NPV, IRR, and the Optimal Holding Period

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An interesting problem in investment analysis is the timing of a decision to sell an asset or, in other words, the optimal holding period. Consider an asset that produces no regular cash flows, so that its owner realizes a return only upon resale (perhaps trees in a forest, wine aging in kegs, or vacant land that may become attractive for development). Suppose further that the value of the asset will rise over time, but that this value will grow at a decreasing rate, and that at some point the value may stop growing (or even begin to decrease). Figure 1 depicts such an asset's value as it relates to the passage of time.

The owner of this asset would seek to identify the holding period that would provide for the greatest increase in wealth. To address this issue, we must identify the quantity that should be maximized. If it is the value of the asset that should be maximized, then the sale should take place at time t^*, as shown in Figure 1. The maximization of value, however, is too simplistic; such a decision rule ignores the *time value of money*. Time value is considered, though, in the two major investment criteria used by financial analysts: *net present value* (NPV) and *internal rate of return* (IRR). Yet while both techniques are based on time value, there are important differences between the NPV and IRR criteria.[1]

Figure 1

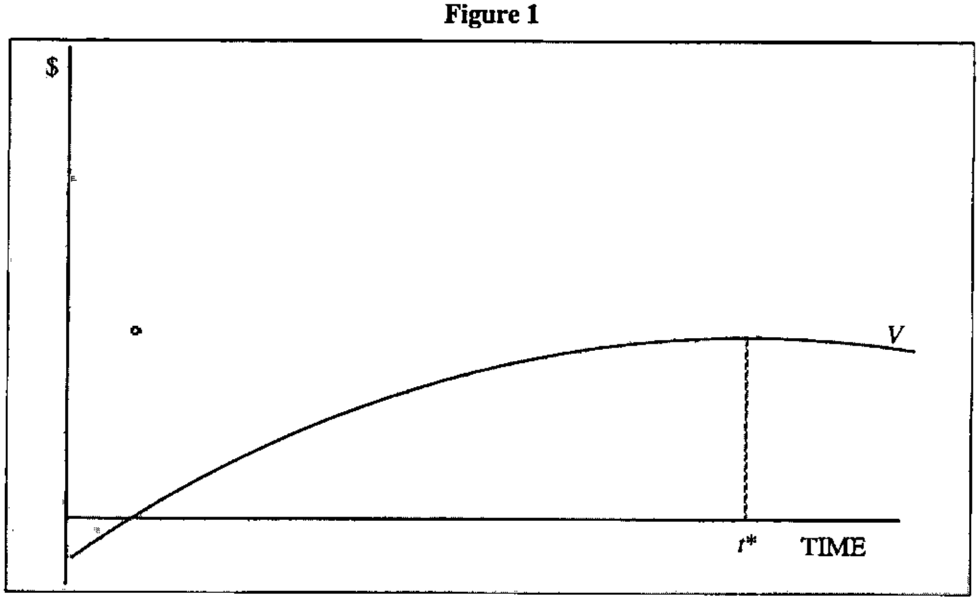
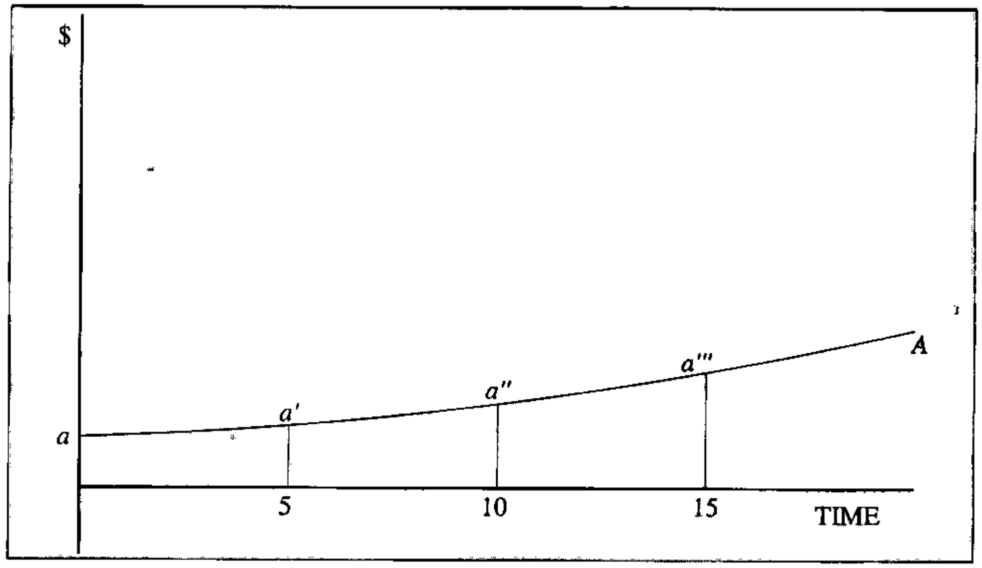


