FIL 260: Topic 12, Mortgage Loan Mechanics – Extra Practice Problems With Solutions

Below are some extra practice problems of a type that are key to the student's knowledge of mortgage loan mechanics for doing well on the exam. Full computational steps are shown, but supporting explanations provided are brief; for more detailed explanations, and calculator solution steps for the various types of problems, see the solutions to the main Topic 12 problem set that you turn in for credit. [But remember that we discourage the use of automated calculator functions on the exam; those who understand and work with the formulas usually have the greatest success.] Your senior citizen instructor keeps all computational steps in calculator memory, so my answers should be free of any rounding influences. If you round (like computing a payment and then re-entering it rounded to whole cents in a multi-step problem) your final answer will be a bit different from the official answer below; do not be concerned about small rounding differences in those situations.

1. A borrower can afford to make a monthly principal-plus-interest payment of about $2,500. If a local credit union is willing to provide a fixed-interest rate, fixed-payment mortgage (FRM) loan at a 7.83% annual percentage rate (APR) of interest with equal end-of-month payments over 30 years, how much can the individual afford to borrow?

Recall that FRM Payment x PV of Annuity Factor = Total Borrowed

Here the number of monthly payment periods n to use in the PV of Annuity Factor is 30 x 12 = 360, and the monthly r to use is .0783 ÷ 12 = .006525, so with a monthly payment of $2,500 we have

$$\$2{,}500\left(\frac{1}{1.006525}\right)^{1} + \$2{,}500\left(\frac{1}{1.006525}\right)^{2} + \cdots + \$2{,}500\left(\frac{1}{1.006525}\right)^{360} = \text{TOT}$$

$$\$2{,}500\left[\left(\frac{1}{1.006525}\right)^{1} + \left(\frac{1}{1.006525}\right)^{2} + \cdots + \left(\frac{1}{1.006525}\right)^{359} + \left(\frac{1}{1.006525}\right)^{360}\right] = \text{TOT}$$

$$\text{Or, because } \left[\left(\frac{1}{1.006525}\right)^{1} + \left(\frac{1}{1.006525}\right)^{2} + \cdots + \left(\frac{1}{1.006525}\right)^{360}\right] = \left(\frac{1-\left(\frac{1}{1.006525}\right)^{360}}{.006525}\right):$$

$$\text{PMT x FAC = TOT}$$

$$\$2{,}500\left(\frac{1-\left(\frac{1}{1.006525}\right)^{360}}{.006525}\right) = \text{TOT}$$

$2,500 x 138.514053 = $346,285.13, or approximately $346,000 can be borrowed

2. What is the monthly payment on a $392,000 fixed-rate, fixed-payment (FRM) home mortgage loan with a 6.54% stated annual percentage rate (APR) of interest, if unchanging payments are to be made at the end of each month for 20 years? Based on this loan's interest rate and number of payment periods, show that a loan payment factor is the sum of the interest rate and sinking fund factor.

Here the number of monthly payment periods n to use in the PV of Annuity Factor is 20 x 12 = 240, and the monthly r to use is .0654 ÷ 12 = .00545, so with $392,000 in principal borrowed we have

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.00545}\right)^{240}}{.00545}\right) = \$392{,}000$$

PMT x 133.702367 = $392,000

So PMT = $392,000 ÷ 133.702367 = $2,931.89 OR

$$\$392{,}000\left(\frac{.00545}{1-\left(\frac{1}{1.00545}\right)^{240}}\right) = \$392{,}000 \times .007479 = \text{PMT} = \$2{,}931.89$$

The loan payment factor for monthly payments, here .007479, is the sum of the monthly interest rate and the monthly *sinking fund factor* (reciprocal of FV of a level ordinary annuity factor for the same periodic interest rate and number of time periods), computed here as

$$\left[1/\left(\frac{(1.00545)^{240}-1}{.00545}\right)\right] = \left(\frac{.00545}{(1.00545)^{240}-1}\right) = \underline{.002029}$$

So adding the .00545 monthly interest rate (which accounts for paying interest on the loan) to the .002029 monthly sinking fund factor (which accounts for repaying principal) gives us the .00545 + .002029 = .007479 monthly loan payment factor.

