Strong vertex-magic and super edge-magic total labelings of the disjoint union of a cycle with $3-$cycles

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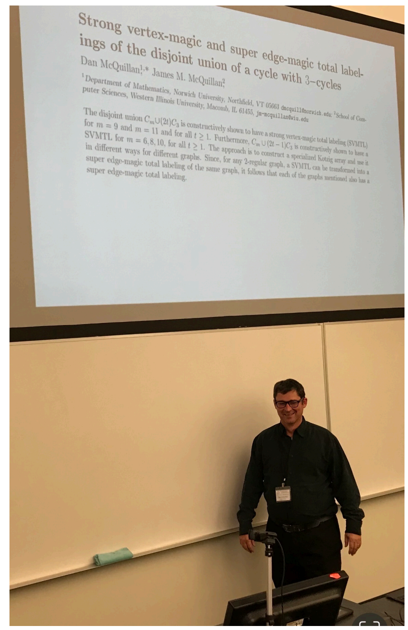
The disjoint union $C_m \cup (2t)C_3$ is constructively shown to have a strong vertex-magic total labeling (SVMTL) for $m = 9$ and $m = 11$ and for all $t \geq 1$. Furthermore, $C_m \cup (2t-1)C_3$ is constructively shown to have a SVMTL for $m = 6, 8, 10$, for all $t \geq 1$. The approach is to construct a specialized Kotzig array and use it in different ways for different graphs. Since, for any 2-regular graph, a SVMTL can be transformed into a super edge-magic total labeling of the same graph, it follows that each of the graphs mentioned also has a super edge-magic total labeling.



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Note: PREPRINT AVAILABLE UPON REQUEST

Vertex-magic / Edge-magic

- SVMTL (Duality for regular)

- odd-order 2-regular

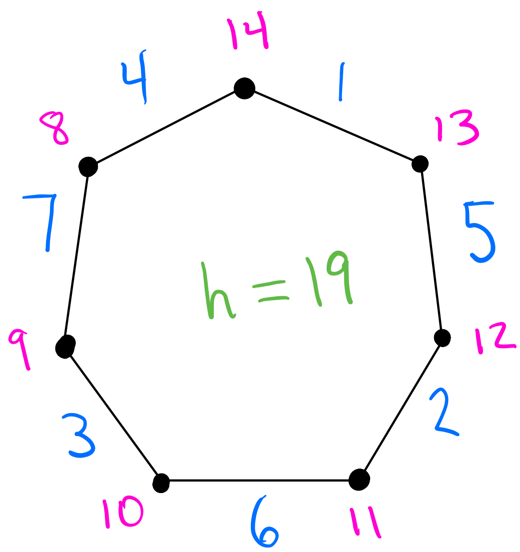
- triangles are hard

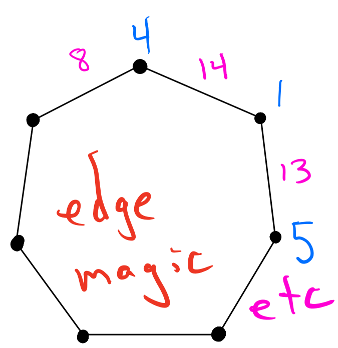
$C_m \cup s C_3$ (odd order)

$3 \leq m \leq 11$

work with J. McQuillan

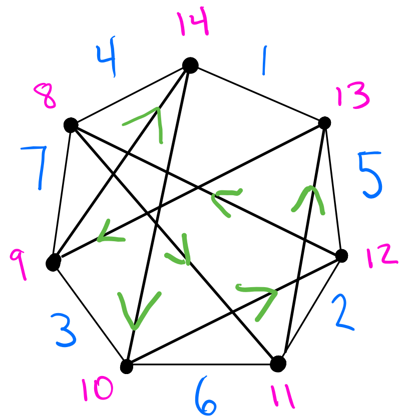
STRONG VMTL for C_7

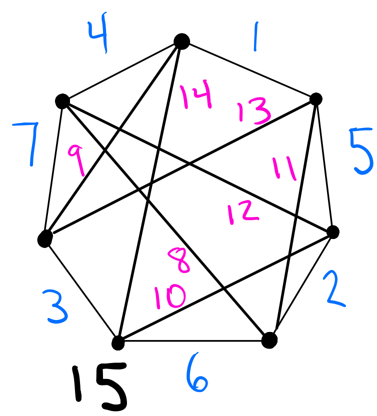




Kotzig + Rosa

which regular graphs (deg 2, 3, 4) have M-valuation?