Figure 2



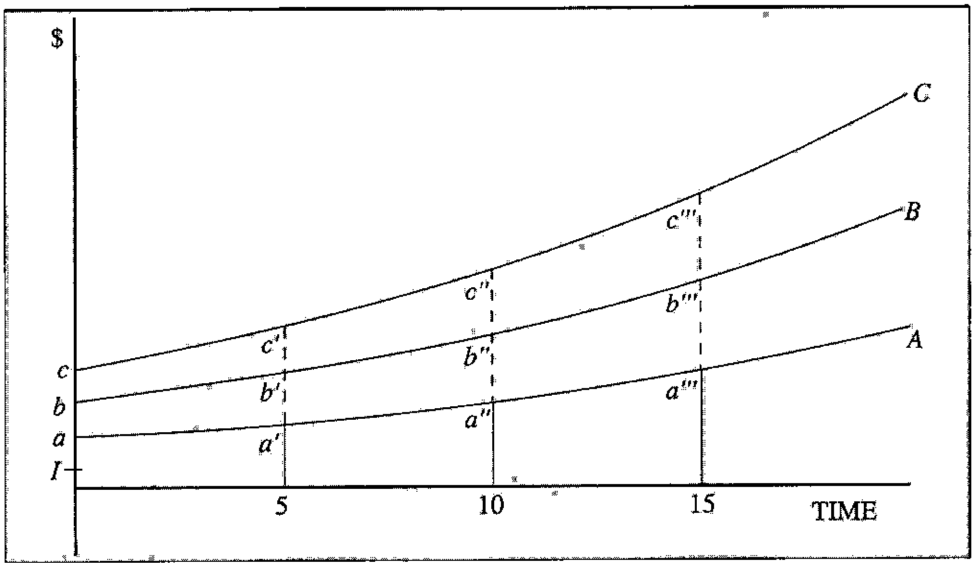
Present Value of Expected Inflows

The *present value* (PV) of a sum of money that an investor expects to receive in the future is the amount that the investor would willingly accept today instead of collecting the expected future payment. In other words, the present value and its corresponding future value provide the investor with equal satisfaction. If the rate at which we discount the future amount in computing the present value is positive (as is typically the case), then the present value is less than the future value. The present value and its corresponding future value, though different in stated dollar terms, offer equal satisfaction to the recipient because of the time value of money. Curve *A* in Figure 2 can be called an "equal-PV" curve, in that all points along the curve represent expected future amounts with equivalent present values. The present value of any point along curve *A* is equal to *a* (perhaps $14,000).

Assume, for example, that an investor's annual discount rate is 6%.[2] Point *a'* represents $18,735 that the investor expects to receive in 5 years, while point *a''* represents a $25,072 inflow to be received in 10 years and *a'''* represents an inflow of $33,552 in 15 years. Each of these possible future inflows has a present value of $14,000 if discounted at a 6% rate.

Figure 3 shows *A* along with two other equal-PV curves. Curves *A*, *B*, and *C* might be thought of as belonging to the same family, in that all are based on the same discount rate (6% in our example). Because the curves are based on the same discount rate, each demonstrates the same rate of increase over time. The difference from one curve to another lies only in the present value of the amount expected to be received later. Thus, along any curve the PV is constant, although a curve with a higher vertical intercept represents a

Figure 3



higher PV; *B* represents a higher PV than that displayed by *A*, while *C* indicates a higher PV than that shown by *B*. For example, curve *B* might show amounts with present values equal to $24,900 (such as the $33,322 corresponding to point *b'*; the $44,592 represented by point *b''*; or the point *b'''* figure of $59,674). *C* illustrates possible future inflows with present values equal to $35,000 ($46,838 corresponding to point *c'*; $62,680 at point *c''*; or the $83,880 point *c'''* figure).

