Weak Sequenceability in Cyclic Groups

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A subset A of an abelian group G is *sequenceable* if there is an ordering $(a_1, \dots, a_k)$ of its elements such that the partial sums $(s_0, s_1, \dots, s_k)$, given by $s_0 = 0$ and $s_i = \sum_{j=1}^{i} a_j$ for $1 \leq i \leq k$, are distinct, with the possible exception that we may have $s_k = s_0 = 0$. In the literature there are several conjectures and questions concerning the sequenceability of subsets of abelian groups, which have been combined and summarized by Alspach and Liversidge into the conjecture that if a subset of an abelian group does not contain 0 then it is sequenceable. If the elements of a sequenceable set A do not sum to 0 then there exists a simple path P in the Cayley graph $Cay[G : \pm A]$ such that $\Delta(P) = \pm A$. In this talk, inspired by this graph-theoretical interpretation, we propose a weakening of this conjecture. Here, under the above assumptions, we want to find an ordering whose partial sums define a walk W of girth bigger than t (for a given $t < k$) and such that $\Delta(W) = \pm A$. This is possible given that the partial sums s_i and s_j are different whenever i and j are distinct and $|i - j| \leq t$. In this case, we say that the set A is t-*weakly sequenceable*. The main result we will present is that any subset A of $\mathbb{Z}_p \setminus \{0\}$ is t-weakly sequenceable whenever $t < 7$ or when A does not contain pairs of type $\{x, -x\}$ and $t < 8$.