Gray: G odd-order

with spanning subgraph H (with SVMTL)

$G - E(H)$ even regular $\to$ G has SVMTL

Motivating Conjecture (Gray)

A 2-regular graph of odd order has a SVMTL iff it isn't

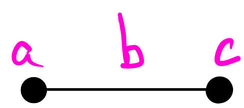
$(2t-1)C_3 \cup C_4$ OR $2t\,C_3 \cup C_5$

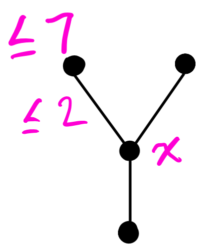
MacDougall's Conjecture:

G regular $\deg G \geq 2$

then G is vertex-magic

unless $G \cong \triangle\,\triangle$





weight of x

$\geq 1+2+3+4$

J. Holden et al.

$(2t-1)C_3 \cup C_4$ OR. $2t\, C_3 \cup C_5$

always has a SVMTL

provided the order is

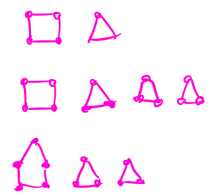
at least 17.

$2t\, C_3 \cup C_7 \quad t \geq 1$

done at the same time.

Kimberley + MacDougall

All 2-regular graphs of odd order (< 30) have SVMTL except



$\therefore$ all odd < 30 reg. graphs are vertex-magic

C_{23} $\quad$ 2.6×10^{10} SVMTLs

$6C_3 \cup C_5$ $\quad$ 1191 SVMTLs

New work:

$C_m \cup sC_3$ (odd order)

has a SVMTL

$m = 5, 6, 8, 9, 10, 11$

$3 \le m \le 11$ settled.

Shifted Kotzig array

each Row a permutation of

$-r, \quad -(r-1), \cdots 0, 1, 2, \cdots, r-1, r$

Kotzig:

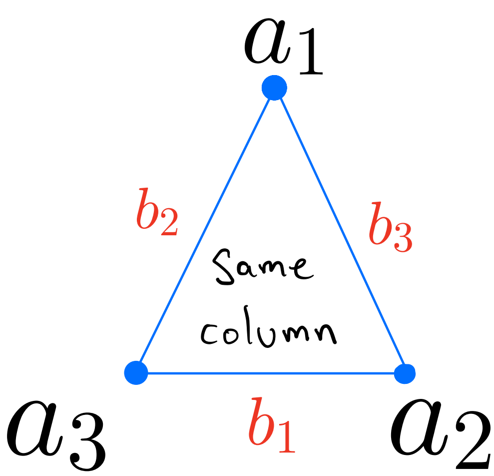
$$\begin{array}{cccc|cccc}
-r, & -(r-1), & \cdots, & 0 & 1, & 2, & \cdots, & r \\
r, & r-2, & & ,-r & r-1, & r-3, & \cdots, & -(r-1) \\
0, & 1, & \cdots, & r & -r, & -r+1, & \cdots, & -1
\end{array}$$

Our techical breakthrough

Different, special, Kotzig arrays

eg

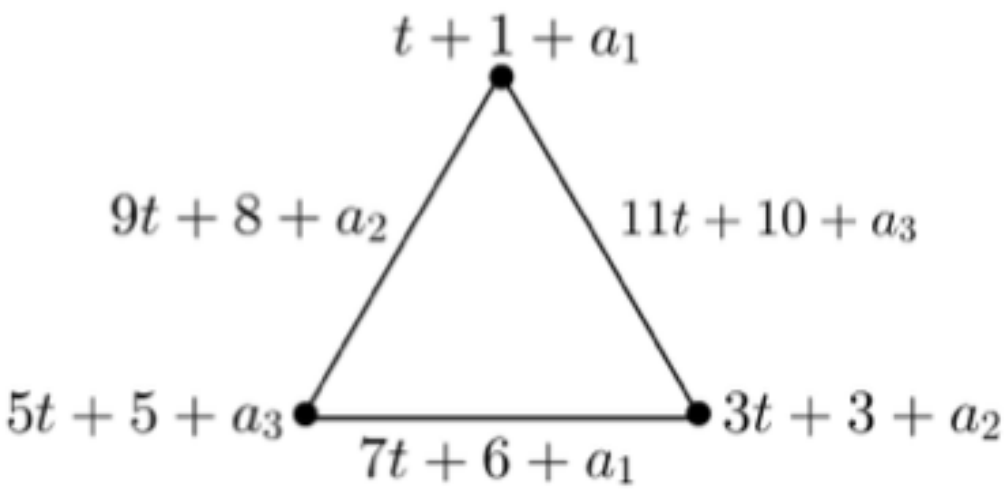
$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$



