Topic 18: Written Problem Set – With Solutions

A separate file has these problems without the detailed solutions. Handwriting need not be the neatest, and you do not have to provide comments, but the computational steps should be fairly complete -- you are earning credit for working the problems carefully so that you will be better prepared for the exam. Points will be taken off for cutting corners. In a few spots you can abbreviate on some of the computational steps (like using $+\cdots+$ or the summation sign when many similar terms are added together), but we want you to show that you understand the relationships – such as that the PV of an annuity factor is just the sum of the so-called PV of $1 factors for the same discount rate and same number of time periods. Our FIL 260 written homework assignments generally are not financial calculator exercises, although it is fine to solve for IRR with the calculator's function keys if you also show the equation the calculator's trial-and-error computations are based on.

Please submit a scanned or photographed copy of your carefully hand-written solutions to these problems. (If you take photographs of individual pages with your phone or other device, please paste all of the pictures into a single Word file in the correct order before submitting. DO NOT submit multiple individual photographs; that kind of submission can be very difficult for the instructor to grade.) Upload your submission to Canvas if you can; attach it to an e-mail to the instructor if you can not. Due date and time are shown with the assignment information on Canvas.

1. A real estate investor buys an income-producing property for $700,000 ($490,000 in debt and $210,000 in equity). The investment is expected to generate after-tax cash flows (ATCF's) to the equity position for six years as follows: $27,000 in year 1; $29,000 in year 2; $31,000 in year 3; $33,000 in year 4; $35,000 in year 5; and $37,000 in year 6. At the end of year 6 the equity investor expects to sell the property for $750,000. After paying off the remaining loan balance and paying all transaction-related fees (including all applicable taxes), the equity investor expects to net $190,000 from the property sale (the after-tax equity reversion, or ATER).

a. What is the net present value (NPV) of this investment if the equity investor requires a 9% annual rate of return?
b. Set up the equation that the equity investor would use in computing her internal rate of return (IRR). [It is not necessary to solve, but you should at least set up the equation.]

a. $$-\$210{,}000\left(\frac{1}{1.09}\right)^0+\$27{,}000\left(\frac{1}{1.09}\right)^1+\$29{,}000\left(\frac{1}{1.09}\right)^2+\$31{,}000\left(\frac{1}{1.09}\right)^3+\$33{,}000\left(\frac{1}{1.09}\right)^4$$

$$+\$35{,}000\left(\frac{1}{1.09}\right)^5+(\$37{,}000+\$190{,}000)\left(\frac{1}{1.09}\right)^6=\text{NPV}\qquad\text{OR}$$

$$\frac{-\$210{,}000}{(1.09)^0}+\frac{\$27{,}000}{(1.09)^1}+\frac{\$29{,}000}{(1.09)^2}+\frac{\$31{,}000}{(1.09)^3}+\frac{\$33{,}000}{(1.09)^4}+\frac{\$35{,}000}{(1.09)^5}+\frac{\$37{,}000+\$190{,}000}{(1.09)^6}=\text{NPV}\qquad\text{OR}$$

$$\frac{-\$210{,}000}{(1.09)^0}+\frac{\$27{,}000}{(1.09)^1}+\frac{\$29{,}000}{(1.09)^2}+\frac{\$31{,}000}{(1.09)^3}+\frac{\$33{,}000}{(1.09)^4}+\frac{\$35{,}000}{(1.09)^5}+\frac{\$37{,}000}{(1.09)^6}+\frac{\$190{,}000}{(1.09)^6}=\text{NPV}$$

– $210,000 + $24,770.64 + $24,408.72 + $23,937.69 + $23,378.03 + $22,747.60 + $22,061.89 + $113,290.79 =

– $210,000 + $254,595.36 = $<u>44,595.36</u> NPV

(because expected ATCFs are unequal we can not use the PV of Annuity factor to group them in computing).

b. $$\frac{-\$210{,}000}{(1+r)^0}+\frac{\$27{,}000}{(1+r)^1}+\frac{\$29{,}000}{(1+r)^2}+\frac{\$31{,}000}{(1+r)^3}+\frac{\$33{,}000}{(1+r)^4}+\frac{\$35{,}000}{(1+r)^5}+\frac{\$37{,}000+\$190{,}000}{(1+r)^6}=\$0$$

Solve for r = IRR with trial and error. (Answer ends up being 13.761038%; notice that

$$\frac{-\$210{,}000}{(1.13761038)^0}+\frac{\$27{,}000}{(1.13761038)^1}+\frac{\$29{,}000}{(1.13761038)^2}+\frac{\$31{,}000}{(1.13761038)^3}+\frac{\$33{,}000}{(1.13761038)^4}+\frac{\$35{,}000}{(1.13761038)^5}+\frac{\$37{,}000+\$190{,}000}{(1.13761038)^6}$$

= -$210,000 + $23,733.96 + $22,408.40 + $21,056.25 + $119,703.34 + $18,369.63 + $104,728.43 = $0.)

