Worst-case Scenarios for the Cycle Graph

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Given a permutation in S_n, the pop-stack sorting method reverses each descending run into an ascending run. We say a permutation is a worst-case scenario if it takes $(n-1)$ pops to reach the identity permutation $123\ldots n$. In this talk we explore the dynamics of the pop-stack sorting method on maximal tubings. Given a graph $G = (V, E)$ we say a tube is a nonempty subset of V whose induced subgraph is connected. A tubing is a collection of tubes which satisfy a certain pairwise compatibility; and a *maximal tubing* is a maximal collection of pairwise compatible tubes. When G is the complete graph, each maximal tubing corresponds to a permutation. When G is a path graph, then the maximal tubings can be partially ordered to give the Tamari lattice. We will explain how to compute the pop-stack sorting method on maximal tubings, focusing on the case where G is the cycle graph. We conjecture that the number of worst-case scenarios are counted by the second difference of Catalan numbers. We describe how to find worst-case scenarios for the path graph in the cycle graph, and finish by describing the remaining worst-case scenarios.