected ultimate losses a claim might cause. Therefore the insurance company needs to know how a claim will develop over time. This process is called the development process. To forecast the development of not yet fully paid claims (see Gamage et al. (2005)), the insurance

company calculates estimates from historical data. To describe the process the ratio of the cumulative paid amounts in two consecutive months can be calculated. These are the so-called development factors. Another possibility is the calculation of completion factors. These factors show the percentage of claims paid today compared to the ultimate month. The calculation of these factors will be shown in more detail later in this chapter.

In the development process of a claim, three main points of time need to be considered. The first important point of time is the date when a claim occurs, the so-called incurred date (Lloyd (2000)). This is the date when the first obligation by a claim is established, e.g. the visit of a health care provider or the first time when the insured receives a treatment in a row of successive treatments related to one claim. Since in health insurance most claims are specified by the month in which they occur the incurred date is often referred as incurred month.

Secondly, the month in which the insurance company gains knowledge about the claim, i.e. the claim is reported to the insurance company, is the so-called reported month. The reported month cannot be before the incurred month.

The third point in time is the paid month, i.e. the month in which the insurance company makes a payment for the claim. As the reported month, the paid month has to be at anytime after the incurred month, because a claim which has not yet been occurred cannot be paid already.

The time between incurred month and paid month which describes the

duration until a payment is made is called the lag period or short lag. For calculation purposes it is assumed that the reported month is equal to the paid month.

To get an overview about the development process, the claim data from previous periods have to be summarized. This is done in the following way which presents the relationship between the incurred month and the development of the claim which is given by the paid month. The claim data will be collected in a table. According to Gamage et al. (2005), each row of the table describes the development of a claim given in an incurred month. The columns of the table display the paid month (see Example 1 - Table 1). Each cell in this table represents the amount of claims paid in a given paid month depending on the incurred month. Since a claim cannot be paid before it occurs, only those cells for which the paid month is not before the incurred month can have non-zero entries. Therefore the table has a triangle structure and the whole data describing process is called a triangle report.

Example 1

Suppose the health insurance company XYZ wants to calculate the claim reserves as of valuation date December, 31st of 2005. Since August 2005 the company XYZ has made the following payments for claims incurred from August 2005 to December 2005. These data were reported in a table (see Table 1: Incurred vs. Paid Month) in which each row represents an incurred month and each column

represents a paid month.

Table 1
Incurred vs. Paid Month

	Aug	Sep	Oct	Nov	Dec
Aug	2,000	1,500	1,000	500	250
Sep		4,000	3,000	2,000	1,000
Oct			1,000	750	500
Nov				3,000	2,250
Dec					5,000

Since the company XYZ is interested in the development structure of payments made to incurred claims, the table is rewritten to show the development (see Table 2: Incurred Month vs Duration). Therefore each row represents an incurred month and each column represents the duration since the first occurrence.

Table 2
Incurred Month vs. Duration

	0	1	2	3	4
Aug	2,000	1,500	1,000	500	250
Sep	4,000	3,000	2,000	1,000	
Oct	1,000	750	500		
Nov	3,000	2,250			
Dec	5,000				

The triangle structure which is inherent to the development process can be seen in both tables. Furthermore the company XYZ assumes that the claims are

fully developed after a period of 5 month, i.e. after a period of five months no more payments have to be made by the company. Therefore the claims occurred in August 2005 are fully paid at the end of December 2005. This property becomes important in the process of calculating reserves (see later in this chapter).

The last assumption made in Example 1 is quite unrealistic. Normally, the claim development of medical claims takes about twelve month. After this period, 98 percent of all claims are paid as Gamage et al. (2005) stated. However, only five months of data will be used in this example to illustrate the claim reserving process to keep the example short and easily understandable.

In general, the development process consists of n incurred and n paid months, i.e. we assume that the valuation date is at the end of incurred month n . However, it is possible to have a different number of incurred and paid months but it is more convenient to work with a quadratic structure. Therefore this thesis will confine itself to only using the number of paid months equal to the number of incurred months. It is assumed that at least the claims of one incurred month are fully developed. Normally, this is the earliest incurred month in the row of incurred months, the incurred month with number 0. As stated above, most medical claims are developed after a period of twelve months. Therefore it can also be assumed that all incurred months with $n - 1 - i > 12$ ($i = 0, \dots, n - 1$) for which i is the number of the incurred month, are fully developed. Nevertheless, this assumption

has to be checked for every new set of data by the actuary. If the observed data show another run-off pattern, the actuary has to use another date until which all claims are fully developed.

Define now for $i, j = 0, \dots, n - 1$

y_{ij} = incremental payment for incurred month i and development duration j ,
 i.e. the claim is paid after a duration of j months from occurrence

Since it is assumed that the valuation date is at the end of incurred month n , the incremental claim amounts for an incurred month $i = 0, \dots, n - 1$ and a lag period $j = 0, \dots, n - 1$ for which $i + j > n - 1$ are not known. These incremental claim amounts have to be estimated, either by the methods described in this chapter or by the methods described in chapter 5. Then the development triangle in which only known incremental claim amounts are implemented is given by Table 3.

Table 3
Development Triangle

	0	1	...	$n - 1$
0	y_{00}	y_{01}	...	$y_{0,n-1}$
1	y_{10}	y_{11}	...	
\vdots	\vdots	...		
$n - 1$	$y_{n-1,0}$			

Since reserves should show the whole amount the insurance company expects to pay, the cumulative amounts paid, i.e. the sum of all amounts already paid until

a given date, are of greater interest than the incremental amounts paid. The methods used which rely mainly on cumulative amounts paid are the so-called chain-ladder method, and the completion factor method which will be described in section "Development Methods".

The cumulative amounts of incurred month i which are paid after a duration of j months are defined by

$$C_{ij} = \sum_{k=0}^j y_{ik} \quad \forall \quad i = 0, \dots, n-1, \quad j = 0, \dots, n-1-i$$

The cumulative development triangle can be written as shown in Table 4.

Table 4
Cumulative Development Triangle

	0	1	...	$n-1$
0	C_{00}	C_{01}	...	$C_{0,n-1}$
1	C_{10}	C_{11}	...	
\vdots	\vdots	...		
$n-1$	$C_{n-1,0}$			

The concepts introduced in this section are also of great interest for the calculation of claim reserves using statistical methods which will be described in chapter 5 of this thesis.

Example 1 (contd.)

With the data of Example 1 the following cumulative development triangle is given:

Table 5
Cumulative Development Triangle for Ex. 1

	0	1	2	3	4
Aug (0)	2,000	3,500	4,500	5,000	5,250
Sep (1)	4,000	7,000	9,000	10,000	
Oct (2)	1,000	1,750	2,250		
Nov (3)	3,000	5,250			
Dec (4)	5,000				

Development Methods

Development methods for claim reserving use a historical development process for the projection of the future behavior of a claim. They are important for the calculation of the estimated ultimate losses and therefore useful in the ratemaking process, too. Here, the assumption that the development patterns of previous periods are the same for the future (Brown (2001)) underlies the calculation of the estimated ultimate losses. For the calculation, development triangles as described in section "Terminology" are used.

The two most popular methods in property/casualty and health insurance are the chain-ladder-method and the completion factor method (or the "age-to-ultimate development factor method").

Chain-Ladder Method

The chain-ladder method uses age-to-age development factors for the calculation of the estimated ultimate losses. As stated in section "Terminology", it is assumed that all claims are fully developed after n time periods, i.e. at least the incurred month 0 is fully developed at valuation date and all development factors for duration longer than n are equal to one (Brown (2001)). This assumption is important since otherwise an estimation of the development of later duration would have to be done without any data. As stated above, the normal development of health claims is completed after 12 months and therefore a calculation can usually be done if one has at least the data of 12 consecutive incurred months.

Formally, the chain-ladder method can be described in the following way (see Dahl (2003)): The goal of the calculation is to estimate the expected ultimate losses

$$\mathbb{E}(C_{i,n-1}|C_{i,n-1-i}; \dots; C_{i0}), \quad i = 1, \dots, n-1.$$

Note that the values for $C_{i,n-1-i}; \dots; C_{i0}$ are known.

For the chain-ladder method we assume that the expected amount of incurred losses of one period is proportional to the amount of incurred losses in the previous period (and the same incurred month). The proportional factor f_j ($j = 1, \dots, n-1$) which is independent of the incurred month is given as the following conditional expectation

$$f_{j+1} := \mathbb{E}(C_{i,j+1}/C_{i,j}|C_{i,j}; \dots; C_{i0}), \quad j = 0, \dots, n-2.$$

This implies for $i = 1, \dots, n - 1$, $j = 0, \dots, n - 2$, that

$$\mathbb{E}(C_{i,j+1}|C_{i,j}; \dots; C_{i0}) = C_{i,j} \cdot f_{j+1} \quad (4.1)$$

and therefore the expected ultimate losses can be calculated as

$$\mathbb{E}(C_{i,n-1}|C_{i,n-1-i}; \dots; C_{i0}) = C_{i,n-1-i} f_{n-i} \cdot \dots \cdot f_{n-1}, \quad (4.2)$$

because the equation (4.1) can be transformed for every incurred month i using the properties of conditional expectations in the following way:

$$\begin{aligned} \mathbb{E}(C_{i,n-1}|C_{i,n-1-i}; \dots; C_{i0}) &= \mathbb{E}(\mathbb{E}(C_{i,n-1}|C_{i,n-2}; \dots; C_{i0})|C_{i,n-1-i}; \dots; C_{i0}) \\ &= \mathbb{E}(C_{i,n-2} f_{n-1} | C_{i,n-1-i}; \dots; C_{i0}) \\ &= \mathbb{E}(C_{i,n-2} | C_{i,n-1-i}; \dots; C_{i0}) f_{n-1} \\ &= \mathbb{E}(\mathbb{E}(C_{i,n-2} | C_{i,n-3}; \dots; C_{i0}) | C_{i,n-1-i}; \dots; C_{i0}) f_{n-1} \\ &= \mathbb{E}(C_{i,n-3} | C_{i,n-1-i}; \dots; C_{i0}) f_{n-2} f_{n-1} \\ &= \dots \\ &= C_{i,n-1-i} f_{n-i} \cdot \dots \cdot f_{n-1}. \end{aligned}$$

Practically, Brown (2001) suggests that the calculation of the chain-ladder estimates for claim reserves can be completed in several consecutive steps which will be described in the following.

The first step is to estimate age-to-age development factors from the given data. These are defined as the ratio of the amount paid up-to-date and the amount

paid up-to-date in the previous period. In mathematical terms and the notation stated in the section "Terminology", the age-to-age development factors λ_{ij} ($i = 0, \dots, n - 1, j = 1, \dots, n - 1 - i$) which describe the ratio of paid claims in lag period j and in lag period $j - 1$ are calculated as follows:

$$\lambda_{ij} = \frac{\text{cumulative amount in } j}{\text{cumulative amount in } j - 1} = \frac{C_{ij}}{C_{i,j-1}}$$

These development factors show the evolution of claim payments from one period to the following.

The second step in the computation of reserves is the calculation of a single development factor f_j for each lag $j = 1, \dots, n - 1$, because the estimation of future development should be based on an "average" development which can be extracted from data of the past. According to Lloyd (2000), special smoothing methods are used for this.

One possibility is the calculation of an arithmetic average of the development factors for different incurred months of the same lag-period. Assume the number of incurred months for which a development factor of lag j is calculated is equal to m with $0 < m \leq n - 1$. Then

$$f_j = \frac{1}{m} \sum_{i=0}^{m-1} \lambda_{ij}, \quad j = 1, \dots, n - 1.$$

Another way of calculating a single development factor is the use of m -month averages with $0 < m \leq n - 1$ (Brown (2001)). This has the advantage that more recent months receive a higher weight than earlier months in the development

process. Therefore the development in recent months has a greater effect on future development as it is assumed that they are more important for the near future.

