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Managing the Tax Liability of a Property-Liability Insurance Company

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Abstract

The income tax burden placed upon a property-liability insurance company creates a variable liability with profound effects on the functioning of the enterprise. It directly affects product pricing and asset investment policies and, therefore, the potential profitability of the insurer. Research has identified fuzzy set theory as a potentially useful modeling paradigm for insurance uncertainty—in claim cost forecasting, underwriting, rate classification, and premium determination. We view the insurance liabilities, properly priced, as a management tool of the short position in the government tax option. To implement that tool, we propose a new method of measuring uncertainty of taxes. Critical parameters of underwriting and investment are modeled as fuzzy numbers, leading to a model of uncertainty in the tax rate, rate of return, and the asset-liability mix.

Introduction

In this work, we analyzes the tax management policy of a property-liability insurance company. Myers' Theorem (1984) implies that the present value of the expected tax liability, the government's tax option, is determined solely by the effective tax rate and the risk-free interest rate. Therefore, controlling the effective tax rate of the firm is crucial in its financial management. A firm that can craft a lower effective tax rate than its competitors does enjoy a competitive advantage, but in competitive equilibrium this lowering of tax rates is achieved by all firms and results in lower premium rates. A prerequisite to effective management of the firm's tax liability is the ability to measure that liability and to forecast movements in that measurement. Uncertainty clearly plays a role in accurate forecasting—uncertainty that stems from both probabilistic and nonprobabilistic sources. Cummins and Derrig (1997) model both types of uncertainty in the pricing and

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underwriting accept/reject context using fuzzy sets methods. We also examine the uncertainty of the firm's tax rate by using fuzzy sets methodology for modeling that uncertainty. Our analysis, using a simplified model of an insurer's asset-liability portfolio, implies that uncertainty is indeed quite great, and may be underestimated under other methodologies. We begin with the tax consequences of the insurance company's investment portfolio.

Myers' Theorem and Its Implications

We assume that an insurance corporation holds an asset portfolio yielding a one-period investment return, and is subject to a tax liability on realized income. We also assume a simple capital asset pricing model (CAPM) market. Let T be the effective tax rate on the investment income, for now taken to be known with certainty.

Myers' Theorem (1984) says that the risk-adjusted present value of the tax liability on investment income from a risky investment portfolio held by a corporation is

$$PV(T\tilde{r}_A) = \frac{Tr_F}{1 + r_F}, \tag{1}$$

where $\tilde{r}_A$ is the rate of return on the risky portfolio, and r_F is the risk-free rate of return. In other words, the present value of the tax liability on the risky return is calculated as if that return were the risk-free rate. The present value of the tax liability is independent of the investment strategy and is determined solely by the effective tax rate and the risk-free rate.

Derrig (1994) notes that the tax liability itself is not risk free. In fact, the beta of the tax can be determined to be

$$\beta_{TAX} = \beta_A \frac{1 + r_F}{r_F}, \tag{2}$$

where β_A is the beta of the risky asset utilized by the company's investment strategy. Note that, unless that asset is risk free or the risk-free rate equals zero, $\beta_{TAX} > \beta_A$.

The present value of the after-tax final investment holdings of the corporation equals

$$PV(1 + (1 - T)\tilde{r}_A) = \frac{1 + (1 - T)r_F}{1 + r_F}, \tag{3}$$

and the after-tax beta of the risky portfolio is

$$\beta_{AFTER-TAX} = \beta_A \frac{(1-T)(1+r_F)}{1+(1-T)r_F}. \tag{4}$$

The implication of these results is that the effective tax rate and the risk-free rate fully determine the present value of the expected investment tax liability, and when combined with the market riskiness of the investment portfolio, the after-tax, effective, riskiness of that portfolio.

Following Myers, we consider a one-period insurance company market value balance sheet at the time a policy is issued:

Assets	Liabilities
Asset Value	Present Value of Expected Losses and Expenses
(Premium + Equity Invested)	Present Value of Underwriting Tax
	Present Value of Investment Tax
	Present Value of Future Profits and Equity Returned

Any firm by virtue of its existence assumes a short position in a security producing cash flows of taxes payable by the firm. The government collecting the tax is long that security. One might naturally expect a firm to develop strategies to manage this short position.

In the case of tax on investment income, we see certain important implications for its management given by Myers' Theorem. The present value of tax can be matched perfectly by investing a portion of assets given by the rate T at the risk-free rate (e.g., if the effective tax rate is 35 percent, invest 35 percent of your portfolio in Treasury bills maturing when taxes are due and use the interest earned to pay taxes). However, from the investor's perspective, the present value of the tax burden imposed on the investor's equity in the insurance firm is transferred to the policyholder through the premium charged (Myers and Cohn, 1987). An increase in the tax liability on the balance sheet—for example, through a higher investment tax rate—results in an increase in the assets acquired from premiums.

