A Digression on Return *On* Investment and Return *Of* Investment

Trefzger, FIL 260 and 360

Someone who commits money expects to earn a percentage periodic rate of return *on* that investment. But it is also essential to receive a return *of* that investment. Consider the extremely simple case of lending $100 and getting $6 back a year later; did you get a 6% annual rate of return? No, to earn a 6% return you would need to get back $106 a year later: a $6 return *on* the investment plus the return, at the end of the year, *of* the $100 invested. Rate of return relates what you get back, on an average periodic basis, to what you initially give up; if you give up $100 and all you ever get back is $6 a year later it should be obvious that you have done poorly, certainly it is not a 6% annual rate of return (actually it is a – 94% annual rate of return, with $94 lost, over the course of a year, on the initial $100 invested).

Let's consider some more complicated examples. The first thing we have to remember is that rates of return or cost almost always are discussed in annual terms, so when an investment lasts for more than one year we typically discuss the compounded or geometric *average annual* return earned. (If payments and compounding occur more frequently than annually we compute based on, *e.g.*, quarterly or monthly figures, but then convert to an average annual figure to talk about.) So let's say you invest $100 today and then get back $6 at the end of each year for five years; think of it as a $100 loan that carries a 6% annual interest rate. If all you get back are the five $6 interest payments (and no principal) it should be clear that you did not get a 6% compounded average annual rate of return; the average annual return is negative because the amount received back is less than the total initially invested (give up $100, get back a total of $6 x 5 = $30 over multiple years). Because it occurs over numerous years, we compute the average annual return as the discount rate that causes the sum of the present values of the expected cash flows to be $0; that rate is the *internal rate of return* (IRR), which must be solved with trial and error if there are multiple expected nonzero cash flows after the initial investment is made:

$$\$0 = \frac{-\$100}{(1+r)^0} + \frac{\$6}{(1+r)^1} + \frac{\$6}{(1+r)^2} + \frac{\$6}{(1+r)^3} + \frac{\$6}{(1+r)^4} + \frac{\$6}{(1+r)^5}$$

The compounded (or geometric) average annual rate of return turns out to be – 30.19% per year (on the Texas Instruments BA II Plus calculator enter $100 +/– PV, $6 PMT, $0 FV, 5 N, CPT I/Y ⇒ screen goes blank for a couple of seconds while it computes with trial and error, and then shows – 30.1933). But if the year-five final payment includes a return *of* the $100 initially lent along with the fifth $6 interest payment, the IRR is computed as

$$\$0 = \frac{-\$100}{(1+r)^0} + \frac{\$6}{(1+r)^1} + \frac{\$6}{(1+r)^2} + \frac{\$6}{(1+r)^3} + \frac{\$6}{(1+r)^4} + \frac{\$106}{(1+r)^5}$$

Here enter $100 +/– PV, $6 PMT, $100 FV, 5 N, CPT I/Y ⇒ screen shows 6.0000, or 6%. (In the two simple cases above we could compute $100 +/– PV, $6 PMT, $0 FV, 1 N, CPT I/Y ⇒ screen shows – 94.0000 for the first; and $100 +/– PV, $6 PMT, $100 FV, 1 N, CPT I/Y ⇒ screen shows 6.0000, or 6% for the second – or since it is a one-period case could also enter as $100 +/– PV, $0 PMT, $106 FV, 1 N, CPT I/Y ⇒ screen shows 6.0000, or 6%. And since it all took place over one period the calculator does not need to do trial and error, thus the screen does not go blank, the answer shows immediately.) It should be intuitively clear that if you get a $6 return every year *on* the $100 invested, and also get a return *of* the $100 at the end of the project's life, then the average annual rate of return is the 6% stated annual interest rate. That is the way a bond with regular coupon interest payments usually works.

Now let's lend $100 and receive $6 in interest at the end of each year for 20 years; IRR is computed as

$$\$0 = \frac{-\$100}{(1+r)^0} + \frac{\$6}{(1+r)^1} + \frac{\$6}{(1+r)^2} + \frac{\$6}{(1+r)^3} + \cdots + \frac{\$6}{(1+r)^{19}} + \frac{\$6}{(1+r)^{20}}$$

Notice that the total amount received back, \$6 x 20 = \$120, exceeds the amount invested. When the amount received back is less than the amount given up the average annual rate of return is negative. Here the total received back is *more* than the amount given up, so the average annual rate of return is positive. But positive is not necessarily good enough; a wealth-enhancing investment's average periodic rate of return must be positive enough to exceed the average periodic cost of capital (or however we characterize the opportunity or *hurdle* rate). The calculator solution is \$100 +/– PV, \$6 PMT, \$0 FV, 20 N, CPT I/Y ⇒ screen shows 1.8030, or 1.803% average annual rate of return. Getting back progressively more relative to what was given up constitutes a progressively higher average annual rate of return; with 30 years of \$6 interest payments (but no separate return of the \$100 initially lent) the IRR is computed as

$$\$0 = \frac{-\$100}{(1+r)^0} + \frac{\$6}{(1+r)^1} + \frac{\$6}{(1+r)^2} + \frac{\$6}{(1+r)^3} + \cdots + \frac{\$6}{(1+r)^{29}} + \frac{\$6}{(1+r)^{30}}$$

