Packing Edge-Colorings

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A proper j-edge coloring of a graph G is an assignment of the colors $\{1_1, 1_2, \ldots, 1_j\}$ to each edge in G such that incident edges receive distinct colors. A strong k-edge coloring of a graph G is an assignment of the colors $\{2_1, 2_2, \ldots, 2_k\}$ to each edge in G such that incident edges receive distinct colors and any two edges that are incident to a common third edge receive distinct colors. In this talk we explore a third type of edge-coloring, a $(1^j, 2^k)$-packing edge-coloring. A $(1^j, 2^k)$-packing edge-coloring of a graph G is an assignment of the colors $\{1_1, 1_2, \ldots, 1_j\}$ and $\{2_1, 2_2, \ldots, 2_k\}$ to the edges in G such that incident edges receive distinct colors and any two edges colored 2_i for the same $1 \leq i \leq k$ are not incident to a common edge. A $(1^j, 2^k)$-packing edge-coloring of a graph can be used as a transition between proper and strong edge-colorings of graphs. We use packing edge-colorings of graphs to approach a conjecture by Erdős and Nešetřil about strong edge-colorings from a new direction. In this talk we will present our result that every graph with maximum degree at most four is $(1^1, 2^{19})$-packing edge-colorable. This research was completed during the 2022 GVSU Summer Mathematics REU with Cicely Henderson under the mentorship of Dr. Michael Santana.