A third method is the calculation of an average without the highest and lowest development factor in the lag period. Suppose we have again m , $2 < m \leq n - 1$ incurred months for lag j . Then the single development factor is given by:

$$f_j = \frac{1}{m - 2} \left(\sum_{i=0}^{m-1} \lambda_{ij} - \max_{i=0, \dots, m-1} \{\lambda_{ij}\} - \min_{i=0, \dots, m-1} \{\lambda_{ij}\} \right), \quad j = 1, \dots, n - 1.$$

This is done to adjust the calculation for catastrophic claims. However, the actuary has to control the structure of claims carefully before using this method. If there was a catastrophic claim within the claims this methods could be appropriate, but if larger (or lower) factors described a new development structure in recent months, this behavior should be included in the calculation of estimated ultimate losses and therefore in the average single development factor.

Another approach to build a single development factor is the usage of weighted averages. Here, the development factors are adjusted with weight factors before the average is calculated. According to Lloyd (2000), this might be done to give the development in recent periods a higher weight in the estimation process than factors from the past.

A last approach is the mean method or the dollar-weighted average method. Here, the estimated development is not calculated as an average of single development factors. It is directly calculated from the cumulative development

triangle. The single development factor is the result of the following calculation:

$$f_j = \frac{\sum_{i=0}^{n-j} C_{ij}}{\sum_{i=0}^{n-j} C_{i,j-1}}, \quad j = 1, \dots, n-1$$

All these methods are appropriate for the calculation of single development factors. Although each of these methods is approved by the regulators, the valuation actuary has to make the decision which method fits the given data for the calculation best and which method gives an appropriate estimate for the claim reserves considered.

Thirdly, the estimated ultimate losses have to be calculated for each incurred month from the single development factors which have been calculated in the second step (Brown (2001)). This is:

$$\text{estimated ultimate losses } (i) = \text{losses paid up-to-date} \cdot \prod_j f_j,$$

where f_j are the single development factors from lag $j-1$ to lag j and $i = 0, \dots, n-1$ describes the incurred months.

If we assume that the development triangle is given as in the section "Terminology", i.e. only the first incurred month (with number 0) is fully developed, we can write the equation above as

$$\text{estimated ultimate losses } (i) = C_{i,n-1-i} \cdot \prod_{j=n-i}^{n-1}, \quad i = 0, \dots, n-1$$

This formula is equivalent to equation (4.2).

The fourth and last step for the calculation of claim reserves with the chain-ladder method is the calculation of the total loss reserve from the estimated ultimate losses. Therefore, at first the loss reserves for each incurred month $i = 0, \dots, n - 1$ are calculated (Gamage et al. (2005))

$$\begin{aligned} \text{loss reserve } (i) &= \text{estimated ultimate loss } (i) - \text{losses paid up-to-date} \\ &= \text{losses paid up-to-date} \cdot \left(\prod_j f_j - 1 \right) \\ &= \text{losses paid up-to-date} \cdot (f_{ULT} - 1) \end{aligned}$$

with $f_{ULT} = \prod_j f_j$.

The total loss reserve as for the valuation date is the sum of the loss reserves for each incurred month, i.e.

$$\text{total loss reserve} = \sum_{i=0}^{n-1} \text{loss reserve } (i).$$

Example 1 (contd.)

In the following the loss reserve will be calculated with help of the chain-ladder method for the data given in Example 1. First of all, the age-to-age development factors for each period have to be calculated. They are given by Table 6.

The next step is the calculation of single development factors. Since the data in this example are chosen to have the same development structure independent of the incurred year, the method that will show this behavior best is the average method. Therefore, the single development factors are chosen as follows:

$$f_1 = 1.75, \quad f_2 = 1.2857, \quad f_3 = 1.1111, \quad f_4 = 1.05.$$

Table 6
Development Factors Ex. 1

	1/0	2/1	3/2	4/3
Aug (0)	1.75	1.2857	1.1111	1.05
Sep (1)	1.75	1.2857	1.1111	
Oct (2)	1.75	1.2857		
Nov (3)	1.75			
Dec (4)				

These single development factors yield to the following estimated ultimate losses for the given incurred months.

$$\begin{aligned}
 \text{estimated ultimate losses (Aug)} &= 5,250 \\
 \text{estimated ultimate losses (Sep)} &= 10,000 \cdot f_4 = 10,500.00 \\
 \text{estimated ultimate losses (Oct)} &= 2,250 \cdot f_3 \cdot f_4 = 2,624.97 \\
 \text{estimated ultimate losses (Nov)} &= 5,250 \cdot f_2 \cdot f_3 \cdot f_4 = 7,874.83 \\
 \text{estimated ultimate losses (Dec)} &= 5,000 \cdot f_1 \cdot f_2 \cdot f_3 \cdot f_4 = 13,124.72
 \end{aligned}$$

Therefore, the following loss reserves are estimated from the data.

$$\begin{aligned}
 \text{estimated loss reserve (Aug)} &= 5,250 - 5,250 = 0.00 \\
 \text{estimated loss reserve (Sep)} &= 10,500 - 10,000 = 500.00 \\
 \text{estimated loss reserve (Oct)} &= 2,624.97 - 2,250 = 374.97 \\
 \text{estimated loss reserve (Nov)} &= 7,874.83 - 5,250 = 2,624.83 \\
 \text{estimated loss reserve (Dec)} &= 13,124.72 - 5,000 = 8,124.72
 \end{aligned}$$

Thus, the insurance company XYZ has a total claim reserve of \$11,624.45 as of the valuation date.

Completion Factor Method

The completion factor method is the most common method for the calculation of claim reserves in health insurance (see Gamage et al. (2005)). Instead of using the relationship of two successive development periods, the percentage of completion of the ultimate amount is calculated. Thus, the so-called completion factor or age-to-ultimate development factor c_{ij} is given as

$$c_{ij} = \frac{\text{cumulative amount in } j}{\text{ultimate cumulative amount}} = \frac{C_{ij}}{C_{i,n-1}}, \quad i, j = 0, \dots, n-1.$$

However, since the ultimate cumulative amount has to be known, the calculation of completion factors is only possible for fully developed months. Normally only a few months are fully developed in the development process. Especially the most recent months are not included in the calculation since only few development data are known. Therefore, this method of calculation of completion factors is not appropriate for the process of estimating loss reserves. For this reason a modified version of completion factor estimation has to be used for the reserving process. Here, the completion factors c_j ($j = 0, \dots, n-1$) are calculated recursively from former data. It is assumed that the incurred month 0 is fully developed, i.e. payments until lag $n-1$ are known. Thus, $c_{n-1}=1$. Then the completion factor for lag $n-2$ is given by

$$c_{n-2} = \frac{\sum_{i=0}^1 C_{i,j-1}}{\sum_{i=0}^1 C_{ij}} c_{n-1}.$$

Further completion factors are calculated in the same way

$$c_j = \frac{\sum_{i=0}^{n-1-j} C_{i,j-1}}{\sum_{i=0}^{n-1-j} C_{ij}} c_{j+1}, \quad j = 0, \dots, n-3.$$

This method provides the input of knowledge of recent data into the calculation of completion factors.

Similar to the chain-ladder method the next step is the calculation of estimated ultimate losses of each incurred month i ($i = 0, \dots, n-1$).

$$\begin{aligned} \text{estimated ultimate losses } (i) &= \text{losses paid up-to-date/completion factor} \\ &= \frac{C_{i;j^*}}{c_{j^*}} \end{aligned}$$

with j^* is equal to the lag of paid claims up-to-date.

The loss reserves for each incurred month $i = 0, \dots, n-1$ are given as the difference of the estimated ultimate losses for the given incurred month and the losses paid up-to-date as it is done in the chain-ladder method. The total reserve is the sum of the single reserves for each incurred month.

Again, the actuary has to make sure that the completion factor method is appropriate for the given set of data. He has to be aware of changes which could influence the development process like seasonal cycles, changes in the claim paying process, inflation, etc.

Example 1 (contd.)

We will show the calculation of loss reserves by using completion factors in this example. For the calculation the modified method is used, because in this example only the claims of August 2005 are fully developed. Since we are assuming a full development after a period of five months, we have $c_4 = 1$. The recursive calculation of the completion factors yield:

$$\begin{aligned}
 c_4 &= 1 \\
 \Rightarrow c_3 &= \frac{5,000}{5,250} \cdot c_4 &= 0.9524 \\
 \Rightarrow c_2 &= \frac{4,500+9,000}{5,000+10,000} \cdot c_3 &= 0.8572 \\
 \Rightarrow c_1 &= \frac{3,500+7,000+1,750}{4,500+9,000+2,250} \cdot c_2 &= 0.6667 \\
 \Rightarrow c_0 &= \frac{2,000+4,000+1,000+3,000}{3,500+7,000+1,750+5,250} \cdot c_1 &= 0.3810
 \end{aligned}$$

These completion factors yield to the following estimated ultimate losses for the given incurred months.

$$\begin{aligned}
 \text{estimated ultimate losses (Aug)} &= 5,250 / c_4 = 5,250.00 \\
 \text{estimated ultimate losses (Sep)} &= 10,000 / c_3 = 10,499.79 \\
 \text{estimated ultimate losses (Oct)} &= 2,250 / c_2 = 2,624.83 \\
 \text{estimated ultimate losses (Nov)} &= 5,250 / c_1 = 7,874.61 \\
 \text{estimated ultimate losses (Dec)} &= 5,000 / c_0 = 13,123.36
 \end{aligned}$$

Therefore, the following loss reserves are estimated from the data.

$$\begin{aligned}
 \text{estimated loss reserve (Aug)} &= 5,250 - 5,250 = 0.00 \\
 \text{estimated loss reserve (Sep)} &= 10,499.79 - 10,000 = 499.79 \\
 \text{estimated loss reserve (Oct)} &= 2,624.83 - 2,250 = 374.83 \\
 \text{estimated loss reserve (Nov)} &= 7,874.61 - 5,250 = 2,624.61 \\
 \text{estimated loss reserve (Dec)} &= 13,123.36 - 5,000 = 8,123.36
 \end{aligned}$$

Thus, the insurance company XYZ has a total claim reserve of \$11,622.59 as of the valuation date.

Criticism of Development Factor Methods

As stated earlier in this section, development factor methods like the chain-ladder method and the completion factor method rely on the implicit assumption that the past development can be used to project the future behavior of claims. Only this assumption allows us to use this method but this assumption implies implicitly that there is no variability in the claim rate of reporting and processing. The only source of variability, as Lynch (2004) stated, is the morbidity rate which can influence the run-out patterns of claims. This is one criticism of development factor methods but since one could try to project some future behavior with help of trends into the expected ultimate losses it is not as severe as the next two disadvantages.

First, Brown (2001) stated that the development methods are not stable because of the huge number of parameters which have to be estimated. Assume, as before, that n incurred months are observed, whereas the month 0 is the only fully developed month. Therefore the loss development matrix has $n - 1$ not fully completed columns. Thus, $n - 1$ single development factors have to be estimated. For the calculation of the n estimated ultimate losses for each incurred month another n calculations have to be done. Therefore the final model consists of

$n + (n - 1) = 2n - 1$ parameters. This number could be very big, since development methods needs a huge amount of data to make reasonable estimations. Lloyd (2000) recommends at least 24 months for medical claims; but he states that 48 to 60 months would be even better. Thus, the number of parameters which define the stability of a model is very big and the model can become very unstable.