The implication is that the effective tax rate on combined investment and underwriting income is an essential parameter in the implementation of theoretical underwriting profit models (Doherty and Garven, 1986; Cummins, 1990; Taylor, 1994). In this work, we will investigate one issue related to the management of the effective tax rate on investment income. Specifically, can fuzzy sets theory be used as a tool for management of uncertainty arising from forecasts of the effective tax rate and after-tax rate of return?

Crafting an Effective Tax Rate

Rational investors seek after-tax risk. In a world with taxes, there is a question of whether true tax advantages exist, when all differences in risk are properly accounted for (Derrig, 1994). Stone (see Derrig, 1990, pp. 7–9) introduced the concept of a regulatory standard investment portfolio in the context of an insurance company—that is, a portfolio of zero-coupon Treasury securities whose maturities are matched to the expected loss payment patterns.[1] If this regulatory standard investment portfolio is used, computation of the effective investment tax rate is simple: all income from Treasury securities is fully taxable at 35 percent corporate tax rate.[2] Further, the short position in the tax liability is fully covered by investing the portion of the policyholder premium equal to the expected tax liability in Treasury securities.

Myers (1984) poses the question of whether some other investment portfolio with lower tax rates is actually superior in all relevant aspects to the regulatory standard portfolio, so that it brings about an additional value to the company holding such a portfolio. If such a portfolio exists, it must contain risky securities. In that case, the short position in the tax liability can be fully covered provided either (1) the effective tax rate of the portfolio is known with certainty, so the tax portion of the policyholder premium will exactly cover the option price of the tax liability, or (2) the uncertainty in the effective tax rate of the portfolio can be measured and eliminated.

Cummins and Grace (1994) determine that insurers perceive a yield advantage for longer maturity tax exempt bonds, implying the existence of a portfolio with an effective tax rate lower than 35 percent. This can be justified only by a tax clientele effect—a marginal buyer with a marginal tax rate of less than the insurers' 35 percent less, at a minimum, their 5.1 percent proration, alternative minimum tax, and capital gains income tax. Of course, the question of comparison of risk characteristics of longer maturity tax exempt bonds with the regulatory standard portfolio, or any other portfolio, remains a complicated issue to resolve.

An insurer, nevertheless, acts as a financial intermediary between, on one hand, the claimholders (policyholders, investors, government) and, on the other hand, the suppliers of securities. What Myers' Theorem implies is that:

- Claims of government (tax liabilities) are transferred to policyholders at the prevailing effective tax rates, so that an economic profit can be earned by crafting a lower effective tax rate (assuming, of course, that this strategy is not available to, or employed by, the competitors of the firm, in which case a lower competitive premium develops);
- Investment tax liability acts to dampen the riskiness of the after-tax investment income of the insurer, so that higher expected profit can be earned by seeking a higher level of risk if sufficient return compensation is available.

Traditionally, the pursuit of a lower effective tax rate has been performed by insurers through investments in tax exempt bonds, as indicated by Cummins and Grace (1994). Other tax-preferred strategies have been employed as well, such as the corporate dividend exemption, or a capital gains preferred tax rate.[3] Compa-

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[1] The Regulatory Standard Company could also utilize immunization techniques—for example, match durations rather than maturities—but this would not produce an exact match of expected cash flows—that is, a match of the net premium, loss, expense, and equity flows.

[2] The marginal corporate tax rate in the United States at the time of this writing is 35 percent.

[3] The current stock dividend exemption available to property-liability insurers is nominally 70 percent. But through the proration provision of the tax code, at least 15 percent of the excluded 70 percent is

nies routinely invest in a variety of assets, each with an accompanying expected tax liability. Taxable and tax exempt bonds dominate the average property-liability insurer's investment portfolio, with stocks accounting for about 20 percent of the assets (Cummins and Grace, 1994, Table 1). The differential treatment of stocks and bonds in the U.S. tax code underlies the importance of the allocation of those types of investments, together with the tax shield of underwriting losses, in determining an overall effective tax rate for the firm (Almagro and Ghezzi, 1988). We now describe the fuzzy sets methodology that will become the modeling context for tax management.

Fuzzy Parameters

As stated above, Myers' Theorem implies that calculation of the effective investment tax rate becomes an essential part of both the ratemaking and portfolio management process. However, that calculation is not only affected by the composition of the insurer's investment portfolio, with varying rates of investment tax on tax exempt bonds, taxable bonds, preferred stock, and common stock, and insurance liabilities but also by future changes in the tax code and IRS interpretations of that code. Derrig (1994) shows how the 1986 Tax Reform Act sharply increased effective tax rates of U.S. property-liability insurers.

