Maximal partial designs

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A few problems related to various combinatorial designs in terms of their maximality will be discussed.

One of the most examined cases concerns maximal partial latin squares. A *partial latin square* of order n is an $n \times n$ array in which each cell is either empty or contains a single symbol from an n-element set S such that each symbol occurs at most once in each row and at most once in each column. A partial latin square is *maximal* if no empty cell can be filled with an element of S without violating latin conditions.

Evidently, a partial latin square of order n corresponds to a proper edge-coloring of a balanced bipartite graph $H = (V, U, E)$ with at most $n = \chi'(K_{n,n})$ colors, where $|U| = |V| = n$. In this way considering maximal edge-colorings of non-bipartite graphs makes an obvious next step. Let G be a graph of order n. A *maximal edge-coloring* of G is a proper edge-coloring with $\chi'(K_n)$ colors such that no edge of the complement $\overline{G}$ can be attached to G without violating conditions of proper edge-coloring. For given n, a *spectrum* $\mathrm{MEC}(n)$ is defined to be the set of all sizes of graphs of order n which admit maximal edge-colorings.

Another class of objects, strongly related to edge-colorings of graphs, are partial Room squares. A *partial Room square* of order n and side $n-1$ on an n-element set S is an $(n-1) \times (n-1)$ array F satisfying the following properties:

(1) every cell of F is either empty or contains an unordered pair of symbols from S,
(2) every symbol of S occurs at most once in each row and at most once in each column of F,
(3) every unordered pair of symbols of S occurs in at most one cell of F.

A partial Room square is *maximal* if no further pair of elements can be placed into any unoccupied cell without violating the conditions that define a partial Room square. A *spectrum* $\mathrm{MPRS}(n)$ is the set of volumes of maximal partial Room squares of order n, where the *volume* means the number of occupied cells.

In all of these cases, the common aim is to determine spectra.