A written solution to the problem should show the equation above. If you want to find the IRR answer on the Texas Instruments BA II Plus financial calculator you have to go into cash flow mode; the row with the five basic time values keys that works with equal annuity payments does not work here. Type in CF 2nd CLR WORK 210000 +/- ENTER ↓ 27000 ENTER ↓↓ 29000 ENTER ↓↓ 31000 ENTER ↓↓ 33000 ENTER ↓↓ 35000 ENTER ↓↓ (37000 + 190000) ENTER ↓ NPV 9 ENTER ↓ CPT IRR CPT (should show 44,595.36 and 13.761038). Notice how the screen goes blank briefly as the calculator computes NPV for all those different cash flow values, and it goes blank for several noticeable seconds as it does the complicated trial and error calculations needed in finding IRR. When there are double down-arrows ↓↓ (typing the down arrow key twice) the first one is telling the calculator to enter the indicated dollar amount, and the second one is saying that cash flow is for only one period, so now move to the next period.

(If there were a series of equal expected cash flows, like if the same $27,000 applied to each of years 1 through 4, followed by $35,000 expected in year 5 alone, then the steps would show as CF 2nd CLR WORK 210000 +/- ENTER ↓ 27000 ENTER ↓ 4 ENTER ↓ 35000 ENTER ↓↓ …) Because NPV and IRR are found with the same equation (just different knowns and unknowns) the calculator computes them with back-to-back steps when in cash flow mode. (In cash flow mode the calculator can handle only one cash flow per period, so we have to combine the year-6 expected ATCF and ATER into one $227,000 value.) Here the cash flows the investor expects to receive after giving up the initial equity investment are enough to provide a return OF the investment, also provide a 9% annual return ON the investment, and also leave something extra: $44,595.36, which is an immediate increase in the equity investor's wealth. Note also that the internal rate of return exceeds the required annual rate of return; a positive net present value results when the rate of return earned per period is higher than the investor's periodic hurdle rate.

2. A real estate investor buys an income-producing property for $1,250,000 ($850,000 in debt and $400,000 in equity). The investment is expected to generate after-tax cash flows (ATCF's) to the equity position of $44,000 per year for ten years, and at the end of year 10 the equity investor expects to sell the property for $1,400,000. After paying off the remaining loan balance and paying all transaction-related fees (including all applicable taxes), the equity investor expects to net $450,000 from the property sale (the after-tax equity reversion, or ATER).

a. What is the net present value (NPV) of this investment if the equity investor requires a 12% average annual rate of return? *Set it up first with year 1 – 10's series of repeated $44,000 cash flows shown individually, and then set it up using the PV of annuity factor, which lets us use the distributive property to group repeated equal payments into one computation*. (You can compute the answer with the briefer annuity approach, but at least show how to set it up with year-by-year dollar amounts.).
b. Set up the equation that the equity investor would use in computing her internal rate of return (IRR).

a. The seven equations below are different ways of saying the same thing; it is merely a matter of whether we look at the ten yearly $44,000 cash flows individually vs. grouping them for computational purposes with the PV of annuity factor. Working with individual years' cash flows tends to be more intuitive, but you need to understand that the much more convenient PV of a level ordinary annuity factor is just the sum of the PV of $1 factors for the same discount rate and same number of time periods (annuity computations are based on the distributive property from junior high school math). If an exam question has 19 years of cash flows you do not want to answer it by adding together 19 individual present values!

$$\frac{-\$400{,}000}{(1.12)^0} + \frac{\$44{,}000}{(1.12)^1} + \frac{\$44{,}000}{(1.12)^2} + \frac{\$44{,}000}{(1.12)^3} + \cdots + \frac{\$44{,}000}{(1.12)^8} + \frac{\$44{,}000}{(1.12)^9} + \frac{\$44{,}000 + \$450{,}000}{(1.12)^{10}} = \text{NPV} \quad \text{OR}$$

$$-\$400{,}000 \left(\frac{1}{1.12}\right)^0 + \$44{,}000 \left(\frac{1}{1.12}\right)^1 + \cdots + \$44{,}000 \left(\frac{1}{1.12}\right)^9 + (\$44{,}000 + \$450{,}000) \left(\frac{1}{1.12}\right)^{10} = \text{NPV} \quad \text{OR}$$