Clearly, the investment tax rate will vary within the range between zero percent (assuming a tax exempt bond portfolio issued completely before 1986) and 35 percent. In practice, the calculation of the effective tax rate, including the implicit tax embedded in the lower yields of tax-exempt bonds, becomes immensely complicated, especially when projecting future income and taxes, where the returns also become uncertain. We believe that we have made a case for estimation of the effective tax rate as an important tool of asset-liability management. However, we also believe that the traditional probabilistic approach may not be appropriate in this context. Uncertainty of taxes goes beyond the standard probability model, in which all outcomes of experiments are clearly defined, and future states of the world are mutually exclusive. Even legislated taxes are subject to interpretations, both in the regulatory context of the Internal Revenue Code and in the practical terms of how the firms perceive them. Thus we propose that the management of the tax liabilities should be undertaken with the use of an alternative uncertainty model. Likewise, the choice among estimates for the expected after-tax returns on risky assets is not amenable to purely probabilistic models.[4]

We propose the use of fuzzy sets theory for estimation of the uncertainty in the tax rate and after-tax rate of return of a property-liability insurer. Zadeh (1965) suggests a methodology for uncertainty radically different from traditional probabilistic models, including the uncertainty caused by vagueness and imprecision of human perception or other human factors.

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taxed at the marginal rate of 35 percent, yielding an overall effective tax rate of at least 14.2 percent. Alternative minimum tax payments can drive that effective rate higher than 14.2 percent.

[4] Good discussions of what has become known as the equity risk premium puzzle can be found in Mehra and Prescott (1985), Ibbotson (1996, pp. 151–161), and Abel (1996).

There may be several reasons for wanting to search for models of a form of uncertainty other than randomness. One is that vagueness is unavoidable. It is caused by the imprecision of natural language or human perception of the phenomena observed. But also when the phenomena observed become so complex that exact measurement involving all features considered significant would be next to impossible, mathematical precision is often abandoned in favor of more workable simple, but vague, "common sense" models. Complexity of the problem may be another cause of vagueness.

These reasons were the motivation behind the development of the fuzzy sets theory (FST). This area has become a dynamic research and applications field, with success stories ranging from a fuzzy logic rice cooker to an artificial intelligence in control of the Sendai subway system in Japan.

Let us define the basic concepts of FST. Recall that a *characteristic function* of a subset E of a universe of discourse U is

$$\chi_E(x) = \begin{cases} 1 \text{ if } x \in E \\ 0 \text{ if } x \notin E. \end{cases} \tag{6}$$

In other words, the characteristic function describes the membership of an element x in a set E. It equals one if x is a member of E and zero otherwise.

Zadeh (1965) suggests that there are sets whose membership should be described differently. One example would be the set of "good drivers." This is an important concept in auto insurance, yet its inescapable vagueness is obvious.

In the fuzzy sets theory, an element's membership in a set is described by the *membership function* of the set. If U is the universe of discourse, and $\tilde{E}$ is a fuzzy subset of U, the membership function $\mu_E : U \rightarrow [0,1]$ assigns to every element x its degree of membership $\mu_E(x)$ in the set $\tilde{E}$. We write either (E, μ_E) or $\tilde{E}$ for that fuzzy set to distinguish it from the standard set notation E. The membership function is a generalization of the characteristic function of an ordinary set. Ordinary sets are termed *crisp sets* in fuzzy sets theory, and they are considered a special case. A fuzzy set is crisp if, and only if, its membership function does not have fractional values.

On the base of this definition, one then develops such concepts as set theoretic operations on fuzzy sets (union, intersection, etc.), as well as the notions of fuzzy numbers, fuzzy relations, fuzzy arithmetic, and approximate reasoning (known popularly as "fuzzy logic"). Pattern recognition, or the search for structure in data, provided an early impetus for developing FST because of the fundamental involvement of human perception (Dubois and Prade, 1980) and the inadequacy of standard mathematics to deal with complex and ill-defined systems (Bezdek and Pal, 1992). A complete presentation of all aspects of FST is available in Zimmerman (1991). Numerical manipulations of FST are amply described in Kaufmann and Gupta (1991).

A *fuzzy number* is a fuzzy subset of the real line such that its membership function has a value of one for at least one point, is zero outside a certain closed interval (finite support), and has a convex area under its graph. If two fuzzy numbers are given, $\widetilde{A}$ with membership function μ_A, and $\widetilde{B}$ with membership function μ_B, then fuzzy addition is performed by defining the membership function of $\widetilde{C} = \widetilde{A} + \widetilde{B}$ as μ_C with $\mu_C(z) = \max\{\min(\mu_A(x), \mu_B(y)) : x + y = z\}$ (Kaufmann and Gupta, 1991). Similar application of the so-called maximin principle (Zadeh, 1965) allows for the creation of other fuzzy arithmetic operations. We will utilize them in the illustrations below.

The first recognition of FST applicability to the problem of insurance underwriting is due to DeWit (1982). Lemaire (1990) sets out a more extensive agenda for FST in insurance theory, most notably in the financial aspects of the business. Under the auspices of the Society of Actuaries, Ostaszewski (1993) assembles a large number of possible applications of fuzzy sets theory in actuarial science. Cummins and Derrig (1993, 1997) complement that work by exploring applications of fuzzy sets to property-liability insurance forecasting and pricing problems. Derrig and Ostaszewski (1995) apply fuzzy clustering algorithms to problems of auto rating territories and fraud detection. Young (1996) models the rate changing decision problem in fuzzy logic terms.