3. What is the monthly payment on a $225,000 fixed-rate, fixed-payment (FRM) home mortgage loan with an interest rate presented as a 5.6618% EAR (effective annual rate), if payments are to be made at the end of each month for 25 years? (Under U.S. federal law the interest rate a lender quotes to a home mortgage loan borrower actually must be an APR-related measure, but for this question we discuss the annual rate as an EAR, the more accurate measure for comparing percentage costs or returns for different financial instruments in annual terms.)

Here the number of monthly payment periods n to use in the PV of Annuity Factor is 25 x 12 = 300, and the monthly r to use when we are given the more complex, compounded EAR as the annual rate measure is $\sqrt[12]{1.056618}$ – 1 = .0046, so with $225,000 in principal borrowed we have

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0046}\right)^{300}}{.0046}\right) = \$225{,}000$$

$$\text{PMT} \times 162.526982 = \$225{,}000$$

$$\text{So PMT} = \$225{,}000 \div 162.526982 = \$1{,}384.39 \quad \text{OR}$$

$$\$225{,}000\left(\frac{.0046}{1-\left(\frac{1}{1.0046}\right)^{300}}\right) = \$225{,}000 \times .006153 = \text{PMT} = \$\underline{\underline{1{,}384.39}}$$

4. A borrower obtains a $310,000 fixed-rate, fixed-payment mortgage loan (FRM) with a 4.68% stated annual percentage rate (APR) of interest and equal end-of-month payments to be made over 30 years. Compute the:
a) unchanging monthly payment, b) amount of interest scheduled to be paid in month 1, c) amount of interest scheduled to be paid in month 2, d) total amount of interest scheduled to be paid in year 1, e) total amount of interest scheduled to be paid over the first six years, and f) total amount of interest scheduled to be paid over years 18 – 23.
[*On an exam question of this type you must show enough steps to assure the grader that you understand the ideas; hitting the BA II Plus AMORT key and ↓ ↓ ↓ shows no understanding of how interest and principal are computed.*]

a) With a 4.68% stated annual percentage rate (APR) of interest and monthly payments, our monthly periodic interest rate r is .0468 ÷ 12 = .0039. Over 30 years we have 30 x 12 = 360 scheduled monthly payment periods. With $310,000 borrowed each scheduled monthly payment is

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0039}\right)^{360}}{.0039}\right) = \$310{,}000$$

$$\text{PMT} \times 193.260472 = \$310{,}000$$

$$\text{So PMT} = \$310{,}000 \div 193.260472 = \$1{,}604.05 \quad \text{OR}$$

$$\$310{,}000 \times .005174 = \text{PMT} = \$\underline{\underline{1{,}604.05}}$$

b) Interest paid in any period is just the periodic (here monthly) rate times the amount of principal owed at the start of the period (working through a month-by-month amortization schedule helps reinforce this idea). Coming into month 1 the entire $310,000 in principal is owed, so interest is very simple to compute, nothing more than .0039 x $310,000 = $<u>1,209</u>.

c) Interest for month 2 is a bit more complicated, but not too scary. First, note that month 1's $1,604.05 payment consists partly of $1,209 in interest, with the remaining $1,604.05 - $1,209 = $395.05 paying down principal. If $395.05 in principal is repaid in month 1, then we end month 1/ enter month 2 owing $310,000 - $395.05 = $309,604.95. So the interest portion of the month 2 payment is just .0039 x $309,604.95 = $<u>1,207.46</u>, slightly lower than the interest component of the month 1 payment (and principal repaid in month 2 is $1,604.05 - $1,207.46 = $396.59, slightly higher than the magnitude of principal repaid in month 1).