The second point of criticism which was stated by Lynch (2004) is even more severe. The completion factor method and also the chain-ladder method are highly variable because of the model inherent "method variance". Since both methods are based on the summation of incurred and paid claims multiplied or divided by a factor and the variance of the sum of random variables is not linear, the variability of the result is not only dependent on the standard deviation of the data, the so-called "process variance" which is inherent to every stochastic model, it also depends on the value of the factors.

For the completion factor method this can be described as follows:

Assume that the amount paid up-to-date for each incurred month i ($i = 0, \dots, n - 1$) is given by a random variable Y_i ($i = 0, \dots, n - 1$). Then the estimated ultimate losses for all incurred month are given by

$$T := \sum_{i=0}^{n-1} \frac{Y_i}{c_i}$$

with completion factors c_i for lag $n - 1 - i$ ($i = 0, \dots, n - 1$). Thus, the estimated ultimate loss T is a random variable, too, and its variance can be calculated. Using

the standard variance calculation for the sum of random variables and under the assumption that the random variables Y_i ($i = 0, \dots, n - 1$) are uncorrelated, we get

$$Var(T) = \sum_{i=0}^{n-1} \frac{Var(Y_i)}{c_i^2}.$$

This shows that the variance of T and therefore the standard error of the modeling process depends not only on the standard deviation of Y_i ($i = 0, \dots, n - 1$) which is the "process variance", but also it is highly dependent on the value of the calculated completion factors since they are considered in the calculation as a squared variable.

According to Lynch (2004), this will become a problem in the calculation of the most recent months for the completion factor method, i.e. incurred months $n - 2$ and $n - 1$, since the completion factors in these months are most of the time very small due to the lack of data. Normally, it is assumed that completion factors less than 40 to 70 percent (Lloyd (2000), p. 29) produce a variance too big to be accepted. Therefore the squared reciprocal of these factors which goes into the calculation of the variance of T becomes very big. For later duration we observe completion factors close to 1 since more and more payments are made. Therefore the "method variance" produced by earlier incurred periods is not as significant as the "method variance" of more recent months. For that reason, alternative methods for the calculation of the most recent months should be considered or, even better, the calculation of estimated ultimate losses should be based on other

methods which completely rely on statistical principles. The approach of using other methods will be discussed in chapter 5.

Expected Loss Ratio Method

As we have seen in the section "Development Methods", the calculation of claim reserves can be done based on the historical development of claims. However, to get precise estimates of reserves an appropriate number of data is needed. But what happens if there is a new line of business and no or not many historical data to analyze? A solution for this question according to Brown (2001) can be given by the expected loss ratio method.

For the calculation of the claim reserves via the expected loss ratio method three steps are needed. First an expected loss ratio is calculated. For example, this could be the ratio of expected dollar losses and the dollar amount of earned premium at current rate, i.e.

$$\text{expected loss ratio} = \frac{\text{expected amount of losses in \$}}{\text{\$ amount of earned premium at current rate}}.$$

This ratio is the same as the expected loss ratio used in the ratemaking process for the premium calculation via the loss ratio method.

Then, the expected ultimate losses are given by:

$$\text{estimated ultimate losses} = \text{expected loss ratio} \cdot \text{earned premium}.$$

Similar to the calculation of loss reserves in the two methods which are based on

the development process, the expected loss reserve is calculated as the difference of the estimated ultimate losses and losses paid up-to-date. Thus,

$$\text{expected loss reserve} = \text{estimated ultimate losses} - \text{losses paid up-to-date.}$$

Although the expected loss ratio method does not need as many data as the development methods, it is not an appropriate method for the calculation of the reserves. Since the expected loss ratio can easily be manipulated by the management of a company without the knowledge of the actuaries, the process can lead to reserves not appropriate for the insurance company. Therefore the actuary should be aware of reserves which do not look satisfactorily. Nevertheless, this method is recommendable for new businesses or businesses with few data. Lloyd (2000) states that this is a good tool of testing the reserves calculated with either the chain-ladder method or the completion factor method.

Bornhuetter-Ferguson Method

A third method commonly used for the calculation of claim reserves was introduced in 1972 by Bornhuetter and Ferguson (1972). This method is a combination of both, the expected loss ratio method and the chain-ladder method.

By the chain-ladder method the expected loss reserve can be written as

$$\text{expected loss reserve} = \text{losses paid up-to-date} \cdot (f_{ULT} - 1)$$

with $f_{ULT} = \prod_j f_j$.

Furthermore we get as estimated ultimate losses by using the chain-ladder method

$$\text{estimated ultimate losses} = \text{losses paid up-to-date} \cdot f_{ULT}.$$

Therefore:

$$\text{expected loss reserve} = \text{estimated ultimate losses} \cdot \left(1 - \frac{1}{f_{ULT}}\right).$$

The last formula is the Bornhuetter-Ferguson formula for calculation of loss reserves. The estimated ultimate losses are not calculated by the development factors. They are estimated with help of the expected loss ratio method, i.e. as the product of expected loss ratio and earned premium. The factors for the calculation of f_{ULT} are extracted from the development process as described in the section "Chain-ladder method". Summarized, the Bornhuetter-Ferguson formula can be written as

$$\text{expected loss reserve} = \text{expected loss ratio} \cdot \text{earned premium} \cdot \left(1 - \frac{1}{f_{ULT}}\right).$$

Brown (2001) describes that the Bornhuetter-Ferguson method has an advantage over both, the chain-ladder method and the expected loss ratio method. It is better than the expected loss ratio method since it does not totally rely on the subjective determination of the expected loss ratio. Only for an early period this loss ratio has to be given. Since this is known and not as easily manipulatable as an every year loss ratio, the Bornhuetter-Ferguson method is more objective. The

advantage over the chain-ladder method is that it does not solely rely on the assumption that the future development is the same as the former development.

The Bornhuetter-Ferguson method is also stable and it allows the input of outside data like the influence of the expected loss ratio.

The main disadvantage of the Bornhuetter-Ferguson method is that an outside data source which provides the information of the expected loss ratio and therefore the estimated ultimate losses is needed for the calculation of an initial expected loss ratio.

Further Methods of Claim Reserving

In this section further claim reserving methods will be described. These methods provide approaches of building IBNR-reserves if the number of observed data is not sufficient for the use of development factor methods as described in section "Development Methods". Furthermore, two methods for the calculation of case reserves, i.e. reserves for already reported claims which are not yet due, will be described and finally a method for estimation of claim reserves for long-term care and long-term disability insurance will be given.

Case Reserve Evaluation

For the case reserve evaluation two methods are most commonly used. These methods are the "examiner's method" (Lloyd (2000), p. 17) or individual claim

estimation method and the average size claim method. Both methods have in common that they are only used for the estimation of remaining payments for already reported claims. Since the data for these claims are known, the estimation is not as difficult as for IBNR-reserves for which both the number of outstanding claims and the amount of payments incurred by those have to be estimated.

The examiner's method is based on the subjective estimation of remaining claims for already reported claims. The estimation can be done by a claim examiner or by qualified personnel of the claim department of the company. The examiner estimates the expected ultimate losses of each reported claim based on specific information about the kind of claim and on historical experience with similar claims. Since each reported claim is estimated by itself, this valuation method is only appropriate for small numbers of reported claims (Lloyd (2000)). However, it can also be used for claims which need a special treatment due to special inherent characteristics to the claim (e.g. large catastrophic claims which would be stated an outlier in the development process, short-term disability claims or claims which are pending because of a lawsuit). Individual evaluation for the reserve of each claim can cause better estimates of the reserve than the calculation by development methods including these cases.

The average size claim method depends on - as the name already indicates - assumptions based on the average claim amount observed in former periods for a class of reported claims. Here, the claim reserve is calculated as the product of an

average estimated claim amount per claim (from historical data) and the number of reported claims in that class lessened by the claims paid up-to-date.

According to Lloyd (2000), the main disadvantage of this model is that the reserve calculation is only possible for already reported claims. An appropriate estimation for the incurred but not reported claims has to be added for the estimate of the total reserve.

Further Methods for the Calculation of IBNR-Reserves

After introducing two methods for case reserves two more methods which can be used for the calculation of IBNR-reserves will be described now, the projection method and the formula method. Both methods are not used as frequently as the development factor method, the expected loss ratio method or the Bornhuetter-Ferguson method, but both methods can become a very important tool if the actuary wants to check his reserve estimates calculated with these methods. Additionally, they can be used if the amount of data is very small and the development methods are not usable.

The projection method (see Lloyd (2000)) bases the estimation of reserves on the estimation of the amount of incurred claims by the development of claim rates from historical data. Therefore the claim rate is given as a function of membership or another measure of exposure, i.e. one compares the amount of claims to the number of exposure units. The most common projection method is the calculation

of the total amount of incurred claims with a per month per member method. Thus, the claim rate is determined as the historical average of "per month per member" claim costs. Then, the estimated claim rate is multiplied with the number of members in the month for which the reserve should be calculated. The reserve is the difference of estimated incurred claims and claims paid up-to-date.

O'Grady (1988) introduces the formula method which is a method similar to the average size claim method which was described in section "Case Reserve Evaluation", but it is not calculated based on the number of reported claims so that it is usable for the calculation of IBNR-reserves, too. Instead of using the number of reported claims, either the number of policies in force, past claim counts or pending claim counts, trended and adjusted for the specific period, are used for the calculation of the estimated loss reserves.

Tabular Method

Tabular methods for calculation of claim reserves are often used for long-term care and long-term disability claims. These claims differ from the claims of other medical insurance contracts because the benefits are paid periodically after the disability has occurred, the elimination period is over and the amount of benefit payments is almost surely known from the insurance contract. In addition, several other factors which determine the final payment, and therefore the reserve, are determined by the insurance contract as elimination period, duration of benefit

payments, etc. The only unknown factor, besides the fact that there could be incurred but not yet reported claims, is the continuance of the disability or the long-term care need. The probability of recovering from a disability or dying is given by continuance tables which can be used for the evaluation of the claim reserves. Therefore, the expected loss reserve is calculated as the present value of amounts not yet due.

Bluhm (2003) distinguishes three types of claims: open claims for which benefits are already paid and the only element of uncertainty is the duration of payment, pending claims which are already reported but a payment has not been made because of the elimination period, or a lacking approval and the incurred but not reported (short: IBNR) claims.

The calculation of reserves for open claims is straightforward by the definition of the reserves above:

$$\begin{aligned} V_n &:= \text{reserve at duration } n \\ &= \sum_{t=n}^{BP} \text{benefit}_t \cdot \text{continuance}_t \cdot \text{interest discount}_t \end{aligned}$$

with $\text{benefit}_t = \text{benefit at period } t$,

$\text{continuance}_t = \text{probability of continuance as of time } t \text{ and benefit period } BP$.

For pending claims the estimation of claim reserves is done similarly to open claims but the calculated amount is adjusted by a pending factor which describes the likelihood of a payment made for the specific claim. We get for a pending

claims in the elimination period

expected loss reserve =

pending factor · tabular claim reserve at end of elimination period

For pending claims which are not yet approved but which pend longer than the elimination period, the reserve is the product of pending factor and the sum of the tabular reserve at current time and the accumulated value of expected payments in the past since the elimination period.

The calculation of IBNR reserves for long-term care and long-term disability insurance is done with the help of development methods or loss ratio methods as described in the previous sections in this chapter. For the calculation of long-term disability reserves for a lag-period of more than two years, the valuation of the reserves is subject to minimum standard assumptions, i.e. the continuance table fulfills the minimum standards of the NAIC. The first two year reserving can be based on the experience of the insurance company.