This article illustrates how FST can be useful in estimation of the effective tax rate and after-tax rate of return on an insurance firm's asset and liability portfolio. Let us begin with a simple model of an insurance firm's expected investment income and tax position. Table 1 displays the expected CAPM results for a simple one-period investment portfolio. We assume a bond/stock allocation of 80/20, approximately the allocation of the U.S. property-liability industry in 1994.[5] We assume only U.S. government bond holdings and diversified (beta = 1) stock holdings. Using corporate bonds, which are taxed at the same rate as Treasuries, would only increase the expected yield (and uncertainty) and, therefore, the bond assessment weight in the tax rate calculation. Using tax exempt bonds with implicit tax rates equal to the effective property-liability rate of less than 30 percent would be the equivalent of using Treasury securities but with a slightly higher beta than we assume here. The estimation of the effective tax rate of tax exempt securities with a positive tax advantage to property-liability insurers, such as perceived by U.S. portfolio managers (Cummins and Grace, 1994) is beyond the scope of this article.

We use CAPM expected yields with a bond beta of 0.049 and stock beta of one. We use an expected market risk premium (MRP), excess of Treasury bills, of 8.6 percent, the 1926 through 1993 average MRP for the U.S. stock market (Ibbotson Associates, 1994). The expected tax rates reflect the dividend exclusion available to U.S. property-liability companies. The capital gain marginal rate, currently equal to the marginal corporate rate, is adjusted downward to reflect the effective tax advantage of annually deferring 50 percent of the unrealized capital

[5] The actual proportion of property-liability insurance company portfolios on an annual statement (amortized bonds, market stocks) basis for third-quarter 1994 is 18.2 (stocks), 75.3 (bonds), 0.7 (mortgages), 4.8 (miscellaneous) and 0.9 (cash) (Board of Governors of the Federal Reserve System, 1994). The stock proportion is much larger (23.4) in the first quarter of 1997, after the large increase in stock prices in 1995 to 1997.

gains. With this set of assumptions the nominal tax rate is 32.4 percent, lower than the marginal rate of 35 percent because of the tax preferences available to stock income. Note that none of the uncertainty of the expected income or tax assumptions is reflected in Table 1.

Table 1
Nonfuzzy Investment Tax Rate Example

	(1)	(2)	(3)	(4)	(5)
Categories	Assets	Expected Return on Assets (%)	Expected Pre-Tax Income (1) × (2)	Tax Rate (%)	Taxes (3) × (4)
U.S. Government Bonds	800	5.70	45.60	35.0	15.96
Stocks	200	13.88			
Dividends		3.81	7.62	14.2	1.08
Capital Gains		10.07	20.14	33.3	6.71
Total	1000	7.34	73.36	32.4	23.75

Note: Asset mix approximates U.S. property-liability company holdings (Board of Governors of the Federal Reserve System, 1994), risk-free return of 5.28 percent is cash-flow weighted Treasury bill and note average yields, November 1993 through October 1994. Bond and stock returns are CAPM with bond beta of 0.049, stock beta of 1.0, and market risk premium of 8.6 percent; dividend yield is ten-year S&P average yield 1984 through 1993; corporate tax rate is 35; dividend and capital gains tax rates reflect property-liability dividend exclusions and deferral of unrealized capital gains of 50 percent per period.

Fuzzy set theory gives us a way to rework Table 1 into a display that reveals the uncertainty in the various input parameters and, hence, in the tax results themselves. Table 2 portrays a version of Table 1 where the tax rates and investment income expectations are suitably uncertain. Admittedly, there are many ways to portray the parameters as fuzzy numbers by incorporating as much or as little of the random and nonrandom uncertainty into the membership function. Generally, we choose to illustrate the FST effect by using triangular (i.e., the shape of the graph of the membership function is triangular) fuzzy numbers, with the uncertainty pegged at plus or minus a value dependent on the uncertainty illustrated.[6] Each fuzzy member is identified by four variables (m_1, m_2, m_3, m_4) representing the left axis, left top, right top and right axis points.[7] The tax rate

[6] The "fuzziness" of stock returns in this example represents the uncertainty in the estimation of the CAPM expected, rather than actual, return. Uncertainty in the expected equity risk premium could arise, for example, in choosing, contrary to Ibbotson's advice, some shorter more recent time period to average equity returns excess of the risk free rate (Ibbotson, 1996, Table A16). Random variation could be illustrated by fuzzy numbers with support equal to one standard deviation about the mean.

[7] Although we do not use the illustration here, $m_2 < m_3$ describes a uniform range of uncertainty for the expected or middle values. This situation often may be the case for nonrandom uncertainty (Babad and Berliner, 1994).

outcome is the fuzzy number (31.0 percent, 32.4 percent, 32.4 percent, 33.6 percent) portraying an uncertain range of about 2.6 percent on the tax rate, arising directly from an assumed 2 percent uncertainty range in the marginal tax rate.