d) Always remember the hugely important lesson that the principal that remains owed, at any point during a long-term loan's life, is the PV of the stream of remaining scheduled payments, discounted at the original loan contract's periodic interest rate. Here at the end of year 1, with 29 years (348 months) of payments left on the original schedule, the PV of the remaining payment stream is

PMT x FAC = TOT

$$\$1{,}604.05\left(\frac{1-\left(\frac{1}{1.0039}\right)^{348}}{.0039}\right) = \$1{,}604.05 \times 190.240838 = \$\underline{305{,}156.35}$$

That part of the original $310,000 principal is still unpaid, so $310,000 – $305,156.35 = $<u>4,843.65</u> of principal has been repaid. Everything else paid during the year has been interest. Therefore

Total payments during year 1: 12 x $1,604.05	$19,248.63
Minus principal portion	<u>4,843.65</u>
Equals interest paid during year 1	$<u>14,404.98</u>

e) Now we have to figure out how much principal is repaid over years 1 to 6. At the end of year 6, with 24 years (288 months) of payments still scheduled, the remaining payment stream's PV is

PMT x FAC = TOT

$$\$1{,}604.05\left(\frac{1-\left(\frac{1}{1.0039}\right)^{288}}{.0039}\right) = \$1{,}604.05 \times 172.833681 = \$\underline{277{,}234.35}$$

That part of the original $310,000 principal is still unpaid, so $310,000 – $277,234.35 = $<u>32,765.65</u> of principal has been repaid during those six years, with everything else paid representing interest. So here

Total payments during years 1 – 6: 72 x $1,604.05	$115,491.80
Minus principal portion	<u>32,765.65</u>
Equals interest paid during first 3 years	$ <u>82,726.15</u>

f) In earlier steps the principal owed at the start of the indicated period was just the $310,000 borrowed. But in this final step we have to compute how much is owed at the beginning of year 18. Coming into year 18 there have been 17 years of payments completed, with 13 years (156 months) of payments still scheduled, so the remaining payment stream's PV is

$$\text{PMT} \times \text{FAC} = \text{TOT}$$

$$\$1{,}604.05\left(\frac{1-\left(\frac{1}{1.0039}\right)^{156}}{.0039}\right) = \$1{,}604.05 \times 116.701209 = \$\underline{187{,}194.90}$$

Then six years later, at the end of year 23, with 23 years of payments completed and 7 years (84 months) of payments still on the schedule, the PV of the stream of remaining payments is

$$\$1{,}604.05\left(\frac{1-\left(\frac{1}{1.0039}\right)^{84}}{.0039}\right) = \$1{,}604.05 \times 71.510117 = \$\underline{114{,}706.00}$$

So the amount of principal included within the total of the year 18 to 23 payments is \$187,194.90 - \$114,706.00 = \$<u>72,488.90</u>. Therefore we have

Total payments during years 18 – 23: 72 x \$1,604.05	\$115,491.80
Minus principal portion	72,488.90
Equals interest paid during years 18 - 23	\$ 43,002.90

The total payments for this fixed-rate, fixed-payment FRM loan are the same \$115,491.80 for the six-year interval of years 1-6 and the six-year interval of years 18-23. However, the year 18-23 period comes much later in the loan's life, when a lot of principal already has been repaid, such that the interest part of year 18-23's \$115,491.80 is considerably smaller (\$43,002.90 vs. \$82,726.15) and the principal portion much bigger (\$72,488.90 vs. \$32,765.65) than seen with the \$115,491.80 paid over years 1-6.

5. A borrower got a fixed-interest rate, fixed-payment mortgage loan (FRM) 11 years ago. The agreement called for equal end-of-month payments for 30 years, and the interest rate was quoted as a 7.56% annual percentage rate (APR). Payments have been made according to the original amortization schedule (there have been no early repayments of principal), and the amount still owed is \$188,641.35. What is the unchanging monthly payment, and how much principal was originally borrowed?

