Fast Algorithms for $(\Delta + 1)$-Edge-Coloring

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Let G be an n-vertex graph of maximum degree Δ. Vizing's Theorem states that G can be properly edge-colored using $\Delta + 1$ colors. We show that for any partial $(\Delta + 1)$-edge-coloring and any uncolored edge e, it is possible to extend the coloring to include e after modifying the colors of at most $\mathsf{poly}(\Delta) \log n$ other edges. This bound is best possible up to the degree of the dependence on Δ, as shown by Chang, He, Li, Pettie, and Uitto. We apply this result together with some other ideas to develop new efficient algorithms for Vizing's theorem. Namely, we design a randomized sequential algorithm that finds a proper $(\Delta + 1)$-edge-coloring of G with probability at least $1 - 1/\Delta^{\Theta(n)}$ in time $\mathsf{poly}(\Delta)\, n$ (which is obviously best possible up to the degree of the dependence on Δ). We also develop a randomized (resp. deterministic) algorithm for $(\Delta + 1)$-edge-coloring in the LOCAL model of distributed computation with running time $\mathsf{poly}(\Delta) \log^3 n$ (resp. $\mathsf{poly}(\Delta) \log^6 n$), which improves recent results of the first author. Finally, we apply our results in the setting of Borel graphs to prove a Borel version of Vizing's theorem for graphs of subexponential growth. The key new idea in our arguments is a novel application of the entropy compression method.