Ramsey, Mahler, Prime, and Concurrencies: Mining Archivable 363-bit Tokens as NFTs

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For a complete ternary tree T of depth n with 0–1 labeled edges, its weight $f(T)$ is the least number of path labels among binary subtrees. The maximum $f(n)$, over all labelings, of these weights starts with 1, 2, 3, 4, 8, and we have $9 \leq f(6) \leq 16$. These were shown by Downey-Greenberg-Jockusch-Milans in their paper in Combinatorica in 2011, where they put bounds on $f(n)$ and solved a problem in computability theory. We show $f(6) \geq 12$, but focus on some challenging computations in still smaller depths. Our main Ramsey-type accomplishments are computations and mining illustrations in depth 5 (with 2^{363} trees): (i) We approximate the percentages for weights 1–8: 0, 1.04, 23.6, 55.0, 18.8, 1.54, 0, 0; (ii) Our NFT supplements include thousdands of trees of the more interesting rare weights 7–8, via mining based on a Ramseyan property in depth 3: the 3000 (resp. 2900) NFT tokens are given 1000-Norm (resp. 500-norm) denomination. Next, to the base-2 uncommonness, we add base-3 rarity. Our Mahler-Ramsey products additionally relate to Mahler's $\frac{3}{2}$-problem and large stopping times showing a number is not a Z-number. Our version is iterated multiplication of integers by $\frac{2}{3}$. For a certain sequence of integer intervals, we present a choice function. The left endpoints of our intervals are "The 1st Round Up in Times by $\frac{2}{3}$": $a(g) = \min\{\ k \mid \{g(\frac{2}{3})^k\} \geq \frac{1}{2} \vee k = \lceil \log_{\frac{3}{2}}(g) \rceil\ \}$, which has $\limsup = \infty$. The right end point of our intervals is a related stopping time, "The 1st Intermediate Rounding Error": multiplying by $\frac{2}{3}$ until rounding down at each step does not match with rounding down just at the end. In contrast, our interpolation is a "No Sudden Death" function: previous value capable of making an alert. Our interest is in simultaneous pecularity in base 2 and base 3: numbers g with large values of $a(g)$ for the size of g, which, additionally, in base 2 give rise to one of the less common Ramsey weights. As $2^{363} \in (3^{229}, 3^{230})$, (iii) we present Mahler-rare numbers (stopping time at least 230) that are simultaneously Ramsey-uncommon ($w = 6$ or 2): serial numbers for our $600 + 300 = 900$ tokens of 625-Norm denomination. Finally (iv) we present triply rare concurrencies by mining $350 + 110 = 460$ serial numbers (given 1250-Norm denomination) that are either in addition to properties in (iii) also prime, or Mahler-rare and prime with the next odd also a prime (a lower twin prime). All the mentioned supplements can be seen at `https://opensea.io/RareConcurrencies`.