This problem is pretty straightforward. Again the key is remembering that principal owed on a loan at any time is the PV of the stream of remaining scheduled payments. With 11 years of payments behind us and 19 years ahead the number of remaining monthly payments n is 19 x 12 = 228, and with a 7.56% APR the monthly r is .0756 ÷ 12 = .0063. So the monthly payment can be computed as

$$\text{PMT} \times \text{FAC} = \text{TOT}$$

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0063}\right)^{228}}{.0063}\right) = \$188{,}641.35$$

$$\text{PMT} \times 120.816502 = \$188{,}641.35$$

$$\text{So PMT} = \$188{,}641.35 \div 120.816502 = \$1{,}561.39 \text{ OR}$$

$$\$188{,}641.35 \times .008277 = \text{PMT} = \$\underline{1{,}561.39}$$

Now that we know the monthly payment, and already knew the 30 year = 360-month original amortization period, we can compute the amount initially borrowed as

$$\text{PMT} \times \text{FAC} = \text{TOT}$$

$$\$1{,}561.39 \left(\frac{1-\left(\frac{1}{1.0063}\right)^{360}}{.0063} \right) = \$1{,}561.39 \times 142.181253 = \$\underline{\underline{222{,}000}}$$

(When the loan was obtained 11 years ago, the monthly payment would have been computed as

$$\text{PMT} \left(\frac{1-\left(\frac{1}{1.0063}\right)^{360}}{.0063} \right) = \$222{,}000$$

$$\text{PMT} \times 142.181253 = \$222{,}000$$

$$\text{So PMT} = \$222{,}000 \div 142.181253 = \$1{,}561.39 \text{ OR}$$

$$\$222{,}000 \left(\frac{.0063}{1-\left(\frac{1}{1.0063}\right)^{360}} \right) = \$222{,}000 \times .007033 = \text{PMT} = \$1{,}561.39)$$

6. Think of a $440,000 fixed-interest rate, fixed-payment (FRM) home mortgage loan with an 8.1% APR interest rate and payments to be made at the end of each month for 25 years. Can we compute the unchanging monthly payment by just computing as though the payments were to be made annually, and then dividing that corresponding annual payment by 12?

The answer is no. The reason is that if a loan has annual payments interest is applied to the full original principal for an entire year before a payment is made that reduces the principal owed, whereas with monthly payments some principal is repaid after just one month. So a monthly payment loan has less principal owed on average, and thus less interest payable in total, relative to a corresponding loan with annual payments. Annual payments on the loan described here would be

$$\text{PMT} \left(\frac{1-\left(\frac{1}{1.081}\right)^{25}}{.081} \right) = \$440{,}000$$

$$\text{PMT} \times 10.584220 = \$440{,}000$$

$$\text{So PMT} = \$440{,}000 \div 10.584220 = \$41{,}571.32 \text{ OR}$$

$$\$440{,}000 \times .094480 = \text{PMT} = \$\underline{41{,}571.32,}$$

and 1⁄12 of that amount is $41,571.32 ÷ 12 = $<u>3,464.28</u>. But the correctly computed monthly payment, with n of 25 x 12 = 300 and r of .081 ÷ 12 = .00675, is a smaller

$$\text{PMT} \left(\frac{1-\left(\frac{1}{1.00675}\right)^{300}}{.00675} \right) = \$440{,}000$$

$$\text{PMT} \times 128.460006 = \$440{,}000$$

$$\text{So PMT} = \$440{,}000 \div 128.460006 = \$3{,}425.19 \text{ OR}$$

$$\$440{,}000 \times .007785 = \text{PMT} = \$\underline{3{,}425.19}$$

(so the approximation of dividing the payment computed annually by 12 is a slight overestimate).