(\$100 +/– PV, \$6 PMT, \$0 FV, 30 N, CPT I/Y ⇒ screen shows 4.3063, or 4.3063% average annual rate of return). But unless there is a separate full return *of* the investment, the IRR will be less than the rate suggested by the periodic payment divided by the initial investment (on a coupon bond investment we call that latter figure the current yield). So while getting back a greater number of \$6 annual interest payments after making a \$100 loan results in a higher IRR, it is if and only if the final annual payment includes a full return of the \$100 lent that the IRR (compounded average annual return) is exactly 6%:

\$100 +/– PV, \$6 PMT, \$0 FV, 40 N, CPT I/Y ⇒ 5.215
\$100 +/– PV, \$6 PMT, \$100 FV, 40 N, CPT I/Y ⇒ 6.000

\$100 +/– PV, \$6 PMT, \$0 FV, 60 N, CPT I/Y ⇒ 5.796
\$100 +/– PV, \$6 PMT, \$100 FV,60 N, CPT I/Y ⇒ 6.000

\$100 +/– PV, \$6 PMT, \$0 FV, 100 N, CPT I/Y ⇒ 5.982
\$100 +/– PV, \$6 PMT, \$100 FV, 100 N, CPT I/Y ⇒ 6.000

\$100 +/– PV, \$6 PMT, \$0 FV, 400 N, CPT I/Y ⇒ 5.999
\$100 +/– PV, \$6 PMT, \$100 FV, 400 N, CPT I/Y ⇒ 6.000

(A perpetuity's IRR actually does equal the stated periodic return – the series of unchanging payments is infinite, so it is as though a specific, separate return *of* the investment will be coming in just after the final periodic payment, but the date when a final payment is received never arrives.)

In each of the examples above any separate return *of* the investment came at the end of the project's life. Getting back all or part of the investment *before* the project ends, while a steady regular interest payment is received each year, increases the money provider's average annual return to something higher than the stated annual interest rate; note that if \$50 of the \$100 lent at a 6% annual interest rate is received at the end of year 2 and the remaining \$50 comes at the end of year 5 the average annual return is computed as

$$\$0 = \frac{-\$100}{(1+r)^0} + \frac{\$6}{(1+r)^1} + \frac{\$56}{(1+r)^2} + \frac{\$6}{(1+r)^3} + \frac{\$6}{(1+r)^4} + \frac{\$56}{(1+r)^5},$$

for 8.2859%. (With changing payments from period to period we can not use the five basic time value keys on the TI BA II Plus, but rather must to go into CF or cash flow mode: enter CF 2nd CLR WORK \$100 +/– ENTER ↓ \$6 ENTER ↓↓ \$56 ENTER ↓↓ \$6 ENTER ↓↓ \$6 ENTER ↓↓ \$56 ENTER ↓ IRR CPT; screen should go blank for a couple seconds of trial and error computing and then show 8.2859.)

It also is possible for the separate return of the investment to occur within regular periodic payments that are bigger than the interest payments (the structure of an *amortizing* loan). If you lend \$100 and receive payments of \$16.10 each year for eight years, the annual IRR on that investment is computed as

$$\$0 = \frac{-\$100}{(1+r)^0} + \frac{\$16.10}{(1+r)^1} + \frac{\$16.10}{(1+r)^2} + \frac{\$16.10}{(1+r)^3} + \frac{\$16.10}{(1+r)^4} + \frac{\$16.10}{(1+r)^5} + \frac{\$16.10}{(1+r)^6} + \frac{\$16.10}{(1+r)^7} + \frac{\$16.10}{(1+r)^8},$$

which can be written more conveniently as

$$\$0 = \frac{-\$100}{(1+r)^0} + \$16.10\left(\frac{1-\left(\frac{1}{(1+r)}\right)^8}{r}\right)$$

Solve as \$100 +/– PV, \$16.10 PMT, \$0 FV, 8 N, CPT I/Y ⇒ close to 6.000 or 6% (the payment more precisely has to be \$16.103594 for the answer to come out *exactly* as 6%). Here the investor gives up \$100 and gets back a total of \$16.10 x 8 = \$128.82, which is more than the initial investment, so we know that the periodic compounded rate of return is positive. And here that positive value is 6% per year. Return *on* investment and return *of* investment are built into the regular amortization payments.

In Topic 6 of FIL 260 we extract an overall capitalization rate by comparing selling price to one year of measured income for each of a group of income-producing properties that are similar to the subject property. That overall rate implicitly (but not explicitly) includes an appropriate periodic rate of return *on* the investment and provision for a return *of* the investment – if it did not, the investors involved in those observed transactions would not have been sufficiently compensated.