

# Coloring Graphs with Forbidden Almost Bipartite Subgraphs

James Anderson\*, Anton Bernshteyn, Abhishek Dhawan

*Department of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332;*  
*james.anderson@math.gatech.edu; bahtoh@gatech.edu; abhishek.dhawan@math.gatech.edu*

A conjecture of Alon, Krivelevich, and Sudakov states that, for any graph  $F$ , there is a constant  $c(F) > 0$  such that  $\chi(G) \leq (c(F) + o(1))\Delta / \log \Delta$  for all  $F$ -free graphs  $G$  of maximum degree  $\Delta$ . The only graphs  $F$  for which this conjecture has been verified so far—by Alon, Krivelevich, and Sudakov themselves—are the so-called almost bipartite graphs, i.e., graphs that can be made bipartite by removing at most one vertex. Equivalently, a graph is almost bipartite if it is a subgraph of the complete tripartite graph  $K_{1,t,t}$  for some  $t \in \mathbb{N}$ . The best heretofore known upper bound on  $c(F)$  for almost bipartite  $F$  is due to Davies, Kang, Pirot, and Sereni, who showed that  $c(K_{1,t,t}) \leq t$ . We prove that in fact  $c(F) \leq 4$  for any almost bipartite graph  $F$ , thus making the bound independent of  $F$  in all the known cases of the conjecture. We also establish a more general version of this result in the setting of DP-coloring (also known as correspondence coloring) introduced by Dvořák and Postle.