7. Commonly observed applications of the "two-step" hybrid mortgage loan are "5/25" and "7/23" (same interest rate and monthly payment for the first five or seven years of a 30-year amortization period, followed by year-to-year changes based on each year's new market interest rate that relates to changes in a verifiable interest rate measure). Think of a borrower getting a "10/20" two-step loan, with an unchanging interest rate and end-of-month payments for the first 10 years, and then annual interest rate changes for however many years there is principal that remains unpaid (up to the remaining 20 years of a 30-year amortization plan). The amount borrowed is $174,000, the initial interest rate is a 6.6% APR, and we predict that the APR will be 7.2% in year 11 and 7.8% in year 12. What is the unchanging monthly payment for all of years 1 to 10, and what are our predicted payments for years 11 and 12?

Step 1: the initial payment, for years 1 – 10, is computed the same way we compute the unchanging payment for a standard fixed-rate, fixed-payment mortgage (FRM) loan. Here the monthly periodic rate r for years 1 – 10 is .066 ÷ 12 = .0055, and we compute:

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0055}\right)^{360}}{.0055}\right) = \$174,000$$

$$\text{PMT} \times 156.578125 = \$174,000$$

So $\text{PMT} = \$174,000 \div 156.578125 = \$1,111.27$ OR

$$\$174,000 \times .006387 = \text{PMT} = \$\underline{\underline{1,111.27}}$$

Step 2: compute the year 11 payment, we first must find principal still owed at the end of year 10 (after 120 months, when 360 – 120 = 240 months of scheduled payments remain):

$$\$1,111.27\left(\frac{1-\left(\frac{1}{1.0055}\right)^{240}}{.0055}\right) = \$1,111.27\ (133.072143) = \$147,878.59\ (\text{"goodbye, old loan"})$$

With a new interest rate of 7.2% APR (for a .072 ÷ 12 = .006 monthly periodic rate r), the year 11 payment is computed as

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.006}\right)^{240}}{.006}\right) = \$147,878.59\ (\text{"hello, new loan"})$$

$$\text{PMT} \times 127.008432 = \$147,878.59$$

So $\text{PMT} = \$147,878.59 \div 127.008432 = \$1,164.32$ OR

$$\$147,878.59 \times .007873 = \text{PMT} = \$\underline{\underline{1,164.32}}$$

(the interest rate rose only a little, so the payment went up just a bit). That's the whole story; now we know the new monthly payment. With the two-step hybrid (loan with both fixed-rate and variable-rate features) the interest rate reset date is determined when the loan is originated; it does not occur at the borrower's discretion (though the borrower is permitted to prepay remaining principal owed at any time). And the new interest rate also is determined at origination, in that it will be some number of basis points above a verifiable index like the 10-year U.S. federal T-Bond rate, and thus can be higher or lower than the initial rate, depending on how the index has changed.

Then to compute the year 12 payment, we again must compute principal still owed at the end of year 11 (after 132 months of payments, when 360 – 132 = 228 months of scheduled payments remain):

$$\$1,164.32\left(\frac{1-\left(\frac{1}{1.006}\right)^{228}}{.006}\right) = \$1,164.32\ (124.056901) = \$144,442.06\ (\text{"goodbye, old loan"})$$

With 7.8% APR as the new interest rate (for a .078 ÷ 12 = .0065 monthly periodic rate r), year 12's payment is

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0065}\right)^{228}}{.0065}\right) = \$144{,}442.06 \text{ ("hello, new loan")}$$

PMT x 118.726924 = $144,442.06

So PMT = $144,442.06 ÷ 118.726924 = $1,164.32 OR

$144,442.06 x .008423 = PMT = $1,216.59

8. Forty-two months ago Reginald Redbird got a fixed-rate, fixed-payment home mortgage (FRM) loan, borrowing $258,000 with an 8.7% annual percentage rate (APR) interest rate and equal end-of-month payments to be made for 30 years. Now interest rates are lower, and today he could refinance to a new loan with a 318-month amortization and a 7.5% APR interest rate (the new loan's principal would be just the balance still owed on the initial loan, and he would expect to make level payments over the next 318 months, the remainder of the original 360-month loan amortization plan, regardless of whether he keeps the current loan or refinances). The cost of refinancing to get a lower interest rate loan (origination fees for a new loan, opportunity cost of time spent searching the market) would be approximately $11,250. Compute the net present value (NPV) of refinancing, using a low-risk discount rate of 3.9% APR to determine the present value of the stream of reduced monthly payments.