Role of the Actuary in the Reserve Valuation Process

The calculation of claim reserves cannot be done only by using the above described methods. It needs the understanding of the whole data environment and the understanding whether the methods used result in appropriate reserves. This is the point where the valuation actuary comes into play. The actuary has to use

her/his knowledge and even a little bit of intuition to choose the right model for the given set of data. Also, the actuary needs to check if the given data are consistent or if some outside influences disturb the data in a way that a proper and stable reserve calculation is not possible.

Before the reserves are calculated it has to be analyzed if some of the following factors have influence on the method used for the calculation for reserves (see Bluhm (2003)). The first thing the actuary should do is to determine whether some internal or external factors influence the data. For example, internal company practices can make the claim development process shorter, such that the lag until the payment is made is not as long as considered. This would be an important factor to know if the completion factor method is used. External influence factors are new laws which influence the claim development process, the number of claims reported or the appearance of an epidemic which will lead to a huge additional number of claims in a short time compared to previous periods.

Furthermore, the type of policy for which the reserves are calculated is important. For disability income insurance the payment lag can be of very long time depending on the type of disability occurred. The actuary has to consider this in the estimation of the claim reserve. However, a long lag period can also be caused by the type of insurance plan and the date of issue. The actuary should be aware of the source of "disturbance".

Although a lot of external factors can influence the data, the actuary has to

make sure that there is no factor which influences the appropriate estimation of the claim reserves inherent in the data, like trends within the data or a seasonal effect. Especially, medical policies with a high out-of-pocket limit tend to be seasonal, since during the first month of a year no claims are due because of the influence of the deductibles but once the out-of-pocket limit is reached claims will be reported to the insurance company. Another seasonal effect is represented by the less frequent use of medical services during holidays, especially over the Christmas holidays in December. Therefore the actuary has to consider other possibilities to get rid of the seasonal factor. For example, (s)he could think of arranging the incurred and paid claimants by other lag periods than months or the claims should be adjusted with weight factors in less frequently used months.

As one could see, the main part of the calculation has to be done before and after the calculation process, when the actuary has to decide the appropriate model and check if the model results in adequate reserves.

CHAPTER V

APPROACHES OF CALCULATING CLAIM RESERVES

The models described in chapter 4 are the most common methods for the evaluation of health claim reserves in the United States, although they are based on distribution-free calculation methods. Therefore no diagnostic tests for the calculated reserves exist. The only possible way to check whether the calculated reserves are appropriate is the knowledge of the valuation actuary. Based on historical experience and the specific data of the claims, (s)he has to decide whether the estimated claim reserve is appropriate or not. However, since the access to health data other the ones used by the methods described in chapter 4 is harder than for property/casualty insurance in which several factors can be determined for each claim (e.g. vehicle number, car type, etc.), it is important to find other methods for the evaluation of claim reserves. Another problem of the development factor methods is given - as described in chapter 4 - by the instability and variability of the data of the most recent months.

One possible approach to solve this problem is to adjust the development methods for the most recent months which produce high variability because of small completion factors. Lynch (2004) suggests to project the estimated ultimate losses

of more recent months by other (distribution-free) methods than the development methods. However, since diagnostic checks would be helpful for the test whether adequate reserves have been calculated, a statistical approach should work even better. Therefore another approach which calculates the estimated incurred losses for incurred months for which the completion factors are small via statistical models would be more appropriate. This is given by the modeling of most recent months via regression based on the estimated incurred losses in earlier periods.

Nevertheless, if only few months are described by a statistical model, diagnostic checks would only exist for these periods. For the other months the actuarial knowledge is again of great importance but since the later months are more developed discrepancies are not as severe as for the earlier months. However, a model based only on statistical methods would be even better, since it would provide diagnostic tests for all months and it would permit the calculation of confidence intervals. These are very important since they allow to get an overview over the range of possible expected losses, because no method can predict the exact outcome of the random behavior of claims. Also, confidence intervals provide a method to include business policies of the respective insurance company into consideration by calculating the confidence interval to a given confidence level. Purely statistical approaches will be discussed in the sections "Extended Link Ratio Family", "Probabilistic Trend Family", "The Chain-Ladder Linear Model" and "Stochastic Model Based on Gamage et al. (2005)".

Calculation Methods for the Most Recent Months

In this section several models based on Lynch (2004) and a regression approach based on Gamage et al. (2005) will be discussed. These models will obtain better estimates for the expected loss reserve with respect to stability and variability than the completion factor method or the chain-ladder method. The models suggested by Lynch (2004) can also be used for the calculation of loss reserves for every incurred month and not only for the most recent ones. Since the most common method of health insurance reserving is the completion factor method, only models for this method are described. However, the same methods described below can be used for the chain-ladder method for which the months with high development factors have to be substituted, i.e. the most recent months, too. A pure statistical approach will be discussed in the sections "Extended Link Ratio Family", "Probabilistic Trend Family", "The Chain-Ladder Linear Model" and "Stochastic Model Based on Gamage et al. (2005)".

Models Based on Lynch (2004)

Since the high method variance of the completion factor method is caused by the division of the known values for claims paid up-to-date by completion factors, other methods which do not use a (multiplicative) factor approach could be more appropriate.

One possible model is described by the so-called "Incurred per month per member"-method or short "Incurred PMPM" method. This method is similar to the projection methods described in chapter 4. The expected loss reserve is calculated as the difference of the total (estimated) amount of incurred losses and the amounts already paid. First, an average amount of incurred claims per member per month is estimated from historical data for which full development is assumed to obtain the total amount of incurred losses. This average PMPM amount is adjusted by a trend factor for the actual development. The result is multiplied by the actual number of members and the number of months in the valuation period to get the total (estimated) amount of incurred losses.

Different to the completion factor method, this model produces a stable estimate of the claim reserve since only few parameters have to be estimated. Nevertheless, as well as the completion factor method, the "Incurred PMPM" model relies on assumptions which do not render the reserve calculation more precisely. Especially, since the average incurred PMPM is only estimated from historical data which are fully developed, most recent developments are not included into the calculation process. Therefore, the trend factor approximation has to be done very accurately to obtain acceptable results.

Although the "Incurred PMPM" method produces stable projections for the total incurred losses, the method is based on the assumption that the only source of variability is the variability in incurred and processed claims. The variability

because of morbidity is not included. Therefore this model is unsatisfactory, too. A further problem of this model is that it produces higher standard errors than the completion factor method. This becomes problematic since the reserves need to be calculated accurately to secure solvability of the insurance company. If a reserve estimate has a very high standard error, the actual amount which has to be paid could have been over- or even worse underestimated which could imperil the solvability.

Therefore, Lynch (2004) introduces a second model, the so-called "Paid claims PMPM" method. This has all properties which are needed for a good estimation: a small model inherent standard error, no underlying assumptions as only one source of variability like the completion factor and the "Incurred PMPM" method and stability.

To calculate the expected IBNR reserve, first an average per month per member amount of incurred but not yet paid claims is calculated from historical data for each lag period. Then, this amount is adjusted by a trend factor for current development. For each incurred month the calculated and trended amounts are summed up over all lag periods. The result is then multiplied with the number of members in the respective month. To get the total IBNR reserve the previous calculated estimates for each incurred month have to be summed up. This model gives an unbiased estimator for the IBNR reserves with a very low variance since it relies only on old IBNR behavior which is trended to current behavior.

Lynch (2004) shows that the "Paid claims PMPM" method always produces the lowest standard error compared to the "Incurred PMPM" method and to the development factor methods but the standard error of the completion factor method converges to the one of the "Paid claims PMPM". This is due to the fact that the completion factors for earlier periods are more precise, since they are better developed. Therefore the completion factors converge to one and the only source of variability is the "process variance" which is inherent to the data.

Although the latest described method seems to be the best in every aspect, a hybrid of "Paid claims PMPM" method and completion factor model in which the most recent months would be estimated with the first one and the later calculation would be based on the development based method, would produce even better estimates.

Nevertheless, since these models are distribution-free, a better approach would be to model the most recent months in the completion factor approach by a statistical model. This is given by a regression approach as in Gamage et al. (2005).

Regression Approach for Most Recent Months

The regression approach for the prediction of the total amount of incurred claims is the idea to create a hybrid model consisting of an evaluation for earlier months done by the completion factor method and the prediction of the incurred claim amounts of most recent months by a regression model (Gamage et al. (2005)). Since the completion factor method for earlier incurred months, for which

the completion factors are closer to one than in more recent months, is not very variable and since a good regression model should also have a low standard error, this hybrid will produce a better estimate for the ultimate incurred losses than the completion factor approach. This model provides not only better predictors for the total amount of incurred claims if the regression model is chosen appropriately, but also methods are provided to determine irregularities in the given set of data as for example outliers. Therefore the actuary does not have to check every data point whether it is an catastrophic claim or not. In the following the procedure to obtain predictions for the incurred claim amount given by this hybrid model will be described.

First, the completion factors have to be calculated from the given data set in the way proposed in chapter 4. Since the regression model will provide a method to check if outliers are present in the given data set, the actuary has not to check in this step whether there are catastrophic claims in the observed data. This will be done later during the evaluation process. With the resulting completion factors the estimated incurred losses are calculated usually as the quotient of paid claims paid up-to-date and the corresponding completion factor. Since it is assumed (as stated in chapter 4, "Criticism on Development Factor Methods") that completion factors less than 40 to 70 percent do not produce appropriate estimates, the amount of incurred claims for these months does not have to be calculated by the completion factor approach. The estimates for these, more recent incurred months will be

provided by the regression approach.

After the incurred claim amount for each incurred month are calculated, it has to be decided whether the dollar amount of incurred claims should be the unit of measurement for the regression or if another unit is better suited. Since the dollar amount of incurred claims can depend on the number of members in the given incurred month and therefore on the normally higher number of incurred claims if more members are observed, a better unit of measurement is represented by the average amount of incurred claims per member per month (PMPM), since this will show major changes in the claim behavior. Therefore the dollar amount of incurred claims in a given incurred month needs to be divided by the number of members in that given month and is then comparable to other months. This adjustment has another advantage than comparability. Since claim estimates for the evaluation of charged premiums or for financial forecasting are also calculated on a per member per month basis, the estimated values can be compared (see Gamage et al. (2005)).

To find an appropriate regression model a graph of the incurred PMPM amount versus the incurred months can be drawn. Since these are the mainly known factors, a regression model will always include the incurred PMPM amount as the dependent variable and the time (the incurred month) as a independent variable. Using the graph the relationship for the regression model can be determined. Possible relationships are for example linear, quadratic or exponential.

Since it is of further interest for diagnostic checking of the model, the general

multiple linear regression model will be described, before more appropriate models for the estimation process in health insurance will be introduced. In general, a multiple linear regression will be described in matrix form. Assume the multiple linear regression model is given by (see Schmidt (2002))

$$Y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} + \varepsilon_i, \quad i = 0, \dots, n-1$$

with independent and identical distributed $\varepsilon_i \sim N(0, \sigma^2)$.

Then the matrix model is given by

$$Y = X\beta + \varepsilon$$

with $Y = (Y_0, Y_1, \dots, Y_{n-1})^T$, $\beta = (\beta_0, \beta_1, \dots, \beta_k)^T$, design matrix

$$X = \begin{pmatrix} 1 & x_{01} & \dots & x_{0k} \\ 1 & x_{11} & \dots & x_{1k} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{(n-1)1} & \dots & x_{(n-1)k} \end{pmatrix}$$

with $\varepsilon = (\varepsilon_0, \varepsilon_1, \dots, \varepsilon_{n-1}) \sim N(0, \sigma^2 I)$.

