Appendix A
SIGNIFICANT FIGURES

Experimental Measurements

Whenever experimental measurements are made, there is always the possibility for errors in the reported value. These errors can arise because of one or more of the following: 1) some type of limitation related to the nature of the measuring device; 2) some error that can be attributed to the individual; and 3) some type of approximation included in the measurement. Therefore, when numerical data are reported or interpreted, care must be exercised so that the reliability of the data is properly portrayed.

Consider the following examples of obtaining the mass of some object as an illustration of the statements made in the preceding paragraph. Suppose that the mass of an object is determined by using a triple-beam balance. The scales of many triple-beam balances are calibrated so that the mass can be read from the balance to the nearest 0.1 gram. Thus, the mass of the object would be reported as 15.3 g.

The mass 15.3 g can be interpreted as follows: The last number represents the least accurate digit in the determination. The mass of the object is definitely 15 g, but there is some uncertainty about the fractional portion. However, the fractional portion of the mass is approximately 0.3 g. A common practice of expressing this uncertainty is to assume or report that the correct value is within some range. The range is determined by taking one-half of the smallest value which can be read from the measuring device and showing that the correct value could deviate by this amount on either side of the reported value. Thus, since the smallest value which can be read from the triple-beam balance is 0.1 g, any weighing will be uncertain within the range of $\pm$ 1/2 (0.1) or $\pm$ 0.05 g. This means that for the weighing of the object which is reported at 15.3 g, the correct mass would be in the range of 15.25 g to 15.35 g.

If the mass of the same object was determined by using an analytical balance instead of the triple-beam balance, then the mass might be reported to be 15.3276 g. This piece of datum indicates that the uncertainty lies in the 0.0001 g place or that the mass of the object lies in the range of 15.32755 g and 15.32765 g.

Significant Figures

The term "significant figures" is used to designate the number of digits contained in some datum which reflect the precision of the datum. The number of "significant figures" is determined by adding the number of digits contained in the datum. For example, when the mass of an object is reported as 15.3 g, there are three significant figures contained in the datum 15.3 g. When the mass is reported as 15.3276 g, there are six significant figures.

When zeros are part of numerical data, the determination of whether the zero is a significant digit or not will depend on the way the number is written and/or the location of the zeroes. Note the examples in Table 1.

Table 1

Number	Significant Figures	Explanation
706	3	------
760	2	Since the decimal is not shown, the last figure with meaning is the 6 and the zero shows the magnitude.
7.6×10^2	2	This is a better way to write 760
760.	3	The zero is now significant because the location of the decimal has been specified!
7.60×10^2	3	Zeroes at the end of fractional numbers are significant.
0.000206	3	The first 3 zeroes only locate the decimal, the middle zero is part of the number and therefore significant.
2.06×10^{-4}	3	A better way to write 0.000206.
2.060×10^{-4}	4	The final zero is significant in that it shows the last digit that can be measured.

Frequently, very large numbers or very small numbers are best written in terms of scientific notation. In this manner, the correct number of significant figures can be expressed most easily.

"Rounding-Off"

Often a number with a large number of significant figures should be expressed using a smaller number of significant figures. Consequently, a procedure called "rounding-off" is employed to reduce the number of significant figures. This procedure involves the dropping of all digits which are not considered to be significant and if required by convention, to add one to the last significant digit.

The following is an outline of the procedure to follow:

1. Determine the desired number of significant digits.

2. Examine the first digit after the last significant figure.

3. If this digit is less than 5, all the numbers after the last significant figure are dropped.

4. If the digit described in 2 is greater than 5, all the numbers after the last significant figure are dropped, and the last significant figure is increased by one.

5. If the digit described in 2 is equal to 5, then the last significant figure is increased by 1 if it is an odd number, or left unchanged if it is an even number. Zero is considered to be an even number. (Note: if additional digits follow the 5, the last significant figure is increased by 1.)

The process is illustrated by the examples in Table 2.

Number	Desired Number of Significant Figures	Rounded-off Number
3.732473	5	3.7325
1.483	3	1.48
8765	3	8760
8775	3	8780
11.637	4	11.64
1.6753	3	1.68

Arithmetic Operations and Significant Figures

A means of recognizing the limitation of some measured quantity is by using the number of significant figures. Thus, the mass of an object as determined to be 15.3 g on a triple-beam balance has only three significant figures and the uncertainty is easily recognized.

Frequently, measured quantities are used to determine other quantities by calculation. Consequently the data are subjected to a variety or sequence of arithmetic operations. A calculated result must only contain the number of digits equal to the least accurate datum used.

Addition and Subtraction

When a group of measured quantities are added or subtracted, the result must be expressed to reflect the least certain value. For example: Suppose that the total mass of a flask, a liquid placed into the flask, and a covering for the flask is to be determined. The values obtained by using *different* balances are:

Flask	257.	g.
Liquid	1.023	g.
Cover	2.360	g.
Total	260.383g.	

In the above, the datum which has the least certainty (least precision) is the mass of the flask, i.e., it is only expressed to the nearest gram. Therefore, the final result can only be expressed to the nearest whole gram, i.e., total mass = 260. g.

Multiplication and Division

The result of any series of multiplications and divisions can only be expressed using the number of significant figures equal to that number with the least significant figures. For example: evaluate the following:

$$\frac{\left(2.7\times10^{-3}\right)\times377}{8.75}=116.33\times10^{-3}$$

When the data are examined, the following can be determined: 2.7×10^{-3} has only two significant figures and the other numbers have three significant figures. Therefore, the result can only be expressed to two significant figures which is 0.12.