The List Color Function Threshold

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Counting proper colorings of graphs is a fundamental topic in enumerative combinatorics that has been extensively studied since the early 20th century. Specifically, the chromatic polynomial of a graph G, denoted $P(G,m)$, is equal to the number of proper m-colorings of G. List coloring is a well-studied generalization of classical coloring that was introduced in the 1970s. The list color function of a graph G is a list analogue of the chromatic polynomial that has been studied since the early 1990s. It is known that for any graph G there is a positive integer k such that $P(G,m)$ is identical to the list color function of G whenever m is at least k. The list color function threshold of G is the smallest k such that $P(G,m)$ is nonzero and identical to the list color function of G whenever m is at least k. In 2009 Carsten Thomassen suggested studying the difference between the list color function threshold of a graph G and the list chromatic number of G. Thomassen also asked whether there is a universal constant that bounds this difference for all graphs. In this talk we develop tools for bounding the list color function threshold of complete bipartite graphs from above and below. We show how these tools allow us to answer Thomassen's question in the negative.