

# EL-Labeling the Shard Intersection Order on the biCambrian Lattice

Reed Acton\*, Emily Barnard

*Department of Mathematics, DePaul University, Chicago, IL 60614, racton1@depaul.edu; e.barnard@depaul.edu*

The *shard intersection order* of a finite Coxeter group  $W$  was introduced by Reading as a lattice structure on the elements of  $W$  containing the poset of noncrossing partitions  $NC(W)$  as a sub-lattice. Building on the work of Bancroft and Petersen in finite Coxeter groups, we look closely at the EL-shellability of the shard intersection order for the *biCambrian lattice*, a lattice quotient of the weak order introduced by Barnard and Reading. We construct a new edge-labeling  $\lambda$  and show that it is an EL-labeling for the bipartite biCambrian lattice in type A. We also show that the edge labeling  $\sigma$  introduced by Bancroft and extended by Petersen fails to be an EL-labeling for all biCambrian lattices in type A. Finally, we extend our EL-labeling to type B, and investigate a type-free edge-labeling.