$$\frac{-\$400{,}000}{(1.12)^0} + \$44{,}000 \left(\frac{1-\left(\frac{1}{1.12}\right)^9}{.12}\right) + \frac{\$44{,}000 + \$450{,}000}{(1.12)^{10}} = \text{NPV} \quad \text{OR}$$

$$\frac{-\$400{,}000}{(1.12)^0} + \frac{\$44{,}000}{(1.12)^1} + \frac{\$44{,}000}{(1.12)^2} + \frac{\$44{,}000}{(1.12)^3} + \cdots + \frac{\$44{,}000}{(1.12)^9} + \frac{\$44{,}000}{(1.12)^{10}} + \frac{\$450{,}000}{(1.12)^{10}} = \text{NPV} \quad \textit{OR}$$

$$-\$400{,}000\left(\frac{1}{1.12}\right)^0 + \$44{,}000\left(\frac{1}{1.12}\right)^1 + \cdots + \$44{,}000\left(\frac{1}{1.12}\right)^9 + \$44{,}000\left(\frac{1}{1.12}\right)^{10} + \$450{,}000\left(\frac{1}{1.12}\right)^{10} = \text{NPV} \quad \textit{OR}$$

$$\frac{-\$400{,}000}{(1.12)^0} + \$44{,}000\left(\frac{1-\left(\frac{1}{1.12}\right)^{10}}{.12}\right) + \frac{\$450{,}000}{(1.12)^{10}} = \text{NPV} \quad \textit{OR}$$

$$-\$400{,}000 + \$44{,}000\left(\frac{1-\left(\frac{1}{1.12}\right)^{10}}{.12}\right) + \$450{,}000\left(\frac{1}{1.12}\right)^{10} = \text{NPV}$$

– \$400,000 + \$44,000 (5.650223) + \$450,000 (.321973) = NPV
– \$400,000 + \$248,609.81 + \$144,887.96 = – \$<u>6,502.23</u> NPV

b. IRR and NPV are solved with the same equation, based on the same cash flow projections. The difference is that in computing NPV we know the discount rate and solve for the difference in the PV's of the cash flows, while in computing IRR we solve for the discount rate that would cause the difference in the PV's of the cash flows to be \$0 (that solution will require doing trial and error if more than one cash flow is anticipated after the initial investment is made, since there will be more than one unknown in a single equation). Here we will consider just one equation with year-by-year figures and one with the annuity factor, but any of the structures shown with NPV above would be equally valid to use:

$$-\$400{,}000\left(\frac{1}{1+r}\right)^0 + \$44{,}000\left(\frac{1}{1+r}\right)^1 + \cdots + \$44{,}000\left(\frac{1}{1+r}\right)^{10} + \$450{,}000\left(\frac{1}{1+r}\right)^{10} = \$0 \quad \textit{OR}$$

$$-\$400{,}000 + \$44{,}000\left(\frac{1-\left(\frac{1}{1+r}\right)^{10}}{r}\right) + \$450{,}000\left(\frac{1}{1+r}\right)^{10} = \$0$$

Solve for r = IRR with trial and error. (Answer ends up being 11.721938%; notice that

$$-\$400{,}000 + \$44{,}000\left(\frac{1-\left(\frac{1}{1.11721938}\right)^{10}}{r}\right) + \$450{,}000\left(\frac{1}{1.11721938}\right)^{10} = -\$400{,}000 + \$44{,}000(5.715121) + \$450{,}000(.330077) = \$0.)$$

A written solution to the problem should show the equation above. If you wanted to find the answer, on the Texas Instruments BA II Plus financial calculator you would be able to use the row with the five basic time value of money keys that work with level annuities, because all of the regular ATCF payments (PMT) are projected to be equal. Type in 400000 +/- PV, 44000 PMT, 450000 FV, 10 N, CPT I/Y; screen goes blank for a noticeable second or two as the calculator does trial and error, and then the screen shows 11.721938.

Now the cash flows the investor expects to receive after giving up the initial equity investment are enough to provide a return OF the investment (the \$450,000 expected from resale in year 10 by itself is enough to cover the \$400,000), but those post-investment cash flows are not also enough in total to provide a 12% annual return ON the investment. The annual rate of return that would be earned is positive (always the case when the unadjusted cash flows total more than the initial investment), but it is not positive enough -- it is less than the 12% annual hurdle rate. Net present value is negative, which means that the investor actually would immediately lose \$6,502.23 in wealth by making this purchase. A negative net present value results when the rate of return earned per period is lower than the investor's periodic hurdle rate.

