A Decomposition Method on Solving the Linear Arboricity Conjecture

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A *linear forest* is a disjoint union of path graphs. The *linear arboricity of a graph* G, denoted by $\mathrm{la}(G)$, is the least number of linear forests into which the graph can be partitioned. Clearly, $\mathrm{la}(G) \geq \lceil \Delta(G)/2 \rceil$ for any graph of maximum degree $\Delta(G)$. For the upper bound, the long-standing *Linear Arboricity Conjecture* (LAC) due to Akiyama, Exoo, and Harary from 1981 asserts that $\mathrm{la}(G) \leq \lceil (\Delta(G)+1)/2 \rceil$. A graph is a *pseudoforest* if each component contains at most one cycle. We prove that *the union of two pseudoforests of maximum degree at most* 3 *can be decomposed into three linear forests.* Combining it with a recent result of Wdowinski on the minimum number of pseudoforests that a graph can be decomposed into, we prove the LAC for the following simple graph classes:

- k-degenerate graphs with maximum degree $\Delta \geq 3k-1$;
- graphs on nonnegative Euler characteristic surfaces provided the maximum degree $\Delta \neq 7$, and graphs on negative Euler characteristic surfaces provided the maximum degree $\Delta \geq 3\left\lceil \left(5+\sqrt{49-24\epsilon}\right)/4 \right\rceil - 1$;
- graphs with no K_t-minor, where $3 \leq t \leq 9$, provided the maximum degree $\Delta \geq 3t-7$; in particular, all graphs with no K_4 minor, and graphs with no K_5 minor with the maximum degree $\Delta \neq 7$;
- graphs with no K_t-minor for large t provided the maximum degree at least $(3\alpha + o(1))t\sqrt{\ln t}$, where $\alpha = 0.319\ldots$;
- graphs with no K_5-subdivision provided the maximum degree $\Delta \neq 7$, and so all planar graphs with maximum degree $\Delta \neq 7$.