The parameter estimate for β is given by

$$\hat{\beta} = (X^T X)^{-1} X^T Y,$$

where X^T describes the transposed designmatrix.

Since only few background information is given for health insurance data, the first approach for an appropriate regression model - according to Gamage et al. (2005) - is to choose a simple model which is only based on the time, i.e. the

incurred month as independent variable x_i ($i = 0, \dots, n - 1$) and the incurred PMPM amount as dependent variable y_i ($i = 0, \dots, n - 1$). Here, $n - 1$ is the number of incurred months for which the completion factors method gives appropriate results. Therefore Gamage et al. (2005) propose the simple regression models

- Simple linear regression:

$$y_i = \beta_0 + \beta_1 i + \varepsilon_i, \quad i = 0, \dots, n - 1$$

- Simple quadratic regression:

$$y_i = \beta_0 + \beta_1 i + \beta_2 i^2 + \varepsilon_i, \quad i = 0, \dots, n - 1$$

- Simple exponential regression:

$$y_i = \exp(\beta_0 + \beta_1 i) + \varepsilon_i, \quad i = 0, \dots, n - 1$$

The model which is appropriate for the given data set depends on the structure of the data. This structure can be seen in the graph of the incurred PMPM amounts vs the incurred month. Since these are only simple models, they can be adjusted for more known information by adding more independent variables to the model or adjust the existing ones.

One approach described in Gamage et al. (2005) is the following. Since not as many people use medical services over the weekend or on holidays, the incurred claim amounts on these days are usually smaller compared to the corresponding amounts on weekdays. Therefore months with many holidays, e.g. December, could show a smaller amount of incurred claims than other months for which the number

of weekends and holidays is not as high. To adjust for this problem weight factors are calculated. First, Gamage et al. (2005) assume that on weekends normally only 35 percent of weekday claim activity occurs. Therefore a weighted month day number for each incurred month is calculated by the summation of weekdays in the month times 1 and weekend / holidays days times 0.35. Then a base month is determined which should get the weight factor 1. By dividing the results from the calculation before by the base month a weight factor compared to this weight month is given. Therefore the following regression models are obtained in which w_i ($i = 0, \dots, n - 1$) is the weight factor for the given month:

- Adjusted linear regression:

$$y_i = \beta_0 + \beta_1(iw_i) + \varepsilon_i, \quad i = 0, \dots, n - 1$$

- Adjusted quadratic regression:

$$y_i = \beta_0 + \beta_1(iw_i) + \beta_2(iw_i)^2 + \varepsilon_i, \quad i = 0, \dots, n - 1$$

- Adjusted exponential regression:

$$y_i = \exp(\beta_0 + \beta_1(iw_i)) + \varepsilon_i, \quad i = 0, \dots, n - 1$$

Other adjustments can be made for example by adding dummy variables which describe changes in medical plans or one can also adjust the model further by adding more parameters to the regression equation if the knowledge about these is given. The adjustments to the described regression models have to be made carefully. Otherwise problems like that of multicollinearity can occur.

According to Gamage et al. (2005), multicollinearity occurs if two columns in

the design matrix X are nearly the same, i.e. the two columns are almost linearly dependent. Thus, the matrix X and X^T are nearly singular and so is $X^T X$. This becomes to a problem when the estimate $\hat{\beta} = (X^T X)^{-1} X^T Y$ of β is calculated, because we have to calculate the inverse of $X^T X$. Since

$$(X^T X)^{-1} = \frac{\text{adj}(X^T X)}{\det(X^T X)}$$

with adjoint matrix $\text{adj}(X^T X)$ and the determinant $\det(X^T X)$ is close to zero because of the nearly singularity of $X^T X$, good parameter estimates would depend on the given data set and the values of Y . Therefore, a very small change in values of Y can imply a very big change in $\hat{\beta}$. Therefore the model should be chosen in such a way that two parameters do not have the same form for nearly every incurred months.

After an appropriate model is chosen and the parameter estimate of β is calculated, it has to be checked whether the model is statistically appropriate. Therefore a closer look on the standard error, the R^2 and the adjusted R^2 as well as checking the behavior of the residuals $\varepsilon = (Y - X\hat{\beta})$ can be helpful (Gamage et al. (2005)).

The standard error

$$s = \sqrt{SSE/n - k - 1}$$

with $SSE = (Y - X\hat{\beta})^T(Y - X\hat{\beta}) = \varepsilon^T \varepsilon$ is a good measure of variability of the model. A low standard error is a sign of low variability.

The R^2 value

$$R^2 = 1 - \frac{SSE}{SST}$$

with SSE as above and $SST = \sum_{i=0}^{n-1} (y_i - \bar{y})^2$ with $\bar{y} = \frac{1}{n} \sum_{i=0}^{n-1} y_i$

and the adjusted R^2 value

$$R^2 = 1 - \frac{SSE/(n - k - 1)}{SST/(n - 1)}$$

are measures for the percentage of variability explained by the model. Therefore a large R^2 value is a characteristic of a good model. Since adding appropriate variables to models increases the R^2 value, a better criterion is a large adjusted R^2 value. If the estimated model for the set of data has a low standard error and large R^2 / adjusted R^2 values and the graph of the residuals shows no other pattern than a random fluctuation around mean level zero, since the residuals should be normal distributed random variables with mean zero, the chosen model could be appropriate to describe the data.

Another thing to be checked is the statistical significance of the coefficients β_j ($j = 0, \dots, k$). The p-values of the estimated coefficients which are used for testing the hypothesis that the estimated coefficient is zero, i.e. $H_0 : \beta_j = 0$, for some $j \in \{0, \dots, k\}$, provides a measure for this (see Gamage et al. (2005)). A parameter is insignificant if the hypothesis H_0 is accepted, i.e. if the p-value of the given coefficient is large. However, sometimes parameters could be useful in the modeling process, although they are statistically insignificant. These

parameters have a practical significance and should therefore still be included into the model.

If a model is declared appropriate, this model might be used for both the calculation of predicted values and for the calculation of corresponding confidence, respectively prediction intervals for the most recent months. Assume $x = (1, x_1, \dots, x_k)$ is the vector of independent variables for which the incurred PMPM amount should be predicted. In case of a simple quadratic regression being used for the calculation of the most recent month, the vector x is given by $x = (1, l, l^2)^T$ where l denotes the incurred month for which the incurred PMPM amount should be predicted. Therefore the predicted value would be given by

$$\hat{y} = x^T \hat{\beta}.$$

A confidence interval to the confidence level $\gamma = 1 - \alpha$ is given by

$$\left(x^T \hat{\beta} - t_{n-k, \alpha/2} s \sqrt{x^T (X^T X)^{-1} x}, x^T \hat{\beta} + t_{n-k, \alpha/2} s \sqrt{x^T (X^T X)^{-1} x} \right)$$

and a prediction interval to the same confidence level is be given by

$$\left(x^T \hat{\beta} - t_{n-k, \alpha/2} s \sqrt{1 + x^T (X^T X)^{-1} x}, x^T \hat{\beta} + t_{n-k, \alpha/2} s \sqrt{1 + x^T (X^T X)^{-1} x} \right)$$

with $t_{n-k, \alpha/2}$ is the t-quantile of a t-distribution with $n - k$ degrees of freedom (see Schmidt (2002)).

The methods for the calculation of claim reserves as discussed in chapter 4 and in the section "Calculation methods for the most recent months" of this

chapter provide no or not many methods for the calculation of a prediction interval of the estimated reserves. Additionally, these methods cannot be used for diagnostic tests which are helpful for the decision whether the chosen model is appropriate for the given data or not. Therefore statistical methods which model the given data of the insurance company to estimate an appropriate reserve are introduced for several years. Most models which are found in the literature are based on property/casualty data. However, since claim reserving of health insurance companies is done with the same deterministic methods, the approaches for property/casualty can be used for health insurance reserving as well but it is important to think about different models for health insurance, too, which describe the lower number of known factors described by the data. One possible idea is to fit regression models which are similar to the deterministic development method described in chapter 4. This is the extended link ratio family approach based on Barnett and Zenwirth (2000). Since these models are not satisfying, as described later in this section, Zenwirth (1994) introduced another statistic modeling family, the probabilistic trend family. These models are only based on the factor time which is good especially for health insurance data, because the time factor is the only almost surely known influenced factor for every data set. Renshaw and Verrall (1998) define a model which can be shown to be directly equivalent to the chain-ladder method but which is based on a distribution assumption that is not appropriate for the calculation of claim amounts. A last approach is a model which

considers the lack of known factors of health insurance data and which was introduced by Gamage et al. (2005). It uses another multiple linear regression approach. These statistical models will be described in the following.

Extended Link Ratio Family

The extended link ratio family as given in Barnett and Zenwirth (2000) is a group of claim reserving models based on the calculation of the expected losses via link ratios as it is done for the deterministic models in chapter 4.

For the definition of the general extended link ratio model it is supposed that $x(i)$, $i \in \{1, \dots, m\}$ defines the cumulative claim amount as of incurred month i and development period $j - 1$, $j \in \{2, \dots, n\}$, i.e. $x(i) = C_{i,j-1}$ and $y(i)$, $i \in \{1, \dots, m\}$ defines the related cumulative claim amount of incurred month i and development month j , i.e. $y(i) = C_{ij}$. The extended link ratio family is defined by

$$y(i) - x(i) = a_0 + a_1i + (b - 1)x(i) + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2 x(i)^\delta). \quad (5.1)$$

δ is a weighting parameter. Therefore, the regression approach for the calculation of the parameters can be interpreted as a weighted regression. σ^2 is the level variance for the whole development period. This base variance is leveled with the weight factor $x(i)^\delta$. The parameter a_0 which describes the intercept is very important for the model. This is because the link ratio of a model without intercept, for which

the data suggests the use of an intercept, would be biased. a_1 describes a trend in the incurred months. This will influence the incremental data of the incurred months and distort the link ratios if it is not included. b is the parameter which describes the link ratio, i.e. the development between two consecutive development years. These models can be used for identifying trends but - according to Barnett and Zenwirth (2000) they cannot estimate the trends for forecast purposes.

Special cases of this model can be directly related to the estimation of expected ultimate losses in the deterministic case.

One special case of the extended link ratio family models is the so called chain-ladder link ratio model (Barnett and Zenwirth (2000)). For this model it is assumed that the weight parameter $\delta = 1$ and $a_0 = a_1 = 0$, i.e. that the only parameters which influence the dependent variable, is the link ratio (the development factor between two successive development months). Thus the equation (5.1) is given by

$$y(i) - x(i) = (b - 1)x(i) + \varepsilon_i = bx(i) - x(i) \quad (5.2)$$

By adding $x(i)$ on both sides of the equation (5.2), the regression equation with dependent variables $y(i)$, i.e. the cumulative amount as of incurred month i and development period j , is given by

$$y(i) = bx(i) + \varepsilon_i, \quad i = 1, \dots, m$$

with $\varepsilon_i \sim N(0, \sigma^2/x(i)^{-\delta})$. To find the best estimate \hat{b} for the parameter b , the

maximum likelihood estimate has to be calculated. Since the error term underlies a normal distribution this is equivalent to the calculation of the weighted least square estimate, i.e. in order to get \hat{b} the expression

$$\sum_{i=1}^m x(i)^{-\delta} (y(i) - \hat{b}x(i))^2$$

has to be minimized. Therefore the least square estimate for this model is given by

$$\hat{b} = \frac{\sum_{i=1}^m x(i)(y(i)/x(i))}{\sum_{i=1}^m x(i)}.$$

This estimate is the same as the single development factor calculated under the chain-ladder method with the help of the dollar-weighted average method.