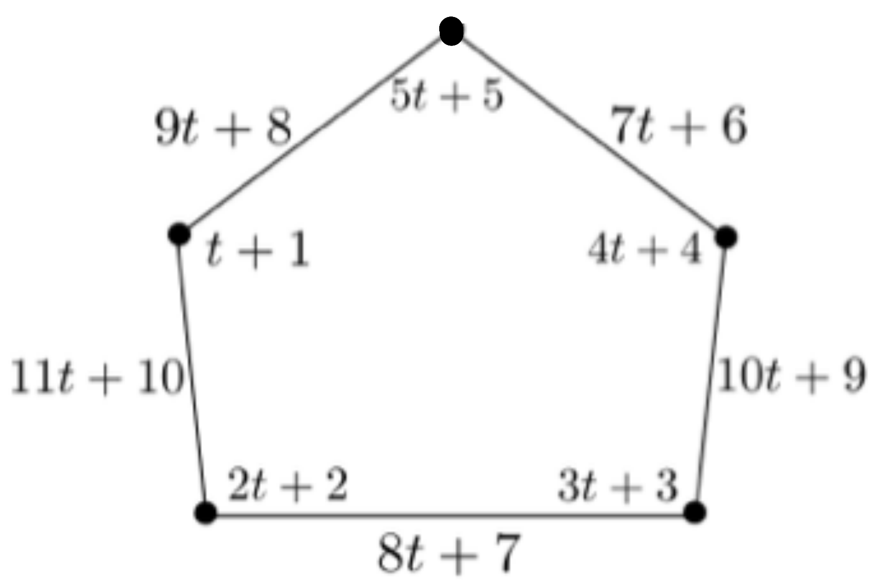
$$b_1 - a_1 = b_2 - a_2 = b_3 - a_3$$

Use $b_i - a_i =$ order of G.

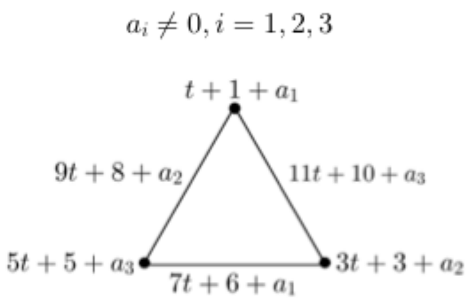
$$a_i \neq 0, i = 1, 2, 3$$



$t + 1 + a_1$
$9t + 8 + a_2$
$11t + 10 + a_3$
$5t + 5 + a_3$
$3t + 3 + a_2$
$7t + 6 + a_1$



$9t + 8$
$5t + 5$
$7t + 6$
$t + 1$
$4t + 4$
$11t + 10$
$10t + 9$
$2t + 2$
$3t + 3$
$8t + 7$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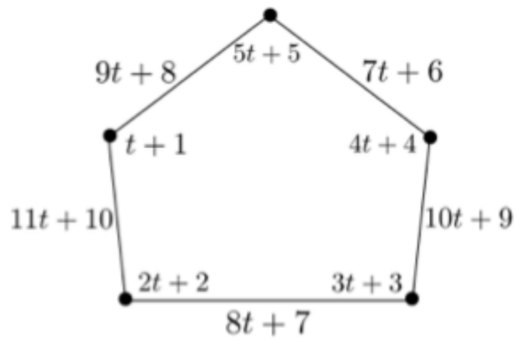
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$7t + 6$
$t + 1$
$4t + 4$
$11t + 10$
$10t + 9$
$2t + 2$
$3t + 3$
$8t + 7$

Lemma 9 *For each positive integer $s \geq 3$, there is a $3 \times (2s+1)$ shifted Kotzig array (using the integers $-s$ to s) such that three of the columns are as shown:*

$$\begin{array}{ccc} -1 & 0 & 1 \\ -(s-1) & 0 & s-1 \\ s & 0 & -s \end{array}.$$

Proof. We will omit the three columns in the statement of the lemma. We will distinguish four cases. In case 1 and case 2, the arrays are of the form $[A, -A]$, with the second half of the matrix being the negative of the first half. As a result we will only provide the first half of each matrix for these cases.