[Just to show how we could use cash flow mode to compute the answers here: let's think of having a series of three $44,000 ATCF's followed by six more $44,000 ATCF's, and then a final $44,000 ATCF plus the $450,000 ATER in period 10. Type in CF 2nd CLR WORK 400000 +/- ENTER ↓ 44000 ENTER ↓ 9 ENTER ↓ (44000 + 450000) ENTER ↓ NPV 12 ENTER ↓ CPT IRR CPT; should show – 6,502.23 and 11.721938. We would get the same answer by entering CF 2nd CLR WORK 400000 +/- ENTER ↓ 44000 ENTER ↓ 3 ENTER ↓ 44000 ENTER ↓ 6 ENTER ↓ (44000 + 450000) ENTER ↓ NPV 12 ENTER ↓ CPT IRR CPT; here we would be pretending that the first three $44,000 annual ATCF's were different from the next six (so that is how we would enter values if the year 4-9 ATCF amounts truly were different from the year 1-3 ATCF's).

An advantage of using cash flow mode is that it generates the NPV for a specified discount rate along with the IRR based on the indicated cash flows, and also allows for computing NPV for any number of different discount rates; just hit the ↓ key twice between attempts. For example, in the case above let's compute NPV using annual discount rates of 15%, 12%, 8%, and 5%, and also IRR: type in CF 2nd CLR WORK 400000 +/- ENTER ↓ 44000 ENTER ↓ 9 ENTER ↓ (44000 + 450000) ENTER ↓ NPV 15 ENTER ↓ CPT (shows – 67,941.06) ↓↓ NPV 12 ENTER ↓ CPT (shows – 6,502.23) ↓↓ NPV 8 ENTER ↓ CPT (shows 103,680.65) ↓↓ NPV 5 ENTER ↓ CPT (shows 216,017.30) IRR CPT (shows 11.721938). Notice once again that NPV is determined by both the discount rate and the cash flows, while IRR is a function only of the cash flows. Here NPV is positive, such that wealth is created for the equity investor, if the discount rate (the required rate of return or hurdle rate or opportunity rate or cost of capital) is lower than the expected rate of return, as with 8% and 5% above providing positive NPV estimates because those discount rates are lower than the 11.72-ish percent IRR.]

3. A land speculator buys some acreage for $120,000, all of which is his own equity money. He plans to sell in four years, at which point he predicts the land will be worth $195,000. During the four holding years he expects to lease part of the land to a farmer and collect just enough in rent to cover property taxes and other annual holding costs for the tract, such that the cash flow during each of years 1 – 4 is expected to be $0 and the only net cash flows to consider are year 0's purchase outlay and the sales price at the end of year 4. The annual cost of capital or hurdle rate the buyer assigns to a project of this type is 8%.

a. What are this investment's net present value and internal rate of return?
b. What would the NPV and IRR be if instead the cost of capital/investment hurdle rate were 15% per year?

a. $$-\$120{,}000\left(\frac{1}{1.08}\right)^0 + \$0\left(\frac{1}{1.08}\right)^1 + \$0\left(\frac{1}{1.08}\right)^2 + \$0\left(\frac{1}{1.08}\right)^3 + (\$0 + \$195{,}000)\left(\frac{1}{1.08}\right)^4 = \text{NPV} \quad \text{OR}$$

$$-\$120{,}000\left(\frac{1}{1.08}\right)^0 + \$195{,}000\left(\frac{1}{1.08}\right)^4 = \text{NPV}$$

$$-\$120{,}000\ (1) + \$195{,}000\ (.735030) = \text{NPV}$$
$$-\$120{,}000 + \$143{,}330.82 = \$\underline{\underline{23{,}330.82}}\ \text{NPV}$$

$$-\$120{,}000\left(\frac{1}{1+r}\right)^0 + \$0\left(\frac{1}{1+r}\right)^1 + \$0\left(\frac{1}{1+r}\right)^2 + \$0\left(\frac{1}{1+r}\right)^3 + (\$0 + \$195{,}000)\left(\frac{1}{1+r}\right)^4 = \$0 \quad \text{OR}$$

$$-\$120{,}000\left(\frac{1}{1+r}\right)^0 + \$195{,}000\left(\frac{1}{1+r}\right)^4 = \$0$$

$$-\$120{,}000 + \$195{,}000\left(\frac{1}{1+r}\right)^4 = \$0$$

$$-\$120{,}000 = -\$195{,}000\left(\frac{1}{1+r}\right)^4 \quad \text{SO} \quad \$120{,}000 = \$195{,}000\left(\frac{1}{1+r}\right)^4$$

