Deciding Whether a Life Insurance Contract Should Be Reinstated

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Abstract

In this article, a process for deciding whether a life insurance should be reinstated is proposed. The present values of net premium of two alternatives are calculated and compared. Our analysis shows that reinstatement of the original policy is not always beneficial. When the holding time of insurance policy before lapse is longer, reinstatement is more favorable, under the assumption of a constant interest rate. However, when the valuation interest rate is a stochastic process, buying a new policy is better than reinstatement of the original one if the equilibrium interest rate and the holding time of insurance policy before lapse have small values; otherwise, reinstatement of the original policy is better.

Key Words: Life insurance, Reinstatement, Decision analysis.

INTRODUCTION

The decision to purchase the "right" life insurance policy is very important to a consumer. Bernheim, Formi and Kotlikoff (2003) find that consumers make poor decisions about life insurance holdings. It appears that development of a better model for those decisions may be a valuable research contribution. And researchers from many areas have contributed to studies on this issue. Puelz and Snow (1991) discuss and establish a model for contractual agreements between insurance firms and their agents with the observation of a high commission rate for a new policy and a low one for a renewed a policy. Carson and Forster (1997) provide an analytical tool to decide whether life insurance should be replaced. Carson and Ostaszewski (2004) explore the actuarial value of life insurance backdating. In this article, we concentrate on deciding whether to reinstate a lapsed policy or to buy a new policy. In such a situation, the consumer must evaluate which alternative is more beneficial. We propose models designed to help the consumer to make a rational selection. This paper is the first study to quantitatively evaluate life insurance reinstatement.

A traditional life insurance policy lapses if the policyholder does not pay the premium by the end of the grace period and he/she does not use a policy loan to pay the premium. The reinstatement provision allows the policyholder to reinstate a life insurance contract that has lapsed within a certain period, typically three or five years in U.S. The policyholder needs to analyze carefully whether it is beneficial to reinstate or to buy a new policy when exercising the right of reinstatement. Rejda

(2004) lists five advantages for a policyholder to reinstate a lapsed policy rather than purchase a new one. The first is that the premium is lower because the reinstated policy was issued at an earlier age. We will perform some quantitative analysis on this issue. The second is that the acquisition expenses incurred in issuing the policy must be paid again under a new policy. The third advantage listed by Rejda (2004) is that the cash values and dividends are usually higher under the reinstated policy, as the new policy may not develop any cash value until the end of the third year (see Rejda, 2004, p. 397). Here, we would like to note that the policyholder can obtain the surrender value of the lapsed policy, which can partly or completely offset the cash value he/she cannot obtain from the new policy until the end of the third year. The other two advantages proposed by Rejda (2004) are unquestionable, as they are:

• The acquisition expenses incurred in issuing the new policy must be paid again under a new policy.

• The contestable period and suicide period under the old policy may have expired. Reinstatement of a lapsed policy does not reopen the suicide period, and a new contestable period generally applies only to statements contained in the application for reinstatement. Statements contained in the original contestable period cannot be contested after the original contestable period expires. Let us note also that the reinstated policy may contain favorable policy provisions, such as a low interest rate on policy loans.

For the first and third advantage proposed by Rejda (2004), we believe that further analysis is warranted. In this article, we calculate the present values of the net premium and the cash values of two alternatives, and analyze which alternative is optimal, and under what conditions.

ASSUMPTIONS AND THE DECISION MODELS

We assume the following

1. The objective that the decision maker hopes to reach is that the premium paid by the policyholder is minimized while the amount of insurance is the same.
2. There are only two alternatives to choose. Alternative 1 is to apply for reinstatement within a period set in the provisions of the contract. Alternative 2 is to buy a new policy that has the same amount of insurance as the original policy.
3. The policyholder buys a whole life insurance policy under either of the alternatives. The policyholder pays the premium at the beginning of each year. The insurer will pay the claim at the end of the year of death.
4. We consider two situations. One is that there is a predetermined interest rate used by the insurer to calculate the premium that is i. Another is that there is a stochastic interest rate used by the insurer to calculate the premium that is r. The age of the insured when the insurance policy is issued is x. The insurance policy lapses after it has been held for h years. The policyholder applies for reinstatement within the allowed period, at time of $h+y$, where y does not exceed the allowed reinstatement period. For the whole life insurance, the insurance term for alternative 1 is $105-x$ and the insurance term for alternative 2 is $105-h-y-x$. In other words, a whole life insurance policy is assumed to

endow at age 105.

