# Multithreshold Multipartite Graphs

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We call a graph $G$ a *$k$-threshold graph* if there are $k$ distinct real numbers $\theta_1, \theta_2, \ldots, \theta_k$ and a mapping $r\colon V(G) \to \mathbb{R}$ such that for any two vertices $u, v \in V(G)$, we have that $uv \in E(G)$ if and only if there are odd numbers $\theta_i$ such that $\theta_i \le r(u) + r(v)$. The least integer $k$ such that $G$ is a $k$-threshold graph is called a *threshold number* of $G$, and denoted by $\Theta(G)$. The well-known family of threshold graphs is a set of graphs $G$ with $\Theta(G) \le 1$. Jamison and Sprague in [Multithreshold graphs, *J. Graph Theory*, 94(4): 518–530, 2020] introduced the concept of $k$-threshold graph, and proved that $\Theta(G)$ exists for every graph $G$. They further obtained a number of interesting results on $\Theta(G)$. In addition, they also proposed several unsolved problems and conjectures, including the following two.

**Problem:** Determine the exact threshold numbers of the complete multipartite graphs.

**Conjecture:** For all even $n \ge 2$, there is a graph $G$ with $\Theta(G) = n$ and $\Theta(G^c) = n + 1$. This is equivalent to that for all odd $n \ge 3$, there is a graph $G$ with $\Theta(G) = n$ and $\Theta(G^c) = n - 1$, where $G^c$ is the complement of $G$.

In this short paper, we give a partial solution of the problem and confirm the conjecture.