

A generalization of Hall's latin rectangle theorem

Amin Bahmanian¹, Anna Johnsen^{2,*}

¹Department of Mathematics, Illinois State University, Normal, IL 61790-4520, mbahman@ilstu.edu;

²Department of Mathematics and Statistics, Georgia State University, Atlanta, GA 30303, ajohnsen2@gsu.edu

In 1945, Hall (*Bull. Amer. Math. Soc.* 51, 387–388) showed that any $r \times n$ latin rectangle can be extended to an $n \times n$ latin square. We generalize Hall's theorem in the following way. Let L be an $n \times n$ array whose top $r \times n$ subarray is filled with k different symbols, each occurring approximately the same number of times in each row of L . We find necessary and sufficient conditions that ensure the remaining cells of L can be filled such that each symbol $\ell \in [k]$ occurs a prescribed number of times in L and approximately the same number of times in each row and column of L . In 2015, Goldwasser, Hilton, Hoffman, and Özkan (*J. Combin. Theory Ser. A* 130, 26–41) solved the case where the prescribed number of times each symbol occurs in L is at most n .