

On 3-Uniform Tight 6-Cycle Designs

Lydia Butters^{1,*} Kylie Cassulo^{2,*} Lauren Falck^{3,*} Levi Neiburger^{4,†,*}

¹University of Northern Iowa, buttelab@uni.edu; ²Illinois State University kyliecassulo@gmail.com;

³North Fayette Valley High School lfalck17@gmail.com; ⁴Normal Community High School leviherbal@gmail.com

Hypergraphs are extensions of the well-studied concept of a graph. For positive integers t and v , a t -uniform hypergraph on v vertices has a set V of size v as its vertex set and a set of t -element subsets of V as its edge set. Thus graphs are 2-uniform hypergraphs. The complete t -uniform hypergraph of order v , denoted $K_v^{(t)}$, has the set of all t -element subsets of a vertex set V of size v as its edge set. For a positive integer λ and a t -uniform hypergraph H , λ -fold H , denoted ${}^\lambda H$, is the multi-hypergraph obtained by replacing each edge e in H with λ copies of e . For graphs (or multigraphs) G and K with G a subgraph of K , it is common to investigate when the edge set (or multiset) of K can be partitioned into isomorphic copies of G . Such partitions are known as G -decompositions of K . A popular case for investigation is when G is an m -cycle. For integer $m \geq 3$, an m -cycle, denoted C_m , is a graph with vertex set $\{v_1, v_2, \dots, v_m\}$ and edge set $\{v_i, v_{i+1} : 1 \leq i \leq m-1\} \cup \{v_m, v_1\}$. Authors have investigated C_m -decompositions of K_n , $K_n - I$, where I is a 1-factor, $K_{m,n}$, and ${}^\lambda K_n$, among many others. There are multiple extensions of the concept of an m -cycle to t -uniform hypergraphs with $t > 2$. For example, for 3-uniform hypergraphs there is the concept of a loose m -cycle, of a Berge m -cycle, and of a tight m -cycle, as well as other cycle-like hypergraphs. A tight 3-uniform m -cycle is commonly seen as the best extension of C_m . For $m \geq 4$, a 3-uniform tight m -cycle, denoted TC_m , is a hypergraph with vertex set $\{v_1, v_2, \dots, v_m\}$ and edge set $\{v_i, v_{i+1}, v_{i+2} : 1 \leq i \leq m-2\} \cup \{\{v_{m-1}, v_m, v_1\}, \{v_m, v_1, v_2\}\}$. Over the past few years, participants in the *Illinois State University Math REU for Pre-service and In-service Teachers* have investigated, and in most cases settled, the decompositions of various 3-uniform hypergraph structures into tight 6-cycles and into tight 9-cycles. Among the settled questions are TC_6 -decompositions of $K_v^{(3)}$, of ${}^\lambda K_v^{(3)}$, of $K_v^{(3)} - I$, where I is a 1-factor, as well as of the complete 3-uniform “bipartite” hypergraph $L_{n,n}^{(3)}$, and of the complete tripartite hypergraph $K_{n,n,n}^{(3)}$. The corresponding TC_9 -decompositions have also been investigated by the REU. In this talk, we focus on TC_6 -decompositions of ${}^\lambda K_v^{(3)}$ and of $L_{n,n}^{(3)}$. This work presents results from a combination of projects that were recently completed at the *Illinois State University REU for Pre-service and In-service Teachers* (National Science Foundation Grant Number A1950357) under the direction of Ryan Bunge, Saad El-Zanati, and William Turner.