Table 2
Fuzzy Investment Tax Rate Example: Corporate Tax Rates and Returns

			Investment Categories				
		Fuzzy Number	U.S. Government Bonds	Stocks	Dividends	Capital Gains	Total
(1)	Investments		800	200			1000
(2)	Expected Return (%)	m_1	4.42	13.08	3.59	9.49	6.15
		m_2	5.70	13.88	3.81	10.07	7.34
		m_3	5.70	13.88	3.81	10.07	7.34
		m_4	6.98	14.68	4.03	10.65	8.52
(3)	(1) × (2) Expected Pretax Income	m_1	35.36		7.18	18.98	61.52
		m_2	45.60		7.62	20.14	73.36
		m_3	45.60		7.62	20.14	73.36
		m_4	55.84		8.06	21.30	85.20
(4)	Tax Rate (%)	m_1	34.0		13.8	32.0	31.0
		m_2	35.0		14.2	33.3	32.4
		m_3	35.0		14.2	33.3	32.4
		m_4	36.0		14.6	34.7	33.6
(5)	(3) × (4) Taxes	m_1	12.02		0.99	6.08	19.09
		m_2	15.96		1.08	6.71	23.75
		m_3	15.96		1.08	6.71	23.75
		m_4	20.10		1.18	7.38	28.66

Note: A fuzzy number is identified by the left axis, left top, right top, and right axis points (m_1, m_2, m_3, m_4). Investment returns are CAPM Table 1 returns with fuzzy risk-free rates, market risk premiums, and crisp betas of 0.049 (bonds) and 1 (stocks).

	Fuzzy Parameter	
	Risk-Free (%)	MRP
m_1	4.00	0.061
m_2	5.28	0.086
m_3	5.28	0.086
m_4	6.56	0.111

INCLUDING THE INSURANCE POLICY TAX SHIELD

The illustrations in Tables 1 and 2 focus on the uncertainty in insurer's investment portfolio. But tax considerations involve the interplay, and uncertainty, of the insurance or liability part of the company's entire portfolio of assets. Table 3 reworks the simple investment illustration of Table 1 to show the interaction with writing insurance liabilities and using the tax shield of those liabilities to offset some of the tax liabilities from investments. This situation, of course, assumes that property-liability insurers are writing to a nominal underwriting loss, a recent historical fact. We assume, in addition to all investment assumptions of Table 1, liabilities written at 2:1 to the surplus (net worth) of the company. We assume an expected underwriting loss of 4.07 percent, a recent value for Massachusetts private passenger automobile insurance rates. The tax rate for liability returns is assumed to be 34.5 percent, a value lower than the marginal rate reflecting the discounting of loss reserves for tax purposes. The expected tax rate for the pre-tax income on the insurer's portfolio drops to 31.1 percent from 32.4 percent because of the effect of the tax shield.

The effects of making the entire insurer portfolio fuzzy, investments and liabilities, are shown in Table 4. In addition to the fuzzy tax rate and investment returns of Table 2, we use a fuzzy underwriting return of about plus or minus 10 percent of the expected.

In addition to showing the effect of these fuzzy numbers on the tax rate, we list the fuzzy expected after-tax returns. The fuzzy tax rate spans 28.6 percent to 33.0 percent, a 4.4 percent gap. While the overall expected tax rate has been reduced by the effect of the tax shield (and policyholder tax hedge), the uncertainty has increased! Likewise, the after-tax rate of return, expected to be 9.56 percent, obtains a wide fuzzy range from 6.77 percent to 12.25 percent—a gap of about 5.5 percent.

Figure 1 displays the effect of a fuzzy tax shield on the fuzzy expected tax rate.

