

The Matrix Representation of the Vertical Recursive Relation of Riordan Arrays, the Rook triangle and the Laguerre triangle

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It is known that the entries of a Riordan array satisfy horizontal recursive relations represented by the A - and Z -sequences. In this paper, we study a vertical recursive relation approach to Riordan arrays. This vertical recursive approach gives a way to represent the entries of a Riordan array (g, f) in terms of a recursive linear combination of the coefficients of g . We also give a matrix representation of the vertical recursive relation. The set of all those matrices forms a group, called the quasi-Riordan group. We present the extensions of the horizontal recursive relation and the vertical recursive relation in terms of c - and C -Riordan arrays with illustrations by using the rook triangle and the Laguerre triangle. Those extensions represent a way to study nonlinear recursive relations of the entries of some triangular matrices from linear recursive relations of the entries of Riordan arrays. In addition, the matrix representation of the vertical recursive relation of Riordan arrays provides transforms between lower order and higher order finite Riordan arrays, where the m^{th} order Riordan array is defined by $(g, f)_m = (d_{n,k})_{m \geq n, k \geq 0}$. Furthermore, the vertical relation approach to Riordan arrays provides a unified approach to construct identities.