Nevertheless, the regression approach has the advantage that this is only a mean estimate and that a standard deviation of the model can be derived. This is very important if a forecast has to be made. Therefore this model gives a more precise estimate than the pure deterministic approach. If the value of δ is changed, other (deterministic) estimates for single development factors can be obtained. For example, choose $\delta = 2$. Then the weighted least square estimate \hat{b} for b is given by the simple arithmetic average of the development factors of one development period, i.e. $\hat{b} = \frac{1}{m} \sum_{i=1}^m y(i)/x(i)$.

This chain-ladder link ratio model relies on several assumptions which have to be checked in order to decide whether the model is appropriate or not. The first assumption is the fact that the normalized error terms $\varepsilon_i/\sigma x(i)^{\delta/2}$ have to be

standard normal distributed. This can be checked by the standard methods for test of normality like the histogram of the residuals, the residual plot, etc. Since normally the loss data are not normal distributed (Barnett and Zenwirth (2000)), this model - as well as every other model of the expected link ratio family - is not appropriate for the calculation of claim reserve estimates. More over, Barnett and Zenwirth (2000) show that the incremental claim amounts are log normal distributed, since the residuals vs. the fitted values are skewed to the right.

Another assumption by the chain-ladder method, which is - according to Barnett and Zenwirth (2000) - not very often satisfied by the model, is

$$\mathbb{E}(y(i)|x(i)) = bx(i) \quad (5.3)$$

i.e. the cumulative amount in development period j depends only on the product of the cumulative amount of the previous period and the trend (link ratio). This can be checked by graphing the values $y(i)$ versus the values of $x(i)$. Sometimes it can be necessary that these values have to be adjusted by a parameter for an intercept, i.e. instead of the chain-ladder model an extended link ratio family model with the parameter $a_0 \neq 0$ has to be estimated. Another test for the validity of the given assumption is the randomness of the weighted standardized residuals versus fitted values. This will show an upward or downward trend, if an intercept should be used. Otherwise the data are over- and / or underpredicted.

This example shows very well the direct relation between the extended link

ratio family and the deterministic models described in chapter 4. The extended link ratio family is useful for the estimation of variability of the data. But why does everybody use the deterministic models while a stochastic model network seems to exist? The reason for this is that the extended link ratio family models have some disadvantages which need to be considered in the process of finding a better model. One main disadvantage is the fact that the error terms are assumed to be normal distributed. This assumption is rarely fulfilled by the loss data of an insurance company, as Barnett and Zenwirth (2000) state. In general, the incremental claim amounts are log normal distributed. This property can be seen in a plot of residuals versus the fitted values of an extended link ratio family model, if the residuals are skewed to the right. Another big disadvantage is that payment month trends (as inflation, etc.) are not included in the model. Therefore these models are only useful, if the data are adjusted for inflation beforehand. This problem is even more severe in the property/casualty insurance than for health insurance, since health insurance claims are normally assumed to be fully developed after a period of twelve months. Therefore a payment months trend as e.g. inflation would not become as important, since most of the inflation factors become more severe after a certain period of time. However, a model which includes a trend in every direction, i.e. incurred month, development month, and payment month, would be even better. These models are described in the following section.

Probabilistic Trend Family

Since the extended link ratio family which describes models close to the deterministic claim reserving methods - as seen in the previous section - are not very satisfactory models for the forecast of reserves, other statistical reserving models are needed which are not limited by implicit assumptions which are not fulfilled by the data. According to Barnett and Zenwirth (2000), an optimal stochastic model should enable the distinction between process and parameter variability and it should provide diagnostic tests whether the model assumptions are satisfied by the data. Additionally it should allow to predict future data as well as an interval of possible values, e.g. prediction and confidence intervals, and it should allow an easy update of the chosen model for new data. Therefore the goal of the modeling process is to create a model for which all inherent assumptions are fulfilled and trends and random fluctuations are also included. Such a family of models is introduced by Zenwirth (1994) in the so-called probabilistic trend family or the stochastic development factor family. This family of models enables the fulfillment of the desired properties.

For the general description of the probabilistic trend family a few notations have to be introduced. As stated in the section "Terminology" of chapter 4, the development process of a set of loss insurance data is dependent on three points in time, the incurred month, the development month and the paid month. These can be interpreted as the directions of the process, i.e. each of these points of time

influences the process in its own distinctive way and therefore each of the three directions should be included in the optimal stochastic model. In chapter 4, the assumption that paid months and development months are equal was used for simplicity reasons. In this section it becomes very important to differ between paid months and development months, since it is possible to observe trends in each of the three direction which are not reflected satisfactorily if only two trends are assumed. Therefore it could be possible to obtain a trend between two consecutive incurred months - this could be interpreted as a level change of the incurred month data -, a trend between two consecutive development months - this could be interpreted as a kind of link ratio between two consecutive development months-, and a trend between two consecutive paid months - this could be caused by inflation.

General Model

For this family of models Zenwirth (1994) defines the incurred months by i ($i = 1, \dots, n$), the development months by j ($j = 0, \dots, n - 1$) and the paid months by t ($t = 1, \dots, n$). The incurred month and the development direction are orthogonal to each other, i.e. a trend in one of those directions does not influence the other. The paid month direction is not orthogonal to either of the other directions, since the paid months t can be given as $i + j$. Therefore a paid month trend will influence both incurred month and development month trend.

As in chapter 4 we define for all $i \in \{1, \dots, n\}$ and $j \in \{0, \dots, n-1\}$

$y_{ij} \hat{=}$ incremental claim amount as of incurred month i and development month j

and furthermore $z_{ij} = \log(y_{ij})$, $y_{ij} > 0$. This second definition is made, since normally insurance loss data follow a log normal distribution. The implicit assumption is made that all data in the loss development triangle which are used in the estimation process are strictly greater than zero. Therefore only positive incremental claim amounts are assumed for the probabilistic trend family. If the log incrementals z_{ij} can be modeled, a model for the incremental claims is obtained.

In general, the probabilistic trend family is given by

$$z_{ij} = \alpha_i + \sum_{k=1}^j \gamma_k + \sum_{t=2}^{i+j} \iota_t + \varepsilon_{ij} \quad \forall i = 1, \dots, n, j = 0, \dots, n-1 \quad (5.4)$$

with independent and identically distributed $\varepsilon_{ij} \sim N(0, \sigma^2)$, parameter α_i which models the effect of incurred month i , i.e. the level change in incurred month i , parameter γ_j which models the trend between two successive development months, and parameter ι_t which describes the trend from one paid month to the next. If the original data are influenced by inflation, the latter one represents the superimposed social inflation. The parameter estimates $\hat{\alpha}_i$, $\hat{\gamma}_k$ and $\hat{\iota}_t$ for α_i , γ_j and ι_t can be estimated with a multilinear regression approach.

For equation (5.4) the multilinear regression equation (described by Zenwirth (1994)) is given by

$$Z = X\beta + \varepsilon \quad (5.5)$$

with

$$Z = (z_{10}, \dots, z_{1,(n-1)}, \dots, z_{n0}, \dots, z_{n,(n-1)})^T$$

$$\beta = (\alpha_1, \alpha_2, \dots, \alpha_k, \gamma_1, \dots, \gamma_l, \iota_1, \dots, \iota_m)^T.$$

The vector β includes

- k ($1 \leq k \leq n$) parameters describing the incurred month trend with

$$\begin{aligned} \alpha_1 & \text{ represents the level for incurred months } 1, \dots, i_1 \\ \alpha_2 & \text{ represents the level for incurred months } i_1 + 1, \dots, i_2 \\ & \vdots \\ \alpha_k & \text{ represents the level for incurred months } i_{k-1} + 1, \dots, n \end{aligned}$$

- l ($0 \leq l \leq n$) parameters describing the trend along the development months

with

$$\begin{aligned} \gamma_1 & \text{ represents the trend along development months } 0, \dots, j_1 \\ \gamma_2 & \text{ represents the level along development months } j_1 + 1, \dots, j_2 \\ & \vdots \\ \gamma_l & \text{ represents the level along development months } j_{k-1} + 1, \dots, n - 1 \end{aligned}$$

- m ($1 \leq m \leq n$) parameters describing the trend along payment months with

$$\begin{aligned} \iota_1 & \text{ represents the trend along paid months } 1, \dots, t_1 \\ \iota_2 & \text{ represents the level along paid months } t_1 + 1, \dots, t_2 \\ & \vdots \\ \iota_m & \text{ represents the level along paid months } t_{k-1} + 1, \dots, n \end{aligned}$$

The designmatrix X for this model is given as

$$X = (x_{10}, \dots, x_{1,n-1}, \dots, x_{n0}, \dots, x_{n,n-1})^T$$

where the vector x_{ij} forms a row of the matrix X and

$$x_{ij} = (\delta_{11}^{(ij)}, \dots, \delta_{1k}^{(ij)}, \delta_{21}^{(ij)}, \dots, \delta_{2l}^{(ij)}, \delta_{31}^{(ij)}, \dots, \delta_{3m}^{(ij)})$$

with

$$\delta_{1r}^{(ij)} = \begin{cases} 1 & \text{if } i_{r-1} + 1 \leq i \leq i_r (i_0 = 0) \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{2r}^{(ij)} = \begin{cases} 1 & \text{if } r = 1 \\ j - j_{r-1} & \text{if } j \geq j_{r-1} + 1 (j \geq 2) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\delta_{3r}^{(ij)} = \begin{cases} i + j - t_{r-1} & \text{if } i + j \geq t_{r-1} \\ 0 & \text{otherwise} \end{cases}$$

The error term ε underlies a normal distribution with mean 0 and variance $\sigma^2 I$ with identity matrix I , i.e. $\varepsilon \sim N(0, \sigma^2 I)$.

Since $\varepsilon_{ij} \sim N(0, \sigma^2)$ for every incurred month i ($i = 1, \dots, n$) and every development month j ($j = 0, \dots, n - 1$) the incremental terms $y_{ij} = \exp(z_{ij})$ are log normal distributed with mean $\mathbb{E}(y_{ij}) = \exp(\alpha_i + \sum_{k=1}^j \gamma_k + \sum_{t=2}^{i+j} \iota_t + \frac{1}{2}\sigma^2)$ and variance $\text{Var}(y_{ij}) = \exp(\alpha_i + \sum_{k=1}^j \gamma_k + \sum_{t=2}^{i+j} \iota_t + \frac{1}{2}\sigma^2)^2 \cdot \exp(\sigma^2 - 1)$.

Special Members of the Probabilistic Trend Family

In the following special members of the probabilistic trend family will be described. They have in common that they do not describe trends in every of the three directions - and therefore are useless for forecasting most of the time - but these models are very good for the process of identifying trends in the direction which is not covered by the model. Thus, they are described in the following.

The Cape Cod model is a basic member of the probabilistic trend family. It is based on the simplifying assumption of homogeneity in the development months

between successive incurred months and the homogeneity in the incurred month, i.e. $\alpha_1 = \dots = \alpha_k = \alpha$. Thus, no payment year trends are covered by the model, i.e. $\iota_1 = \dots = \iota_m = 0$. Therefore the Cape Cod model is given by

$$z_{ij} = \alpha + \sum_{k=1}^j \gamma_k + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2).$$

Because of the simple structure of this model example 2 shows how this model can be used for the identifying of trends in the paid month direction.

The CCI-model is an extension of the Cape Cod model. It assumes homogeneity of incurred and development month but additionally a parameter for constant payment month inflation is added to the model. Therefore the CCI-model is given by

$$z_{ij} = \alpha + \sum_{k=1}^j \gamma_k + \iota \cdot (i + j - 1) + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2).$$

This can be written as

$$z_{ij} = \alpha + i\iota - \iota + \sum_{k=1}^j (\gamma_k + \iota) + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2).$$

Thus, a paid month trend influences both incurred month and development month, as it was assumed for the three trends.