Figure 1
Fuzzy Investment Tax Rates: Effect of Liability Tax Shield

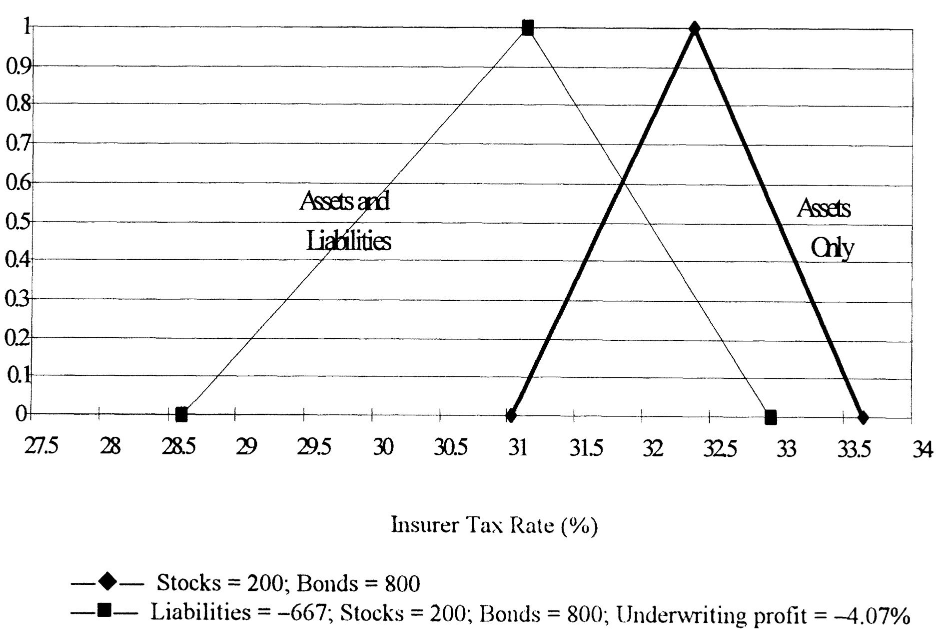


Table 3
Nonfuzzy Portfolio Tax Rate Example

	(1)	(2)	(3)	(4)	(5)
Categories	Portfolio Weights	Expected Return (%)	Expected Pre-Tax Income (1) × (2)	Tax Rate (%)	Taxes (3) × (4)
Liabilities	−667	4.07	−27.15	34.5	−9.36
U.S. Government Bonds	800	5.70	45.60	35.0	15.96
Stocks	200	13.88			
Dividends		3.81	7.62	14.2	1.08
Capital Gains		10.07	20.14	33.3	6.71
Surplus/Totals	333	13.88	46.21	31.1	14.39

Note: Investment returns and tax rates are defined in Table 1. Expected return on liabilities as in expected underwriting profit margin for Massachusetts private passenger automobile liabilities, tax rate for liabilities reflects discounting of loss reserves.

Asset Allocation

A common method of tax management in property-liability insurance companies is to balance the tradeoff of increased risk from a larger stock allocation with the decreased tax rate that emanates from the stock income preferences. Figure 2 shows the fuzzy range of tax rates as the asset allocation changes from 80/20 bond/stock to 20/80. If we measure the uncertainty of the *difference* between two fuzzy expected tax rates by the height of their intersection (the point at which they cross), one can observe the increasing uncertainty in distinguishing tax outcomes as the asset allocation moves to a larger stock position. Thus, while 80/20 and 20/80 are clearly distinct, even in the fuzzy sense, 50/50 and 40/60 retain a high degree (0.7 to 0.8) of uncertainty in differentiation of results.

The fuzzy tax effect of adding the insurance liabilities to the invested asset portfolio is demonstrated in Figure 3. Leverage ratios of 1:1 to 3:1, liabilities to surplus, provide for lower crisp expected tax rates. But those lower rates have little to distinguish them from one another on a fuzzy (uncertain) basis on either end of the assets allocation spectrum.

After-Tax Rates of Return

The fuzzy after-tax rates of return are displayed in Table 2. They reflect, of course, the uncertainty in the tax rates, expected investment yields and in the liabilities. Figure 4 shows the portfolio effect on after-tax rates of return for different leverage ratios and the extremes of the asset allocation illustrations (80/20, 20/80). Note that the ability to distinguish the fuzzy outcomes at the low investment risk level (80/20) for different leverage ratios but not to distinguish at the high investment risk level (20/80) lends the interpretation that the fuzzy after-tax rates of return reflect *total* uncertainty.

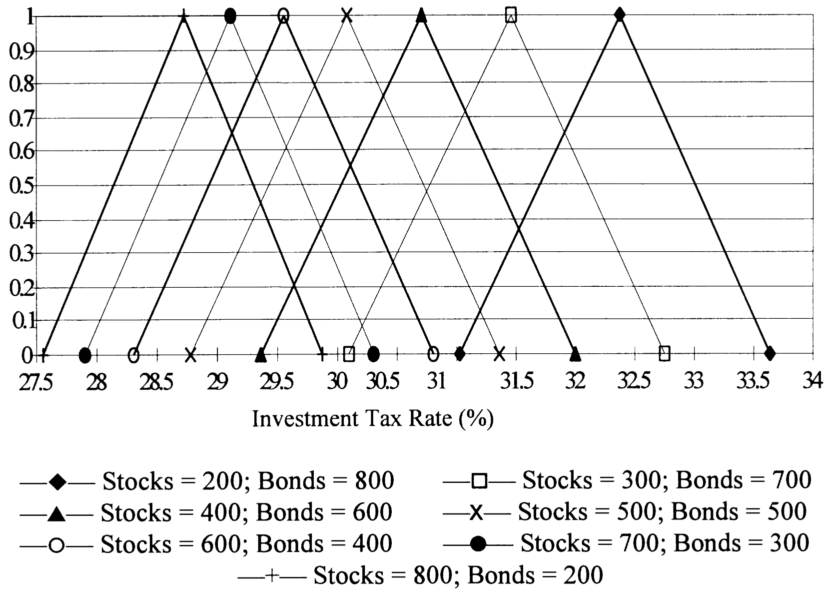
Table 4
Fuzzy Portfolio Tax Rate Example: Corporate Tax Rates and Returns

