

On Hypergraph Designs

Isabel Brady-Myerov^{1,*} Clare Jones^{2,*} Daniel Kessinger^{3,*} Julie Kirkpatrick^{4,†,*}

¹Vassar College, isabel.bradymyerov@gmail.com;

²Colorado State University, Clare.Jones@colostate.edu;

³Greenville University, daniel.m.kessinger@gmail.com;

⁴Cedar Falls Community School District, jkirkpatrickmath@gmail.com

A commonly studied problem in combinatorics concerns decompositions of graphs into edge-disjoint subgraphs. For graphs G and K , a G -decomposition of K is a partition of the edge set of K into subgraphs isomorphic to G . Such decompositions are generally known as graph designs. The problem is of most interest when both G and K are complete graphs. Other cases though have also attracted attention. Decompositions of complete graphs into cycles, stars and trees in general are among the popular questions. Corresponding questions for decompositions of t -uniform hypergraphs are of interest. In a t -uniform hypergraph, the edges are t -element subsets of a vertex set (thus graphs correspond to the case $t = 2$). The complete t -uniform hypergraph of order v , denoted $K_v^{(t)}$, has a set V of size v as its vertex set and the set of all t -element subsets of V as its edge set. For $t > 2$, the only general question on H -decompositions of $K_v^{(t)}$ that is settled completely is the case where H is a matching. Though, it is now known that the necessary conditions for H -decompositions of $K_v^{(t)}$ are asymptotically sufficient for all t -uniform hypergraphs H . A seminal 2014 paper by Bryant, Herke, Maenhaut, and Wannasit [Decompositions of complete 3-uniform hypergraphs into small 3-uniform hypergraphs, *Australas. J. Combin.* **60** (2014), 227–254] gives a template for constructing 3-uniform hypergraph designs. Over the past several years, participants in the *Illinois State University Math REU for Pre-service and In-service Teachers* have used the Bryant et al. approach to investigate decompositions of complete t -uniform hypergraphs into various configurations of interest. Several results have focused on t -uniform structures that can be visualized as “cycle-like” structures. In this talk, we show a generalization of the approach by Bryant et al. and spotlight decompositions of $K_v^{(3)}$ into loose 6-cycles and decompositions of $K_v^{(4)}$ into 4-colorable 3-cycles. This work presents results from a combination of projects that were recently completed at the *Illinois State University REU for Pre-service and In-service Teachers* (National Science Foundation Grant Number A1950357) under the direction of Ryan Bunge, Saad El-Zanati, and William Turner.