On monochromatic pairs with nondecreasing diameters

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Our problem comes from the field of combinatorics known as Ramsey theory. Ramsey theory, in a general sense, is about identifying the threshold for which a family of objects, associated with a particular parameter, goes from never or sometimes satisfying a certain property to always satisfying that property. Research in Ramsey theory has applications in design theory and coding theory. For integers m, r, and t, we say that a set of n integers colored with r colors is (m,r,t)-permissible if there exist t monochromatic subsets $B_1, B_2, \ldots, B_t$ such that (a) we have $|B_1| = |B_2| = \cdots = |B_t| = m$, and (b) the largest element in B_i is less than the smallest element in B_{i+1} for $1 \leq i \leq t-1$, and (c) the diameters of the subsets are nondecreasing. We define $f(m,r,t)$ to be the smallest integer n such that every string of length n is (m,r,t)-permissible. In this thesis, we first look at some preliminary results for values of $f(m,r,t)$, specifically when each individual parameter is 1 as the others vary. We then show that $f(m,r,t)$ exists for all possible positive parameters. We proceed by determining $f(2,2,t)$ for all positive integers t. We conclude by considering colorings with more than two colors and monochromatic sets that have more than 2 elements, as well as investigating an enumeration of the number of ways a string could be realized as (m,r,t)-permissible.