Step 1: compute the initial/current monthly loan payment, based on an n of the original 360-month amortization plan and monthly periodic interest rate r of .087 ÷ 12 = .00725:

PMT x FAC = TOT

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.00725}\right)^{360}}{.00725}\right) = \$258{,}000$$

PMT x 127.692387 = $258,000

So PMT = $258,000 ÷ 127.692387 = $2,020.48 OR

$258,000 x .007831 = PMT = $2,020.48

Step 2: compute the monthly payment that would be made on a new loan, based on a 318-month life and .075 ÷ 12 = .00625 monthly periodic interest rate. Principal still owed 42 months into the original loan's 360-month life (with 360 – 42 = 318 months left) is

$$\$2{,}020.48\left(\frac{1-\left(\frac{1}{1.00725}\right)^{318}}{.00725}\right) = \$2{,}020.48 \times 124.063214 = \$250{,}667.33 \text{ ("goodbye, old loan")}$$

[Recall that on the day he got the loan he owed $\$2{,}020.48\left(\frac{1-\left(\frac{1}{1.00725}\right)^{360}}{.00725}\right) = \$258{,}000$.]

The monthly payment on a new $250,667.33, 318-month n, .00625 monthly periodic rate r loan would be

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.00625}\right)^{318}}{.00625}\right) = \$250{,}667.33 \text{ ("hello, new loan")}$$

PMT x 137.937949 = $250,667.33

So PMT = $250,667.33 ÷ 137.937949 = $1,817.25 OR

$250,667.33 x .007250 = PMT = $1,817.25

Notice that these first two steps are structurally identical to the first two steps in computing the first and second level payments on the two-step hybrid loan, as in the previous problem. But there are some important differences in the underlying stories. A decision on whether and when to refinance is made entirely at the borrower's discretion, rather than being set from the beginning to occur after five/seven/ten or some other number of years. And a refinancer's new interest rate is not determined with reference to a verifiable index, but rather is simply a function of financial market conditions at the decision date; the availability of a lower interest rate might prompt a borrower to consider refinancing (though refinancing could occur even after market interest rates have risen, perhaps a "cash out" refinance in which more is borrowed than remains owed on the original loan). Also, whereas after the second computational step with the two-step loan we say well ok, that's the new payment, the refinancing decision necessitates some further analysis, as follows:

Step 3: by refinancing after 42 months with a new 318-month loan, Reginald would realize a monthly payment reduction of $2,020.48 – $1,817.25 = $<u>203.23</u>, thereby saving $203.23 monthly for an expected 318 months. Using a low-risk monthly periodic reinvestment or opportunity rate r of .039 ÷ 12 = .00325, we compute the present value of this savings stream (the PV of an annuity) as

$$\$203.23\left(\frac{1-\left(\frac{1}{1.00325}\right)^{318}}{.00325}\right) = \text{TOT}$$

$$\$203.23 \times 198.044214 = \$\underline{40{,}249.25}$$

Final step 4: The roughly $11,250 cost to Reginald of getting the new loan if he refinances is already a present value figure, since the search costs and new loan application fees would be borne today (or over a brief period around the present time).

So refinancing has a net present value of PV Savings Stream minus PV Costs of Refinancing = $40,249.25 – $11,250 = $<u>28,999.25</u>. NPV is a very high positive value, so <u>refinancing makes economic sense</u> (locking into the lower payments going forward creates almost $29,000 in wealth today for borrower Reginald – his option to refinance the loan is in-the-money). Here the NPV of refinancing is high because the decline in interest rates has been fairly substantial (120 basis points, more than a full percentage), the amount of principal that remains owed is large (so the monthly payments drop by quite a bit), and the borrower expects to benefit from the lower monthly payments over a lengthy 318-month span.