Net Present Value

The *net* present value is the present value of the expected inflow *minus* some initial investment *I*; this difference provides a dollar measure of the investment's profitability. In Figure 3, any point along *C* represents a higher NPV than any point along *B* or *A*, because *I* (perhaps $5,300) is shown as an amount that does not vary with respect to the expected inflows. In this case, the project would have a computed NPV of $29,700 if the asset were expected ultimately to be generate an inflow as indicated along curve *C* ($35,000 – $5,300); $19,600 if the expected inflow were shown by a point along curve *B* ($24,900 – $5,300); or $8,700 if the inflow would be a point along curve *A* ($14,000 – $5,300). Of course, a higher PV of inflows does not necessarily correspond to a higher NPV if the associated initial investment is higher.

Internal Rate of Return

The IRR criterion is a percentage measure of profitability. Curve *D* in Figure 4 is an "equal-IRR" curve; all points along the curve exhibit equal internal rates of return. The IRR is the discount rate that equates the PV of a project's expected inflows to the initial outlay. In Figure 4, amounts *d'* and *d''* have the same internal rates of return; perhaps *d'* represents an expected inflow after 5 years of $14,921, while *d''* corresponds to an expected inflow of $42,008 at the end of year 10. In either case, the IRR is 23%; note that the PV of a year-5 inflow of $14,921, like the PV of a year-10 inflow of $42,008, is $5,300 (our assumed initial investment *I*) if the discount rate is 23%.

Figure 5 shows curve *D* along with two other equal-IRR curves. All three curves emanate from the same initial investment point, because each curve represents discounting to a present value equal to the initial cost of $5,300, but the three represent different internal rates of return on the asset. Because a lower value is associated with a higher discount rate, curve *F* represents a higher IRR than does curve *E*, while *E* represents a higher IRR than *D*. For example, curve *E* might show amounts that represent IRRs of 30% (such as the $19,679 represented by point *e'* or the $73,065 shown at *e''*), whereas *F* illustrates possible future prices that correspond to IRRs of 40% (such as $28,505 if received in 5 years, shown at point *f'*).

Optimal Time to Sell

A financial analyst might use either the NPV or the IRR criterion in determining the optimal time at which to sell an asset whose value has been growing. There is no intuitively clear method for choosing the better decision rule, although it should be intuitively obvious that, without a properly functioning rule, the investor could easily sell too early or too late. Our goal is to identify the precise time when we should sell an asset that has been increasing in value, and to identify what our choice of NPV or IRR has to do with this decision.

As Figure 6 indicates, exactly one equal-NPV curve and exactly one equal-IRR curve are *tangent* to (touching at only one point) the asset's value curve *V*. Each of these tangency points is critical to our

