

Finding a Lower Bound for k -Unbounded Hamiltonian Cycles

Albert Jiang

Plano West Senior High School, Plano, TX 75024, galarblemarble@gmail.com

Methods to determine the existence of Hamiltonian Cycles in graphs have been extensively studied. However, little research has been done following cases when no Hamiltonian Cycle exists. Let a vertex be “unbounded” if it is visited more than once in a path. Furthermore, let a k -Unbounded Hamiltonian Cycle be a path with finite length that visits every vertex, has adjacent start and end vertices, and contains k unbounded vertices. I consider a variant of the Hamiltonian Cycle Problem in which the objective is to find an m -Unbounded Hamiltonian Cycle where m is the minimum value of k such that a k -Unbounded Hamiltonian Cycle exists. I provide an exponential-time brute-force algorithm for the determination of such a path. Furthermore, I present a polynomial-time heuristic for the determination of an m -Unbounded Hamiltonian Cycle that also functions as an effective heuristic for the original Hamiltonian Cycle Problem. I also consider the task on trees and discuss efficient approaches for this subclass of graphs.