5. Adverse selection is not considered.[1]

6. Of course, there are other factors to affect the decision, such as: the financial strength of the insurers, the coverage of the insurance contract provisions and so on. To simplify our analysis, we assume that the difference of the strength of the insurers and the coverage of the insurance contract provisions can be ignored.

7. To simplify our analysis, we assume that there are no guaranteed benefits or other warranted benefits. And we also do not consider the cases when the policyholders are rejected to be reinstated because of their health conditions. The notation used in this paper is listed below:

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i: a predetermined interest rate used by the insurer to calculate the premium

r: a stochastic interest rate used by the insurer to calculate the premium

h: the holding time of the insurance policy before it lapses

y: the period between the time of lapse and the time of applying for reinstatement

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x: the age of the insured when the insurance policy is issued

P_1: the single premium for a \$1 benefit for the Alternative 1

Alternative 1: to apply for reinstatement within a period set in the provisions of the

[1] The insured who is healthy can do significant comparison shopping with other companies and thus has less incentive to consider reinstatement. The result is that the group of former policyholders who return for reinstatement are likely to be of higher mortality than the general population. Arguably, however, if they are clearly aware of any deterioration in their health, and they have the means to pay the premium, they would not lapse the original policy in the first place. Thus, while adverse selection among those who reinstate is likely, it is reasonable to expect it to be of lesser significance than with respect to the process of lapses and withdrawals in general, where clearly good risks are more likely to leave. Furthermore, if there does exist adverse selection, we can assume that the underwriting process will serve to minimize it.

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P_2 : the single premium for alternative 2

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Alternative 2: to buy a new policy that has the same amount of insurance as the original policy

A_1 : the annual level premium paid by the policyholder when he/she reinstates the lapsed policy

A_2 : the annual level benefit premium for the new policy

${}_hCV_x$: the cash value at time t

K : the ratio of the cash value to the reserve of reinstated policy

K_{new} : the ratio of the cash value to the reserve for the new policy

σ : the volatility of interest rates

μ : the long run equilibrium rate of interest

$\mu - r$: the gap between its current and long-run equilibrium level

κ : a measure of the sense of urgency exhibited in financial markets to close the gap

l_x : The number of persons alive from the initial birth cohort at age x

d_x : The number of deaths at age x

The Decision Models

1. Assume that the interest rate is a constant

Let P_1 be the single premium for a \$1 benefit for the Alternative 1 and P_2 be the single premium for alternative 2. The decision process is about comparing these two values, and possible cash values. The first premium, P_1, is equal to the single premium applicable for Alternative 1 with possible adjustment for interest due to differences in timing of payments of the two premiums. The second premium, P_2, is the single premium for buying a new life insurance policy, but adjusted for any surrender value received from the lapsed policy. Let i be the constant interest rate. Based on the actuarial equivalence principle, we have

$$\begin{aligned} P_1 &= \sum_{j=1}^{y} A_1(1+i)^{y-j} + \sum_{j=0}^{105-y-h-x-1} A_1/(1+i)^j \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} \\ &= A_1\left[\sum_{j=1}^{y}(1+i)^{y-j} + \sum_{j==0}^{105-y-x-h-1} 1/(1+i)^j \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}}\right], \end{aligned} \tag{1}$$

where A_1 is the annual level premium paid by the policyholder when he reinstates his/her lapsed policy, so that

$$A_1 = \frac{\sum_{j=0}^{105-x-1} 1/(1+i)^{j+1} \cdot \frac{d_{x+j}}{l_x}}{\sum_{j=0}^{105-x-1} 1/(1+i)^j \cdot \frac{l_{x+j}}{l_x}}. \tag{2}$$