Figure 4

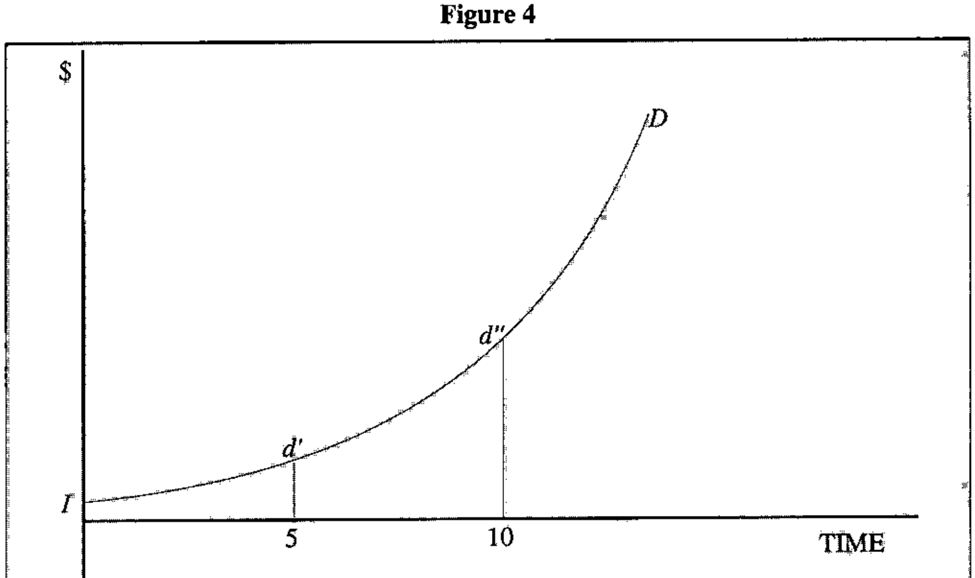
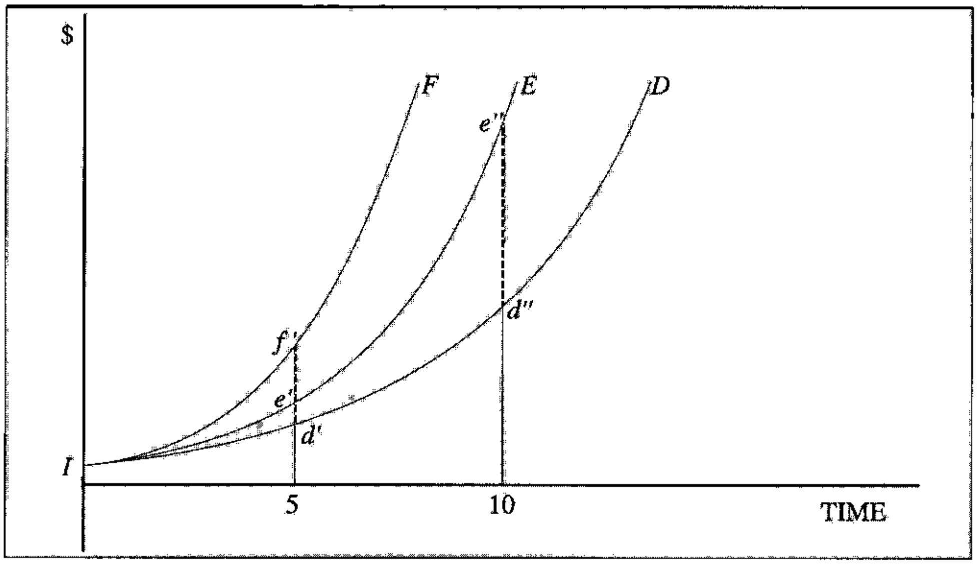


Figure 5



analysis, in that each point of tangency indicates the time in the asset's life when NPV or IRR, and not the asset's value, will have been maximized. The asset should be sold when the date corresponding to the maximum NPV or IRR is reached, because holding for any longer time fails to increase the asset's returns.

Do the two methods yield similar answers? For the NPV analysis, the relevant curve is B ($19,600 NPV in our example), which attains a tangency with value function V and therefore corresponds to the maximum amount that could be added to the investor's wealth. The appropriate selling date is t^*_{NPV}. (Note that we could not choose a date such that the value curve would touch C, with its $29,700 NPV.) We would, therefore, wish to sell long before the t^* date when the asset value is maximized (after which the trees begin to rot or the wine begins to sour, for example). The maximum attainable NPV is $[b - I]$. If the expected sale price is discounted back for t^*_{NPV} periods to a present value, and if this PV exceeds initial investment I, then NPV is positive; the investment contributes to the owner's wealth. For holding periods ending before t^*_{NPV} the potential contribution to wealth is still growing, whereas for longer periods it begins to decrease. For sale dates sufficiently distant in time from t^*_{NPV}, NPV becomes negative.

With regard to IRR analysis, the curve tangent to V is E, which corresponds in our example to a 30% IRR. No holding period that we could select would offer a higher IRR than would one ending at t^*_{IRR}; consider earlier t_1 or later t_2 selling dates, each of which corresponds to a 23% internal rate of return along curve D. Note also that the indicated holding period under IRR analysis is shorter than that suggested by the NPV criterion, ending at t^*_{IRR} rather than t^*_{NPV}.

We can draw an important inference regarding an investment in which we must wait for a specified interval before an inflow is realized: If the decision is based on NPV, and if the NPV is positive, then the chosen sale date should be later than that selected under the IRR criterion. The reverse is true if the highest attainable PV for the asset's expected inflows is less than the initial cost. If the highest attainable NPV is zero, then the optimal holding periods identified by the two techniques should be identical. ■

Footnotes

1. The concepts discussed in this article are not new ones. In fact, economists have studied present value and rate of return phenomena through "point-input, point-output" models for well over a century. UCLA economist Jack Hirshleifer reviewed the economics of present value and rate of return investment criteria in his now-classic book *Investment, Interest, and Capital* (Prentice Hall, 1970). The analysis offered in the paragraphs below follows the earlier work of Hirshleifer and others.

2. Numbers used for illustrative purposes are based on an assumption of annual compounding. The curves shown in the figure actually reflect continuous compounding.

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Figure 6