Example 2 (see Zenwirth (1994), p. 484)

Assume a simple Cape Cod model as the chosen model for the estimation of loss reserves for a given set of data, i.e. it is assumed that incurred month and

development month trends between two successive incurred months are homogeneous. If no paid month trend is added, the residuals of this model should only be given as a random error which occurs since we are assuming the data as a sample path of a random variable. This implies that the residuals versus each direction show randomness.

If a paid month trend of p percent is added to the given data, the residuals will be given as

$$\text{residual} = \text{estimate of error} + p\% \text{ paid month trend.}$$

This is because the Cape Cod model would not adjust for paid month trends.

Therefore the graph of residuals versus paid months will show an upward or downward trend - depending on the value of p - instead of random behavior.

However, the graphs of residuals versus incurred months and development months should show randomness since these factors are estimated by the Cape Cod model.

If the CCI-model is used instead of the Cape Cod model, the graph of residuals versus paid months should again show randomness since the CCI-model includes a parameter for a constant paid month trend.

If two paid months trends are assumed to be included in the data, i.e. a paid month trend of $p\%$ is added in one month and another (different) paid month trend of $q\%$ is added to another month, the residuals of both models would not show randomness versus paid months. For the Cape Cod model the residuals will be

given by

$$\text{residual} = \text{estimate of error} + p\% \text{ paid month trend} + q\% \text{ paid month trend} .$$

Thus, the graph of the residuals versus paid months will show both distinct trends.

The CCI-model will estimate the paid month trends as one average paid month trend. Therefore the residuals will be given as the random error added to the difference of the given trend in the paid month to the average estimated trend.

Since due to this some values in the data are over- or underpredicted, the graph of the residuals will show a strong V-shape. This obviously shows the point of time when the trend change occurs.

These two models are very simple models for the stochastic loss reserving process but they are not very useful for an exact forecasting or for diagnostic tests. This fact distinguishes them from the next three special cases of the probabilistic trend family which will be introduced in the following. According to Zenwirth (1996), they are powerful tools for the identification of trend changes in one of the three directions - depending on the model chosen - and the choice of an optimal model for the loss reserving process.

First the stochastic chain-ladder model by Zenwirth (1996) will be described. As the deterministic chain-ladder model, this model assumes homogeneity of development factors of different incurred months but heterogeneity of different

incurred months. This is different to the previous described Cape Cod model for which the incurred months are assumed to be homogeneous, too.

The stochastic chain-ladder model is given by

$$z_{ij} = \alpha_i + \sum_{k=1}^j \gamma_k + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2).$$

The main disadvantage of this model is that no paid month trends can be predicted. However, this lack of parameter for paid months makes it possible to identify paid month trends. This knowledge can be used in finding the optimal stochastic model. Since the stochastic chain-ladder model estimates trends in the two directions incurred month and development month, the residuals of these periods should be random and the mean of the standardized residuals should be equal to zero for every incurred / development month. Different to this, the residuals versus paid months should not be random and will show a trend development between different paid months if this exists.

The second model, the so-called separation model (see Zenwirth (1996)), describes the log incremental payment as a sum of a constant parameter for the incurred month, i.e. it assumes that no trends in the incurred months exist, trends in the development months are homogeneous over consecutive incurred months and trend changes between two consecutive paid months. Thus, the model is given by

$$z_{ij} = \alpha + \sum_{k=1}^j \gamma_k + \sum_{t=2}^{i+j} \iota_t + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2).$$

Since this model only adjusts the data in paid month and development month

direction, it is useful for the identification for incurred month trends with help of the graph of residuals versus incurred months.

The incurred month / payment month model assumes that the development process of loss data is only based on incurred month and payment month trends and not on trends in the development months. Therefore it is appropriate to identify development month trends. The model is given by

$$z_{ij} = \alpha_i + \sum_{t=2}^{i+j} l_t + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2).$$

Properties of the Probabilistic Trend Family

Zenwirth (1994) introduces the probabilistic trend family because it covers every possible trend direction of the loss data and it enables the establishment of a distribution over the given trends. Since

$$\begin{aligned} z_{ij} - z_{i,j-1} &\stackrel{(5.4)}{=} \alpha_i + \sum_{k=1}^j \gamma_k + \sum_{t=2}^{i+j} l_t + \varepsilon_{ij} - \left(\alpha_i + \sum_{k=1}^{j-1} \gamma_k + \sum_{t=2}^{i+j-1} l_t + \varepsilon_{i,j-1} \right) \\ &= \gamma_j + l_{i+j} + \varepsilon_{ij} - \varepsilon_{i,j-1}, \end{aligned}$$

the mean trend between two consecutive development months of the same incurred month is given by

$$\mathbb{E}(z_{ij} - z_{i,(j-1)}) = \gamma_j + l_{i+j} + \mathbb{E}(\varepsilon_{ij}) - \mathbb{E}(\varepsilon_{i,j-1}) \quad (5.6)$$

$$= \gamma_j + l_{i+j} \quad (5.7)$$

because ε is normal distributed with mean 0. Equation (5.7) describes exactly the

trend expected under the assumption that a development month and paid month trend exist. Additionally the mean trend between two incurred months at the same development status is given by

$$\mathbb{E}(z_{i+1,j} - z_{i,j}) = \alpha_{i+1} - \alpha_i + \iota_{i+1+j} \quad (5.8)$$

with the same argumentation as above. Equation (5.8) describes the expected trend under the assumption that both incurred month and paid month trend exist.

These are from Zenwirth (1994) point of view good properties. Additionally, the probabilistic trend family fulfills other properties which will be stated in the following.

First, most insurance loss data are not - as assumed by the models of the extended link ratio family - normal distributed. Instead they often show a log normal behavior. Therefore, this should be modeled. The models of the probabilistic trend family are based on the estimation of log incremental claim amounts via regression and thus it assumes the log normality of the incremental loss data. Since the log incrementals are normal distributed, the regression approach for obtaining the maximum likelihood estimators is an easy way to calculate the estimated log incrementals, because only the sum of the least squares has to be minimized. Therefore the incrementals of the loss triangle are obtained by exponentiating the predicted log incrementals.

Additionally, the use of different members of the probabilistic trend family

can be helpful for the identifying process of the optimal model as mentioned before in the description of special members of the probabilistic trend family.

The third property the probabilistic trend family fulfills are the following tests which Zenwirth (1994), p. 489, states as important properties a good statistical model should satisfy.

Test 1 If the incremental claim payments behave randomly under the assumed underlying distribution and there are no trends inherent to the data, an appropriate statistic loss reserving model should show this behavior.

Test 2 If a loss development triangle is given and a second loss development triangle is created from the first one by adding some trends, e.g. for example two different payment month trends to different paid months, then an appropriate statistic loss reserving model will enable a quick determination of the differences of the two triangles.

The probabilistic trend family fulfills both tests, since it provides several different models which used together allow a quick determination of different trends but which also show a random behavior of data since the models are assumed to project a log normal distribution onto the incremental loss data. Test 2 fails for the extended link ratio family since the models include no paid month trend. Therefore only the analysis of trends in the paid month direction would be possible. The analysis in the other two directions would fail since it is not known whether paid month trends influence trends in the other two directions or not.

After analyzing the different models and identifying the trends in all three directions, an appropriate model can be found. However, before a forecast can be made, some criteria should be checked, whether the model is appropriate or not.

Criteria for a Good Model

To choose the "best possible" model within the probabilistic trend family for a given set of data, Zenwirth (1994) proposes several criteria. The first criteria which has to be checked is Ockham's Razor parsimony. The best model between equal models is the simplest, i.e. a model with the fewest parameters. A complicated model with lots of parameters does not always have to be preferred to a parsimonious model with a little higher variance. More complicated models can produce the problem of multicollinearity. If the more simple model describes all important facts of the data well, it could still be better.

Another criteria which should be considered is the Akaike information criteria for the goodness-of-fit of the chosen model. For the probabilistic trend family this criteria is given by (see Zenwirth (1994), p. 515)

$$AIC = N \cdot (2 * \prod s^2(MLE)) + N + 2P$$

with

N = number of observed data

$s^2(MLE)$ = maximum likelihood estimator for σ^2

P = number of parameters in the model

The best model among "equal ones" should have a minimum AIC-value.

To test if the chosen model is optimal for the given set of data an analysis on stability of the model should be done. In a stable model the incremental losses estimated from the set of data without the most recent incurred months should not differ from those with the last months data by more than one standard error.

Zenwirth (1994) proposes a validation test for this analysis. Here, a subset of the data (without the most recent years) is chosen and parameter estimates for the chosen model are calculated with less data. If the model gives the same estimates as the model for the whole set of data, the model is appropriate.

If a stable model which seems to be best compared to other models is found, forecasts based on the chosen model can be done and the estimated ultimate losses can be predicted. This evaluation includes the variability of parameters as well as the whole process inherent variability. Since the loss reserves of an insurance company are an important sign of the solvability of the company an accurate estimation is important and an overview of the possible variability of the given data has to be given. This can be done with the help of confidence intervals as

described for the probabilistic trend family in the following way (see Zenwirth (1994)). Assume y is the incremental payment in a given incurred month and a given development month. Define $x := \log y$. Furthermore suppose $\mu = \mathbb{E}(x)$ and $\sigma^2 = \text{Var}(x)$. Then the $100(1 - \alpha)\%$ confidence interval for x is given by

$$(\mu - \sigma z_{\alpha/2}, \mu + \sigma z_{\alpha/2})$$

where $z_{\alpha/2}$ is the $1 - \alpha/2$ quantile of the standard normal distribution. Then the $100(1 - \alpha)\%$ confidence interval for y is given by

$$(\exp \mu - \sigma z_{\alpha/2}, \exp \mu + \sigma z_{\alpha/2}).$$

Therefore the actuary can describe the variability of the expected loss reserve and can use it for the reserve stated in the balance sheet.

Since the probabilistic trend family relies only on assumptions about the factor time, i.e. the trend parameters for the given incurred months, development months and payment months, it seems appropriate as a model for the calculation of health claim reserves, since the only factor which has to be known is the factor time. One problem of these models is that they require a broad statistical knowledge from the valuation actuary who has to decide which of the several models given by the probabilistic trend family (s)he should choose. Another disadvantage which can become a problem in the estimation process is that the models of the probabilistic trend family are assumed to have a log normal

distribution. Therefore it is not possible to calculate estimates if negative incremental claim amounts occur in the development triangle.

Therefore, other models which do not rely on this implicit assumption of positivity are better. Two possible models serving this requirement will be described in the following.

The Chain-Ladder Linear Model

Renshaw and Verrall (1998) suggest a model which is different to the approach of modeling the claim development process with help of the probabilistic trend family or with help of the extended link ratio family. Their model is based on a Poisson distribution and they show that it is exactly equivalent to the distribution-free chain-ladder model for which the single development factors are calculated by the dollar-weighted average approach as described in chapter 4. Since it is based on a distribution, it provides tools for the determination of variability of the model over the given data and also diagnostic checks.

The model underlying structure is equivalent to the structure described in the section "Terminology" in chapter 4. The incremental claims as of incurred month i ($i = 0, \dots, n - 1$) and development month j ($j = 0, \dots, n - 1$) which describe the development triangle (see Table 3). The cumulative claim amounts are given as

$$C_{ij} = \sum_{k=0}^j y_{ik} \quad \forall i = 0, \dots, n - 1, j = 0, \dots, n - 1 - i$$

and are described in the cumulative development triangle (see Table 4). The goal of this model is to estimate the missing values in the development process, i.e. to calculate C_{ij} for $i = 1, \dots, n - 1$ and $j = n - i, \dots, n - 1$. Especially, we are interested in the calculation of the total amounts of every not yet fully developed incurred month, i.e. to calculate $C_{i,n-1}$, $i \in \{1, \dots, n - 1\}$, if it is assumed that claims are fully developed after a period of n months.