Case 1: $s = 4r$, $r \geq 1$

If $r = 1$:

$$\begin{array}{ccc} 2 & 3 & 4 \\ 1 & -4 & -2 \\ -3 & 1 & -2 \end{array}$$

If $r > 1$:

$$\begin{array}{cccc|cc} 2 & 3 & \cdots & 2r & 3r & 4r \\ 4r-3 & 4r-5 & \cdots & 1 & -4r & -2r \\ -4r+1 & -4r+2 & \cdots & -2r-1 & r & -2r \end{array}$$

$$\begin{array}{cccc|cccc} 2r+1 & 2r+2 & \cdots & 3r-1 & 3r+1 & 3r+2 & \cdots & 4r-1 \\ -2 & -4 & \cdots & -2r+2 & -2r-2 & -2r-4 & \cdots & -4r+2 \\ -2r+1 & -2r+2 & \cdots & -r-1 & -r+1 & -r+2 & \cdots & -1 \end{array}$$

Case 2: $s = 4r-1$, $r \geq 1$

If $r = 1$:

$$\begin{array}{cc} 2 & 3 \\ -3 & -1 \\ 1 & -2 \end{array}$$

If $r > 1$:

$$\begin{array}{cccc|cc} 2 & 3 & \cdots & 2r-1 & 3r-1 & 4r-1 \\ 4r-4 & 4r-6 & \cdots & 2 & -4r+1 & -2r+1 \\ -4r+2 & -4r+3 & \cdots & -2r-1 & r & -2r \end{array}$$

$$\begin{array}{cccc|cccc} 2r & 2r+1 & \cdots & 3r-2 & 3r & 3r+1 & \cdots & 4r-2 \\ -1 & -3 & \cdots & -2r+3 & -2r-1 & -2r-3 & \cdots & -4r+3 \\ -2r+1 & -2r+2 & \cdots & -r-1 & -r+1 & -r+2 & \cdots & -1 \end{array}$$

Case 3: $s = 4r - 2$, $r \geq 2$.

If $r = 2$:

$$\begin{array}{cccccccccc}
-6 & -5 & -4 & -3 & -2 & 2 & 3 & 4 & 5 & 6 \\
4 & 2 & -1 & 6 & 3 & -6 & -4 & 1 & -3 & -2 \\
2 & 3 & 5 & -3 & -1 & 4 & 1 & -5 & -2 & -4
\end{array}$$

If $r > 2$:

$$\begin{array}{cccc|c}
-4r+2 & -4r+3 & \cdots & -2r-1 & -2r \\
4r-4 & 4r-6 & \cdots & 2 & 4r-2 \\
2 & 3 & \cdots & 2r-1 & -2r+2
\end{array}$$

$$\begin{array}{ccc|cccc}
-2r+1 & -2r+2 & -2r+3 & -2r+4 & -2r+5 & \cdots & -2 \\
-2 & -4 & -3 & -7 & -9 & \cdots & -4r+5 \\
2r+1 & 2r+2 & 2r & 2r+3 & 2r+4 & \cdots & 4r-3
\end{array}$$

$$\begin{array}{cccc|cccc}
2 & 3 & \cdots & 2r & 2r+1 & 2r+3 & \cdots & 4r-3 \\
4r-5 & 4r-7 & \cdots & -1 & -6 & -10 & \cdots & -4r+2 \\
-4r+3 & -4r+4 & \cdots & -2r+1 & -2r+5 & -2r+7 & \cdots & 1
\end{array}$$

$$\begin{array}{c|cccc}
2r+2 & 2r+4 & 2r+6 & \cdots & 4r-2 \\
-5 & -8 & -12 & \cdots & -4r+4 \\
-2r+3 & -2r+4 & -2r+6 & \cdots & -2
\end{array}$$