Similarly, for alternative 2

$$P_2 = \begin{cases} A_2 \sum_{j=0}^{105-y-h-1} 1/(1+i)^j \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} & \text{when } h \le 2 \\ A_2 \sum_{j=0}^{105-y-h-1} 1/(1+i)^j \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} - {}_hCV_x & \text{when } h > 2 \end{cases} \tag{3}$$

where A_2 is the annual level premium for the new policy, i.e.,

$$A_2 = \frac{\sum_{j=0}^{105-y-h-x-1} 1/(1+i)^{j+1} \cdot \frac{d_{x+h+y+j}}{l_{x+h+y}}}{\sum_{j=0}^{105-y-h-x-1} 1/(1+i)^j \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}}}, \tag{4}$$

and $${}_hCV_x = K\,{}_hV_x = K\left[1 - \frac{\sum_{j=1}^{105-x-h} 1/(1+i)^j \cdot l_{x+h+j}}{\sum_{j=1}^{105-x} 1/(1+i)^j \cdot l_{x+j}}\right], \tag{5}$$

where K is the ratio of the cash value to the reserve.

The value of K will affect the decision, because it is a measure of surrender charge. When K and the reserve are larger, the cash value owned by the policyholder will be larger and the conditions of buying a new policy are more favorable.

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Note that the quantity $\min(P_1, P_2)$ affects the optimal alternative for the policyholder.

In the following, we will perform sensitivity analysis. Let $x = 20, y = 5, K = 0.6$. Table 1 lists the values of single premium for one dollar of benefit of P_1 and P_2 when the interest rate i and the holding time of insurance policy before it lapses h take different values.

Table 1: The Values of Single Premiums (P_1 and P_2) For One Dollar of Benefit For Varying Parameters i and h

$i = 0.03$								
h	1	2	3	4	5	6	7	8
P_1	0.6373	0.6351	0.6329	0.6306	0.6282	0.6258	0.6233	0.6207
P_2	0.5948	0.6103	0.6263	0.6428	0.6597	0.6772	0.6951	0.7136
$i = 0.05$								
h	1	2	3	4	5	6	7	8
P_1	0.1824	0.1822	0.1819	0.1817	0.1814	0.1812	0.1809	0.1806
P_2	0.1719	0.1803	0.1867	0.1949	0.2036	0.2128	0.2223	0.2323
$i = 0.10$								
h	1	2	3	4	5	6	7	8
P_1	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157
P_2	0.0136	0.0147	0.0157	0.0169	0.0182	0.0196	0.0212	0.0229

Notes: i is the interest rate, h is the holding time of the insurance policy before it lapses, and $P_1(P_2)$ is the single premium for one dollar of benefit of the reinstated policy (a new policy). The mortality data is from China Life Table (2000).