9. This final question is not like one that would directly appear on an exam, but the computations involved should help reinforce our overall understanding of mortgage loan mechanics. When a loan is "recast" (as discussed in Topic 11), the borrower reduces subsequent monthly payments by giving the lender a lump sum of principal. The interest rate and remaining amortization period remain unchanged; the monthly payment is simply recomputed or recast based on the smaller remaining principal owed. Not all lenders offer the recasting option, but those that do tend not to charge high fees because paper work with outside parties is minimal (no new appraisal or title work is required, unlike when a loan is refinanced), and the lender actually faces less default risk after a recasting because less principal remains owed. Of course, if current market interest rates are below the loan's contract rate the lender has to re-lend that lump sum of principal received at a lower rate, so the lender likely charges at least something for making the change. And if rates have fallen the borrower might be more likely to refinance the loan, to gain wealth through a positive NPV, rather than recasting. But it also could be that rates have fallen yet the borrower does not want to do a full refinance, instead saying: "we are better off owing $10,000 less on a loan that costs 7% per year than earning 3% per year on that $10,000 in a savings account" (a small positive NPV from recasting would likely result). A borrower facing an opportunity rate above the loan's contract rate might consider recasting for budgeting/ disciplinary reasons even though the NPV of doing so would be negative; think: "we just inherited $10,000 from late Aunt Matilda, and to prevent ourselves from frittering it away let's pay down some principal on our mortgage loan."

a. Six years and three months (75 months) ago Mortgagor obtained a $233,000 FRM loan with a 7.32% APR interest rate and equal payments to be made at the end of each month for 25 years. Today she gets an unexpected $11,000 profit-sharing bonus at work, which leaves her with $8,000 after federal and state income taxes. What will the reduction in her monthly payment be if she recasts the loan today by paying down $8,000 in owed principal?

Step 1: compute the initial/current monthly loan payment, based on an n of the original 300-month amortization plan and monthly periodic interest rate r of .0732 ÷ 12 = .0061:

$$\text{PMT} \times \text{FAC} = \text{TOT}$$

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0061}\right)^{300}}{.0061}\right) = \$233{,}000$$

$$\text{PMT} \times 137.490529 = \$233{,}000$$

$$\text{So PMT} = \$233{,}000 \div 137.490529 = \$1{,}694.66 \quad \text{OR}$$

$$\$233{,}000 \times .007273 = \text{PMT} = \$\underline{1{,}694.66}$$

Step 2: compute the monthly payment that would be made on a new loan of the remaining principal owed minus $8,000, based on an unchanged .0732 ÷ 12 = .0061 monthly interest rate and 300 – 75 = 225-month life. Principal still owed 75 months into the original 300-month amortization period is

$$\$1{,}694.66\left(\frac{1-\left(\frac{1}{1.0061}\right)^{225}}{.0061}\right) = \$1{,}694.66 \times 122.207957 = \$\underline{207{,}101.20} \text{ (“goodbye, old loan”)}$$

Then when $8,000 is prepaid (home mortgage borrowers generally can repay large or small amounts of principal at any time, with no direct prepayment penalty) the new loan is based on principal owed of $207,101.20 – $8,000 = $<u>199,101.20</u>. The monthly payment on the recast loan (no change in interest rate or periods remaining until maturity, just lower remaining principal balance) would be

$$\text{PMT}\left(\frac{1-\left(\frac{1}{1.0061}\right)^{225}}{.0061}\right) = \$199{,}101.20 \text{ (“hello, new loan”)}$$

$$\text{PMT} \times 122.207957 = \$199{,}101.20$$

$$\text{So PMT} = \$199{,}101.20 \div 122.207957 = \$1{,}629.20 \quad \text{OR}$$

$$\$199{,}101.20 \times .008103 = \text{PMT} = \$\underline{1{,}629.20}$$

So by recasting the loan the borrower reduces her monthly payment for the 225 remaining months of the loan amortization period by $1,694.66 – $1,629.20 = $<u>65.46</u>.

b. What would be the implications of recasting this loan if the lender charges $300 in fees, and the opportunity rate (what the borrower could earn by investing available money in a low-risk savings instrument) is a 3.6% APR?

