

On completing partial latin squares with prescribed diagonals

Lars Døvling Andersen,¹ Stacie Baumann^{2,*} Anthony J. W. Hilton^{3,4} C. A. Rodger²

¹*Department of Mathematical Sciences, Aalborg University, Aalborg, Denmark, lda@math.aau.dk;* ²*Department of Mathematics and Statistics, Auburn University, Auburn, Alabama, USA, szb0131@auburn.edu; rodgerc1@auburn.edu;*

³*Department of Mathematics, University of Reading, Reading, UK, a.j.w.hilton@reading.ac.uk;* ⁴*Department of Mathematics, Queen Mary University of London, London, UK*

Necessary and sufficient numerical conditions are known for the embedding of an incomplete latin square L of order n into a latin square T of order $t \geq 2n + 1$ in which each symbol is prescribed to occur in a given number of cells on the diagonal of T outside of L . This includes the classic case where T is required to be idempotent. If $t < 2n$ then no such numerical sufficient conditions exist since it is known that the arrangement of symbols within the given incomplete latin square can determine the embeddibility. All examples where the arrangement is a factor share the common feature that one symbol is prescribed to appear exactly once in the diagonal of T outside of L , resulting in a conjecture over 30 years ago stating that it is only this feature that prevents numerical conditions sufficing for all $t \geq n$. We prove this conjecture, providing necessary and sufficient numerical conditions for the embedding of an incomplete latin square L of order n into a latin square T of order t for all $t \geq n$ in which the diagonal of T outside of L is prescribed in the case where no symbol is required to appear exactly once in the diagonal of T outside of L .