Multiplying each side of that equation by $(1 + r)^4$ gives

$$\$120{,}000\ (1 + r)^4 = \$195{,}000$$
$$(1 + r)^4 = 1.625$$
$$\sqrt[4]{(1 + r)^4} = \sqrt[4]{1.6250}$$
$$1 + r = 1.625^{\frac{1}{4}} = 1.625^{.25} = 1.129050$$
$$\text{SO } r = \underline{\underline{12.905043}}\% \text{ per year IRR}$$

[In this example you can directly compute the IRR, because there is only one nonzero cash flow expected after the initial investment. If you wanted to find the answer on the BA II Plus you could enter 120000 +/- PV, 0 PMT, 195000 FV, 4 N, CPT I/Y; the screen does not go blank but instead immediately shows 12.905043.] The idea that IRR relates dollars expected back to dollars given up applies just as it did in the two earlier examples. But here the calculator does not have to use trial and error to solve for IRR (and neither do we), because there is only one unknown, r^4, in the one equation, so we can directly solve by taking the appropriate (fourth) root. In this example the investor's 12.9% expected annual rate of return is greater than the 8% required annual rate of return, so the purchase of the land immediately increases the investor's wealth – here, by a little more than $23,000.

b. An investment project's IRR is a function solely of its expected cash flows; as we computed above: giving up $120,000 and then getting back $195,000 four years later constitutes a <u>12.905043% IRR</u> compounded average annual rate of return. The IRR is not affected by the cost of capital; rather, we COMPARE the IRR to the cost of capital in determining whether an investment is worth making because it increases the investor's wealth. Here it should be clear that with the 15% periodic cost of capital exceeding the 12.905043% periodic rate of return the investment should be avoided; making it would reduce the project owner's wealth. That reduction in wealth would be equal to the investment's NPV of

$$-\$120{,}000\left(\frac{1}{1.15}\right)^0 + \$195{,}000\left(\frac{1}{1.15}\right)^4 = \text{NPV}$$
$$-\$120{,}000\ (1) + \$195{,}000\ (.571753) = \text{NPV}$$
$$-\$120{,}000 + \$111{,}491.88 = -\$\underline{\underline{8{,}508.12}}\ \text{NPV}$$

- Required annual rate of return < 12.9% annual IRR $\Rightarrow$ NPV is positive
- Required annual rate of return > 12.9% annual IRR $\Rightarrow$ NPV is negative
- Required annual rate of return = 12.9% annual IRR $\Rightarrow$ NPV is $0, which is minimally acceptable because if return earned equals return required the equity investor's wealth is sustained

4. The following scenarios relate to a $7,000,000 investment property. Compute the owner's annual return on equity (ROE) in each case, and indicate in parts b and d whether positive or negative financial leverage is present.

a. The property generates net rents minus operating costs minus income taxes of $830,000 per year, and the entire purchase was financed with equity (there is no debt financing):

$$\text{ROE} = \$830{,}000 \div \$7{,}000{,}000 = \underline{\underline{11.8571\%}}$$

b. The property generates net rents minus operating costs minus income taxes of $830,000 per year, and 75 percent of the purchase price was financed with an interest-only loan that carries a 9.5% after-tax annual interest cost:

Debt is .75 x $7,000,000 = $5,250,000; Equity is .25 x $7,000,000 = $1,750,000

Cash flow to property is	$830,000
Interest is .095 x $5,250,000 =	$498,750
Cash flow to equity is	$331,250

ROE = $331,250 ÷ $1,750,000 = 18.9286% ⇒ Positive Fin. Leverage (higher ROE with debt financing)

c. The property generates net rents minus operating costs minus income taxes of $565,000 per year, and the entire purchase was financed with equity (there is no debt financing):

ROE = $565,000 ÷ $7,000,000 = 8.0714%

d. The property generates net rents minus operating costs minus income taxes of $565,000 per year, and 75 percent of the purchase price was financed with an interest-only loan that carries a 9.5% after-tax annual interest cost:

Debt is .75 x $7,000,000 = $5,250,000; Equity is .25 x $7,000,000 = $1,750,000

Cash flow to property is	$565,000
Interest is .095 x $5,250,000 =	$498,750
Cash flow to equity is	$ 66,250

ROE = $66,250 ÷ $1,750,000 = 3.7857% ⇒ Negative Fin. Leverage (lower ROE with debt financing)