The "chain-ladder linear model" - as Renshaw and Verrall (1998) called it - is defined as follows:

The incremental claim amounts for incurred month i ($i = 0, \dots, n - 1$) and development month j ($j = 0, \dots, n - 1$) are given as independent random variables with

$$Y_{ij} \sim Poi(m_{ij}) \quad \forall i, j$$

and

$$\log m_{ij} = \log (\mathbb{E}(Y_{ij})) = \mu + \alpha_i + \beta_j$$

with $\alpha_0 = \beta_0 = 1$. Similar to the approach of the extended link ratio family only a trend in incurred month, described by the parameter α_i ($i = 0, \dots, n - 1$), and a trend in development months, given by the parameter β_j ($j = 0, \dots, n - 1$), is assumed. Since a Poisson distribution is the underlying distribution, it is better to assume that the number of claims is modeled instead of the claim amounts. As it can be seen later, it is not the Poisson distribution which is important for the

modeling process, but the form of the Likelihood function. Therefore the model can be used for the calculation of claim amounts, too. Additionally, this model differs from the model of Zenwirth (1994) and other authors, because it does not use the logarithms of incremental claim amounts for the modeling process, but it uses the logarithm of the expected values. This is an advantage because there is no requirement needed that every incremental claim amount is positive (which is quite unrealistic). However, this model is restricted to the use of only such data for which the sum of the incremental claims of each development period is positive, i.e.

$$\sum_{i=0}^{n-j-1} y_{ij} \geq 0 \quad \forall j = 0, \dots, n-1.$$

The necessity for this assumption becomes important in the process of calculating predicted values. The estimated total number of claims in a given incurred month i ($i = 1, \dots, n-1$) is given by

$$\hat{C}_{i,n-1} = C_{i,n-1-i} + \sum_{j=n-i}^{n-1} \exp \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j$$

where $\hat{\mu}$, $\hat{\alpha}_i$ and $\hat{\beta}_j$ are the maximum likelihood estimators of μ , α_i and β_j .

Renshaw and Verrall (1998) show that this approach for calculating the estimated ultimate losses is equivalent to the calculation of maximum likelihood estimators of the following conditional likelihood function which is conditioned on the latest row totals just like the chain-ladder method is:

$$L_C = \prod_{i=0}^{n-1} \left(\frac{C_{i,n-i-1}!}{\prod_{j=0}^{n-i-1} y_{ij}!} \prod_{j=0}^{n-i-1} p^{(i)j} \right) \quad (5.9)$$

$p_{(i)j}$ is the conditional probability that a claim incurred in month i is reported in development month j , i.e.

$$p_{(i)j} = P(\text{claim reported in } j \mid j \leq n - i - 1) = \frac{p_j}{\sum_{k=0}^{n-i-1} p_k}$$

with

$$p_j = P(\text{claim is reported in development month } j), \quad j = 0, \dots, n - 1.$$

Thus, the estimated ultimate loss of incurred month i is given as

$$\hat{C}_{i,n-1} = \frac{C_{i,n-i-1}}{1 - \sum_{j=n-i}^{n-1} \hat{p}_j} \quad (5.10)$$

where \hat{p}_j is the maximum likelihood estimator for p_j obtained from (5.9). For the equivalence proof of the chain-ladder method and their proposed model, Renshaw and Verrall (1998) rewrite equation (5.10) to the following equivalent equation which calculates the estimated ultimate losses for the incurred months i for which the last known development month is the development month j

$$\hat{C}_{n-j-1,n-1} = \frac{C_{n-j-1,j}}{1 - \sum_{k=j}^{n-1} \hat{p}_k}. \quad (5.11)$$

Furthermore, instead of calculating the maximum-likelihood estimates of (5.9) for p_j a recursive calculation is used. Therefore the estimates \hat{p}_j of p_j are given as

$$\begin{aligned} \hat{p}_{n-1} &= \frac{y_{0,n-1}}{C_{0,n-1}} \\ \hat{p}_j &= \frac{y_{0j} + y_{1j} + \dots + y_{n-j-1,j}}{C_{0,n-1} + \frac{C_{1,n-2}}{1-\hat{p}_{n-1}} + \dots + \frac{C_{n-j-1,j}}{1-\hat{p}_{j+1}-\dots-\hat{p}_{n-1}}}. \end{aligned}$$

With help of the latest estimate of p_j Renshaw and Verrall (1998) prove the equivalence of the distribution-free chain-ladder method as of chapter 4 and the conditional likelihood function approach and because of the equivalence of the latter one with the chain-ladder linear model to this model.

Since the approach using the conditional likelihood function and the chain-ladder linear model are equivalent and therefore only the form of the conditional likelihood function is relevant, the assumption that only the number of claims can be calculated is released. Thus, also the total claim amounts can be calculated with this model. However, Renshaw and Verrall (1998) state that this model is not appropriate for the calculation of claim amounts, although it is exactly equivalent to the distribution-free chain-ladder method, and that one could find better models for this calculation which are not relying on a Poisson distribution. These models can be described by generalized linear models of claim reserves but they do not have to be exactly equivalent to the chain-ladder method. One possible approach is given by England and Verrall (2001). Instead of calculating the total amount of claims, a risk-theoretical approach is used to calculate the expected number of claims by a Poisson distribution and the expected claim amount per claim by a Gamma distribution. The expected total amount of claims is then given by the product of these two.

Stochastic Model Based on Gamage et al. (2005)

Although previous described models like the probabilistic trend family and the chain-ladder linear model do not seem to be too bad for the modeling process, they depend on some problematic assumptions. For the probabilistic trend family it is assumed that the loss data are log normal distributed - being, according to Barnett and Zenwirth (2000), the reality for most claim data. But what happens if there are negative incremental claim amounts? The idea of Renshaw and Verrall (1998) and Gamage et al. (2005) is to model the incremental claim data but as seen above the chain-ladder linear model is restricted by the assumption of a Poisson distribution and therefore only appropriate for the modeling of the number of claims instead of the claim amounts. Therefore Gamage et al. (2005) proposes a stochastic approach which is based on a simple multiple linear regression model. The model is based on only one influencing factor, the factor "time" given by the incurred months and the development months. This is done, since time is the only factor which is always known for health insurance data.

For the modeling process it is assumed that the incremental data are derived from incurred month i ($i = 1, \dots, n$) and development month j ($j = 1, \dots, n$) by y_{ij} . Until the valuation date only such incremental claims are known, for which $y_{ij} \in \{y_{ij} : i + j \leq n\}$. The other incremental claim amounts have to be estimated from the known values to predict the ultimate loss amount after full development,

i.e. it is to calculate

$$C_{in} = \sum_{j=1}^n y_{ij} \quad \forall i = 2, \dots, n.$$

The general multi-linear regression model for the claim-modeling proposed by Gamage et al. (2005) is given by

$$y_{ij} = \beta_0 + \beta_1 y_{i-1,j} + \beta_2 y_{i,j-1} + \beta_3 i + \beta_4 j + \beta_5 i^2 + \beta_6 j^2 + \beta_7 ij + \varepsilon_{ij}$$

with independent and identically distributed $\varepsilon_{ij} \sim N(0, \sigma^2)$. It is assumed that the values of y_{ij} are not only dependent on the incurred month and the development month but that they are also dependent on the incremental claim amounts of the previous incurred month and the previous development month. Therefore parameters for $y_{i-1,j}$ and $y_{i,j-1}$ are included in the model. Therefore the known incremental claim amounts are used to calculate parameter estimates $\hat{\beta}_0, \dots, \hat{\beta}_7$ for β_0, \dots, β_7 . Therefore it is assumed that $y_{0,j} = 0 \quad \forall j = 1, \dots, n - 1$ and $y_{i,0} = 0 \quad \forall i = 1, \dots, n - 1$. To check if all parameters are significant for the calculation of parameter estimates for the given set of data, the p-values can be used. If some parameters are insignificant, the actuary has to decide whether (s)he should use a model without those parameters or whether they are practical significant and should be included into the estimation process. Also, it has to be checked whether the model is appropriate or not with the same methods as described in the section "Regression approach for most recent months". If other factors which influence the claims are known, the model can be improved by adding

additional parameters but for health insurance data not many additional factors are known. After an appropriate model has been chosen, the missing incremental claim amounts can be estimated successively for each payment year, i.e. for each year p with $p = i + j$. Most parameter estimates are then calculated based on the estimated values of the previous incurred month and the previous development month. Therefore this model could cause a higher "method variance" than other models.

Additionally, Gamage et al. (2005) proposes a way of including negative incremental under a log normal distribution which is - as Barnett and Zenwirth (2000) stated - most appropriate for loss data. The incremental claim data are adjusted in such a way that no negative incremental claims occur. Therefore a positive constant which is bigger than the absolute value of the smallest negative claim amount, i.e. $c = |\min_{i,j} \{y_{ij}\}| + 1$ is added to the incremental claim data in each cell. Then a regression based on the log incrementals as dependent variables is used to calculate parameter estimates and predict the expected values of the development triangle. Finally, the constant is subtracted of each cell and one can calculate the original ultimate claim amount based on the data.

CHAPTER VI

CONCLUSIONS

Health insurance reserving is, as reserving in general, one of the most important tasks for an actuary of a health insurance company. This thesis describes mainly the calculation of claim reserves which are different to expense and policy reserves, since the possible benefits the insured person receives from the insurance company are unknown. Nowadays the evaluation of claim reserves is usually done based on distribution-free methods which do not provide any measures of variability.

Most of the time the completion factor method is used for the calculation of health reserves for which the variability of the estimated claim amounts in the most recent months is very high. This method does not provide any diagnostic checking for the given set of data. Therefore, there has been a search for calculation methods which can predict estimated ultimate losses, describe its variability and provide diagnostic checking. Hybrids of the completion factor method and a regression approach for most recent months have been proposed by Gamage et al. (2005). These models provide diagnostic checking for the most recent months and they reduce the variability in the most variable recent months, but purely

statistical methods as the extended link ratio family, the probabilistic trend family, the chain-ladder linear model or the multilinear regression model based on Gamage et al. (2005) are even better. However, as we have seen in this thesis, these models are based on underlying assumptions which are not described by the data, e.g. the normality assumption or the Poisson distribution assumption, or a method which makes restrictive assumptions about the data, e.g. the log normality assumption which confines the actuary to only using positive incremental claim data. Therefore these models are not always useful. Another problem of statistical models for health insurance data is the fact that the only really well-known factor is the factor time. Thus, the statistical models should be appropriate and only based on the factor time and sometimes vary by some adjustments as a weight factor for weekdays / weekends.

Thus, the statistical methods for health insurance reserving should be based only on simple approaches. As it can be seen, the probabilistic trend family as described by Zenwirth (1994) is the best model, since it covers each time direction. The only disadvantage is the assumption that every incremental claim amount has to be positive because of the log normal distribution. Therefore an adjustment as described by Gamage et al. (2005) (adjusting the data by adding a positive constant such that no claim amount is negative) would perhaps provide a better model. Nevertheless, possible other approaches have to be discussed. One possibility of another model could be a time series model, since the main factor influencing the

data is the time. Another possible approach could be a non-parametric model as proposed by Gamage et al. (2005) and England and Verrall (2001).

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