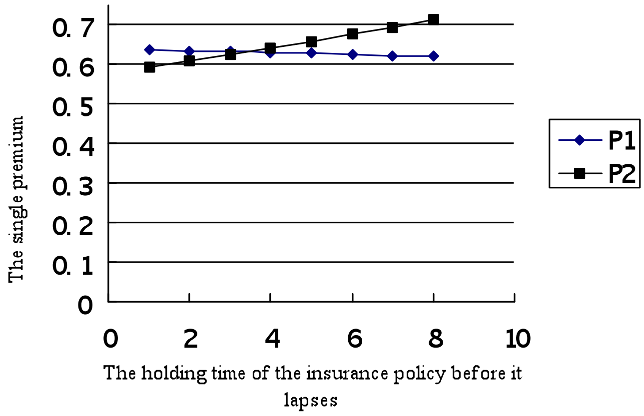

Fig. 1. The values of the single premium of P_1 **and** P_2 **when** $i = 0.03$

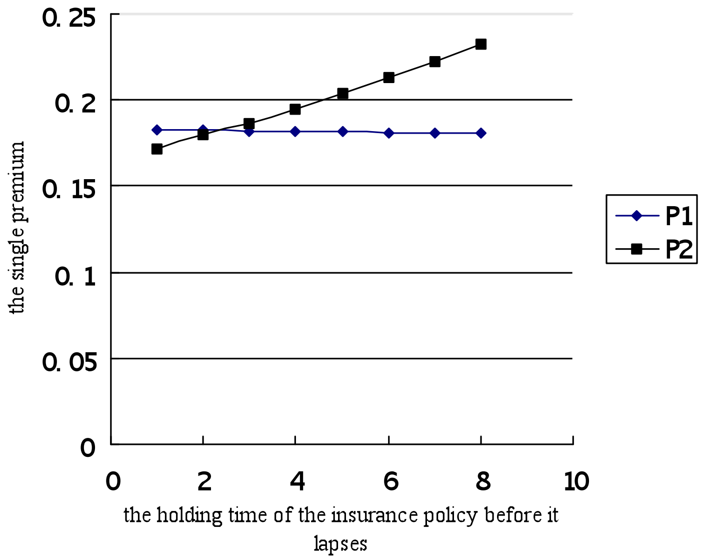

Fig. 2. The values of single premium of P_1 **and** P_2 **when** $i = 0.05$

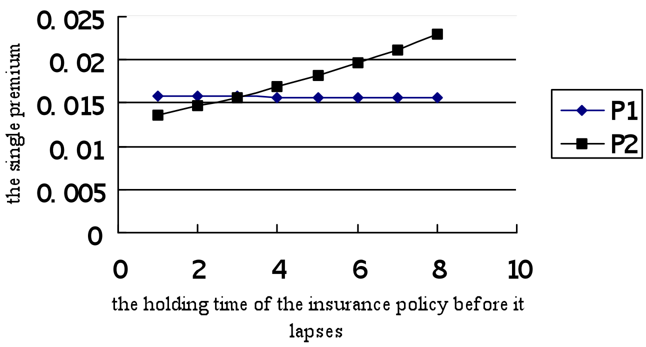

Fig. 3. The values of single premium of P_1 and P_2 when $i = 0.10$

Figure 1, Figure 2, and Figure 3 show the relationships between the single premium for one dollar benefit of reinstatement, P_1, the single premium for one dollar benefit of buying a new policy, P_2, and the holding time of the insurance policy before it lapses, h, when the interest rate $i = 0.03,\ 0.05$ and 0.10 respectively. From these three figures, we can see that there exists break-even point $(h^*, P_1^*, P_2^*,)$ in all these three figures. The value of P_1 is greater than the value P_2 when h is smaller than h^* and vice versa. Thus, buying a new policy is better than reinstating original one when h is smaller than h^*, but reinstating the original policy is better than buying a new one when h is larger than h^*. Therefore, reinstating a lapsed contract is not always better than buying a new policy from the aspect of net premium paid by policyholder. From Table 1, we can see that the values of premiums for two alternatives are very sensitive to the change of the interest rate and the premiums for two alternatives are negatively related to the interest rates. From Figure 1 to Figure 3, we can also find that the lines of the single premium of Alternative 1 are much flatter

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than the lines of the single premium of Alternative 2, which illustrates that new policy is much sensitive to the change of the holding time of the insurance policy before it lapses than that of reinstating the original one.

Table 2 lists the break-even points when interest rate $i = 0.10$, while the ratio of cash value to the reserve K takes different values.

Table 2: The Break-even Points When $i = 0.10$ for Varying K

K	0.1	0.3	0.5	0.7	0.9
$h*$	5	6	6	6	6
$P_1*(P_2*)$	2.3442	2.3518	2.3414	2.3311	2.3270

Note: i is interest rate, K is the ratio of cash value to reserve, and $P_1(P_2)$ is the single premium of the reinstated policy (a new policy).

