# Sum and Difference Sets in Generalized Dihedral Groups

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Given a group $G$, we say that a set $A \subseteq G$ has more sums than differences (MSTD) if $|A+A| > |A-A|$, has more differences than sums (MDTS) if $|A+A| < |A-A|$, or is sum-difference balanced if $|A+A| = |A-A|$. Recently, Miller and Vissuet studied the frequency of these types of subsets in arbitrary finite groups $G$ and proved that almost all subsets $A \subseteq G$ are sum-difference balanced as $|G| \to \infty$. For the dihedral group $D_{2n}$, they conjectured that of the remaining sets, most are MSTD. Some progress on this conjecture was made by Haviland et al. in 2020, when they introduce the idea of partitioning the subsets by size: if, for each $m$, there are more MSTD subsets of $D_{2n}$ of size $m$ than MDTS subsets of size $m$, then the conjecture follows. We extend the conjecture to generalized dihedral groups $D = \mathbb{Z}_2 \ltimes G$, where $G$ is an abelian group of size $n$ and the nonidentity element of $\mathbb{Z}_2$ acts by inversion. We make further progress on the conjecture by considering subsets with a fixed number of rotations and reflections. By bounding the expected number of overlapping sums, we show that the collection $\mathcal{S}_{D,m}$ of subsets of the generalized dihedral group $D$ of size $m$ has more MSTD sets than MDTS sets when $6 \leq m \leq c_j\sqrt{n}$ for $c_j = 1.3229/\sqrt{111+5j}$, where $j$ is the number of elements in $G$ with order at most 2. We also analyze the expectation for $|A+A|$ and $|A-A|$ for $A \subseteq D_{2n}$, providing an explicit formula for $|A-A|$ when $n$ is prime.