

# On Decompositions of the $\lambda$ -fold Complete 3-Uniform Hypergraph into the $3 \times 3$ Grid Graph

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The  $\lambda$ -fold complete 3-uniform hypergraph of order  $v$ , denoted  ${}^\lambda K_v^{(3)}$ , has a set  $V$  of size  $v$  as its vertex set and the collection of all 3-element subsets of  $V$ , each subset repeated  $\lambda$  times, as its edge multiset. Let  $H$  denote the hypergraph with vertex set  $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$  and edge set  $\{\{v_1, v_2, v_3\}, \{v_4, v_5, v_6\}, \{v_7, v_8, v_9\}, \{v_1, v_4, v_7\}, \{v_2, v_5, v_8\}, \{v_3, v_6, v_9\}\}$ . Then  $H$  is 3-uniform and 2-regular and can be visualized as the  $3 \times 3$  grid graph. We give necessary and sufficient conditions for the existence of a decomposition of the  $\lambda$ -fold complete 3-uniform hypergraph of order  $v$  into isomorphic copies of the grid graph. This work was completed at the *Illinois State University REU for Pre-service and In-service Teachers* (National Science Foundation Grant Number A1950357) under the direction of Ryan Bunge and Saad El-Zanati.