If the borrower's monthly opportunity rate is .036 ÷ 12 = .003, the PV of the stream of reduced monthly payments is

$$\$65.46\left(\frac{1-\left(\frac{1}{1.003}\right)^{225}}{.003}\right) = \text{TOT}$$

$$\$65.46 \times 163.442942 = \$\underline{10{,}699.33}$$

To get an assured series of 225 end-of-month inflows of $65.46 each, the borrower would need to have $10,699.33 in a low-risk savings account today (we have to look at a low-risk opportunity or alternative plan, because the loan payments are essentially risk-free; we know they will be due every month.) Or just consider that the borrower could take the $65.46 monthly payment savings and, without crossing risk classes, deposit it into a savings plan that pays a 3.6% APR rate or .3% per month. By the end of month 225 the balance would grow to a future value of

$$\$65.46 \left(\frac{(1.003)^{225}-1}{.003}\right) = \$65.46 \times 320.683121 = \$20{,}992.62$$

and the present value of that amount is

$$\$20{,}992.62 \left(\frac{1}{1.003}\right)^{225} = \$20{,}992.62 \times .509671 = \$10{,}699.33$$

(a future value of an annuity's present value is the present value of an annuity based on the same periodic discount rate and number of time periods:

$$\left(\frac{(1+r)^n-1}{r}\right)\left(\frac{1}{1+r}\right)^n = \left(\frac{((1+r)^n-1)\left(\frac{1}{1+r}\right)^n}{r}\right) = \left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right)).$$

To obtain that $10,699.33 benefit the borrower gives up $8,000 to reduce the loan balance plus $300 in fees = $8,300 today, for a net present value of recasting the loan of $10,699.33 - $8,300 = $2,399.33. Mortgagor actually gains a slight amount of wealth by recasting.

c. What would the implications of recasting be if interest rates in the marketplace have risen considerably since the loan was originated, and the rate that could be earned on a savings account is a 6.9% APR?

If the borrower's monthly opportunity rate is .069 ÷ 12 = .00575, the present value of the stream of reduced monthly payments is

$$\$65.46 \left(\frac{1-\left(\frac{1}{1.00575}\right)^{225}}{.00575}\right) = \text{TOT}$$

$$\$65.46 \times 126.042002 = \$\underline{8{,}250.98}$$

To get an assured series of 225 end-of-month inflows of $65.46 each, the borrower would need to have only $8,250.98 in a low-risk savings account today, with higher interest rates available on savings. To obtain that $8,250.98 benefit the borrower gives up $8,300 today, for a recasting net present value of $8,250.98 - $8,300 = - $49.02. By this measure Mortgagor loses a slight amount of wealth by recasting when the $8,000 profit sharing windfall could earn a high rate of return in another low-risk application.

However, perhaps Mortgagor loses sleep worrying about losing her home to foreclosure through an inability to make mortgage loan payments, and gains peace of mind through reducing the required monthly payment even by a small amount. We could say she is paying something to achieve that added peace of mind. Alternatively we might say that her personal opportunity rate is lower than the 6.9% she could earn on a bank account, leading to a positive imputed NPV. That idea could be consistent with Mortgagor valuing the assured savings aspect of recasting (no worry that she will spend the $8,000 on something else on a whim), and also might reflect her view that interest paid on savings accounts (which are fairly short-term by nature) will be declining in the years ahead. •