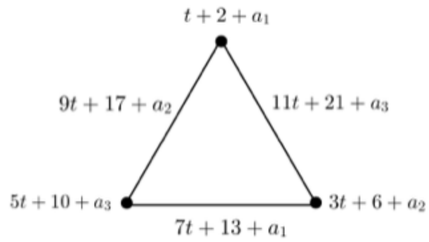
Case 4: $s = 4r - 3$, $r \geq 2$

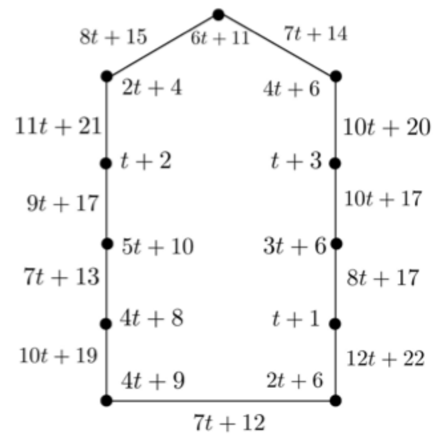
$$\begin{array}{cccc|cccc}
-4r+3 & -4r+5 & \cdots & -2r-1 & -4r+4 & -4r+6 & \cdots & -2r \\
4r-5 & 4r-9 & \cdots & 3 & 4r-3 & 4r-7 & \cdots & 5 \\
2 & 4 & \cdots & 2r-2 & -1 & 1 & \cdots & 2r-5
\end{array}$$

$$\begin{array}{c|cccc}
-2r+1 & -2r+2 & -2r+3 & \cdots & -2 \\
2 & -2 & -4 & \cdots & -4r+6 \\
2r-3 & 2r & 2r+1 & \cdots & 4r-4
\end{array}$$

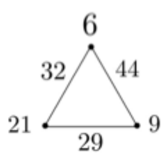
$$\begin{array}{cccc|c|cccc}
2 & 3 & \cdots & 2r-3 & 2r-2 & 2r-1 & 2r & \cdots & 4r-3 \\
4r-6 & 4r-8 & \cdots & 4 & -4r+3 & 1 & -1 & \cdots & -4r+5 \\
-4r+4 & -4r+5 & \cdots & -2r-1 & 2r-1 & -2r & -2r+1 & \cdots & -2
\end{array}$$

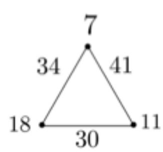
$$a_1 \neq -1, 0, 1$$
$$a_2 \neq -(s-1), 0, s-1$$
$$a_3 \neq s, 0, -s$$

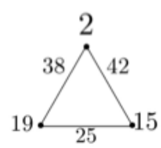


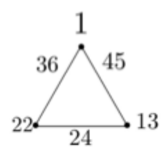


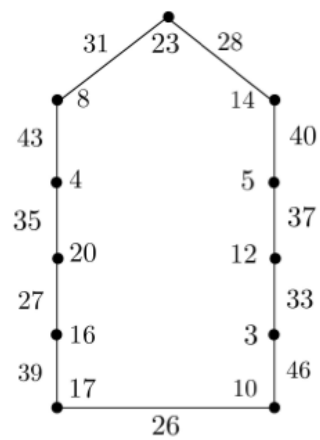
$$\begin{array}{rr|ccc|rr} 2 & 3 & -1 & 0 & 1 & -2 & -3 \\ -3 & -1 & -(s-1) & 0 & s-1 & 3 & 1 \\ 1 & -2 & s & 0 & -s & -1 & 2 \end{array}$$











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I. Gray, Vertex-magic total labellings of regular graphs, SIAM J. Discrete Math., Vol. 21, Issue 1 (2007), pp. 170-177.

J. Holden, D. McQuillan and J. M. McQuillan, A conjecture on strong magic labelings of 2-regular graphs, Discrete Math. 309 (2009), 4130-4136.

J. S. Kimberley and J. A. MacDougall, All regular graphs of small order are vertex-magic, Australas. J. Combin. 51 (2011) 175-199.

W. D. Wallis, Vertex magic labelings of multiple graphs, Congr. Numer., 152 (2001), pp. 81-83.

Preprint available upon request.
Also possibly of interest:
Magic Labelings of triangles, Dan and Jim, Discrete Mathematics, (309) 2009 pp. 2755-2762.

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