			Investment Categories					
		Fuzzy Number	Liabilities	U.S. Government Bonds	Stocks	Dividends	Capital Gains	Total
(1)	Portfolio Weights		−667	800	200			333
(2)	Expected	m_1	3.65	4.42	13.08	3.59	9.49	9.48
	Pre-Tax	m_2	4.07	5.70	13.88	3.81	10.07	13.88
	Return (%)	m_3	4.07	5.70	13.88	3.81	10.07	13.88
		m_4	4.49	6.98	14.68	4.03	10.65	18.27
(3)	(1) × (2)	m_1	−29.95	35.36	26.16	7.18	18.98	31.57
	Expected	m_2	−27.15	45.60	27.76	7.62	20.14	46.21
	Pre-Tax	m_3	−27.15	45.60	27.76	7.62	20.14	46.21
	Income	m_4	−24.35	55.84	29.36	8.06	21.30	60.85
(4)	Tax	m_1	33.6	34.0		13.8	32.0	28.6
	Rate (%)	m_2	34.5	35.0		14.2	33.3	31.1
		m_3	34.5	35.0		14.2	33.3	31.1
		m_4	35.4	36.0		14.6	34.7	33.0
(5)	(3) × (4)	m_1	−10.06	12.02	7.07	0.99	6.08	9.03
	Taxes	m_2	−9.36	15.96	7.79	1.08	6.71	14.39
	Paid	m_3	−9.36	15.96	7.79	1.08	6.71	14.39
		m_4	−8.61	20.10	8.56	1.18	7.38	20.05
(6)	(3) − (5)	m_1	−19.89	23.34	19.09	6.19	12.90	22.54
	Expected	m_2	−17.79	29.64	19.97	6.54	13.43	31.82
	After-Tax	m_3	−17.79	29.64	19.97	6.54	13.43	31.82
	Income	m_4	−15.74	35.74	20.80	6.88	13.92	40.80
(5)	(6) ÷ (1)	m_1	2.36	2.92	9.55	3.10	6.45	6.77
	Expected	m_2	2.67	3.71	9.98	3.27	6.71	9.56
	After-Tax	m_3	2.67	3.71	9.98	3.27	6.71	9.56
	Return (%)	m_4	2.98	4.47	10.40	3.44	6.96	12.25

Note: A fuzzy number is identified by the left axis, left top, right top, and right axis points (m_1, m_2, m_3, m_4). Investment returns are CAPM with fuzzy risk-free rates, market risk premiums, and crisp betas of 0.049 (bonds) and 1 (stocks).

	Fuzzy Parameter	
	Risk-Free (%)	MRP
m_1	4.00	0.061
m_2	5.28	0.086
m_3	5.28	0.086
m_4	6.56	0.111

Figure 2
Fuzzy Invesment Tax Rates with Selected Asset Mixes



The Beta One Company

As a further illustration of the value of the fuzzy approach to tax liability management, we consider the case of a beta one company.[8] Using the asset allocation of 80 (bonds) and 20 (stocks) and the three leverage ratios 1:1, 2:1, 3:1 liabilities to surplus (or 2:1, 1.5:1, 1.33:1 assets to liabilities), we can calculate the target fuzzy underwriting profit for the overall beta one company. Stated differently, with the 80/20 asset allocation and three leverage ratios, underwriting returns of (–6.26, –6.04, –6.04, –5.62 percent), (0.36, 0.78, 0.78, 1.20 percent), and (2.62, 3.04, 3.04, 3.46 percent) will result in three fuzzy after-tax returns, all "centered" on 13.88 percent—the beta one expected return. Figure 5 shows those fuzzy after-tax returns and their ranges of uncertainty. Note that the intuitive result of more uncertainty in the higher leveraged firm obtains even when the target after-tax return is the same.

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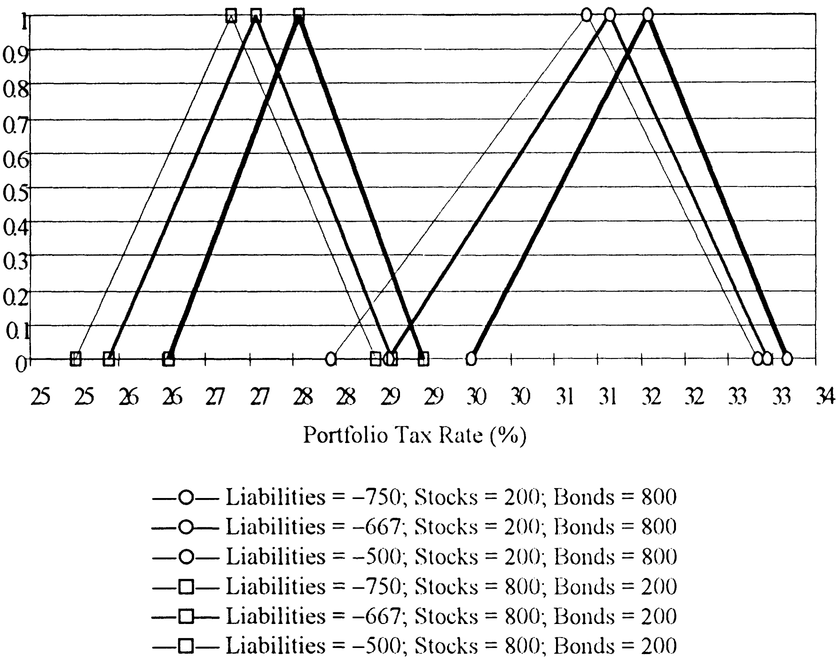
[8] U.S. property-liability insurers are often thought of as being of average (beta) risk. Unfortunately, this view does not necessarily take into account the vast distribution of the capitalization of those companies (Ibbotson Associates, 1996, pp. 125–141). Our simplifying assumption is used regardless of leverage of the firm.

Figure 3
Fuzzy Portfolio Tax Rates with Selected Investment Mixes



CONCLUSION

This article explores a simple model for the management of the government's short position for tax liabilities in the context of a property-liability insurance firm. We view the writing of the insurance liability as covering that short position under certain circumstances.

By virtue of Myers' Theorem, the tax management focus falls upon the effective tax rate of the investment portfolio. We show the ability of fuzzy set theory to illustrate not only the parametric interactions, but also the uncertainty, random and nonrandom, in the key parameters and outcomes. The advantages of the underwriting tax shield and the effects of parametric uncertainty on tax rate and after-tax return uncertainty are illustrated. Outcomes generally follow intuitive results; the benefit is the quantification, and graphic display, of the uncertainty of those results.

Figure 4
Fuzzy Portfolio After-Tax Returns with Selected Investment Mixes

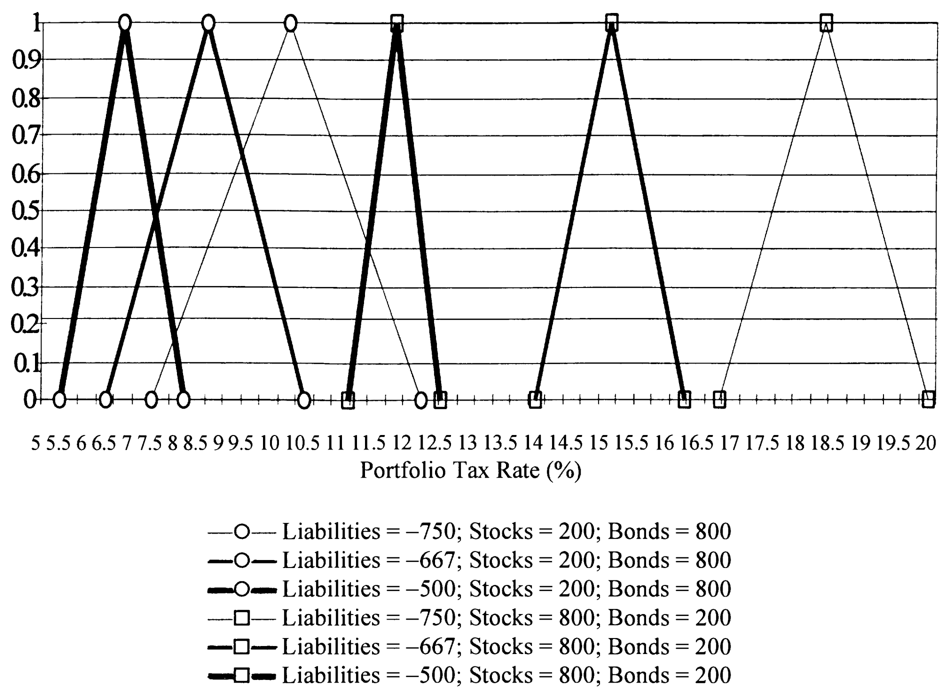
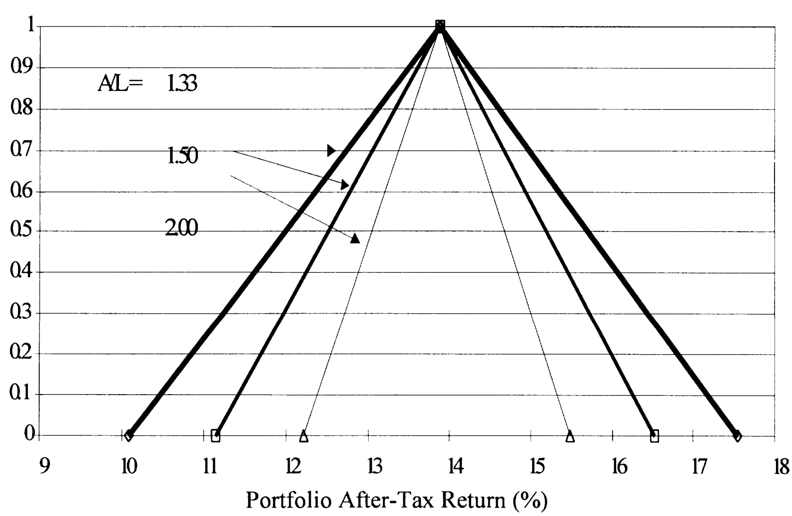


Figure 5
Fuzzy After-Tax Returns: Effect of Leveraged Investments



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