In what follows now, we will give formulas for calculating cash values of these two alternatives, and further find the crossover points. Let ${}_tCV_x^1$ and ${}_tCV_x^2$ express the cash values of alternative 1 and alternative 2 at time *t* respectively. Then we have

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$$ {}_tCV_x^1 = K\left[1 - \frac{\sum_{j=1}^{105-x-t} 1/(1+i)^j \cdot l_{x+t+j}}{\sum_{j=1}^{105-x} 1/(1+i)^j \cdot l_{x+j}}\right], \qquad (6) $$

and

$$ {}_tCV_x^2 = K_{new}\left(1 - \frac{\sum_{j=1}^{105-x-t} 1/(1+i)^j \cdot l_{x+t+j} / l_{x+t}}{\sum_{j=1}^{105-x-h-y} 1/(1+i)^j \cdot l_{x+h+y+j} / l_{x+h+y}}\right) + {}_hCV_x^1(1+i)^{t-h-y}, \quad t > h+y+2, \qquad (7) $$

where K_{new} is the ratio of the cash value to the reserve of new policy.

Table 3 lists the change patterns of cash values of two alternatives with the changes of surrender time t when the ratios of the cash values to the reserves take different values.

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Table 3: The Cash Values of Two Alternatives ($h=10, y=5, i=0.05$)

K=0.3, $K_{new}=0.2$								
t	60	61	62	63	64	65	66	67
${}_tCV_x^1$	0.2254	0.2336	0.2419	0.2503	0.2586	0.2685	0.2754	0.2837
${}_tCV_x^2$	0.2235	0.2334	0.2437	0.2545	0.2658	0.2670	0.2903	0.3070
K=0.5, $K_{new}=0.4$								
t	17	18	19	20	21	22	23	24
${}_tCV_x^1$	0.0455	0.0484	0.0514	0.0545	0.0578	0.0612	0.0648	0.0684
${}_tCV_x^2$	0.0438	0.0469	0.0502	0.0537	0.0573	0.0611	0.0651	0.0692
K=0.7, $K_{new}=0.6$								
t	12	13	14	15	16	17	18	19
${}_tCV_x^1$	0.0464	0.0496	0.0529	0.0563	0.0599	0.0637	0.0677	0.0719
${}_tCV_x^2$	0.0436	0.0473	0.0521	0.0552	0.0595	0.0639	0.0685	0.0734

Notes: t is the time, $K(K_{new})$ is the ratio of cash value to reserve used in reinstated policy (a new policy), and ${}_tCV_x^1({}_tCV_x^2)$ is the cash value of the reinstated policy (a new policy).

In Table 3, we list three cases, and there exists break-even point for all these three cases. But, for different values of K and K_{new}, the break-even points are different. The corresponding values of t^* in break-even points decrease with the increase of K *and* K_{new}. Therefore, the selection of optimal plan is dependent on the values of

K *and* K_{new}.

2. Assume that the interest rate is a stochastic process

Assume that the volatility of interest rates is constant, and the Cox, Ingersoll, and Ross (1985) Model[2] is used in valuation. Interest rate r satisfies a stochastic differential equation

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$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}dw,$$

where w is a Wiener process, σ denotes the volatility of interest rates, μ is the long run equilibrium rate of interest, (μ - r) is the gap between its current and long-run equilibrium level, and κ is a measure of the sense of urgency exhibited in financial markets to close the gap, i.e., it gives the speed at which the gap is reduced, where the speed is expressed in annual terms.

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The premium of Alternative 1 can be expressed as follows:

$$\begin{aligned} P_1 &= \sum_{j=1}^{y} A_1 e^{\int_j^y r(u)du} + \sum_{j=0}^{105-y-h-x-1} A_1 e^{-\int_0^j r(u)du} \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} \\ &= A_1 \left[\sum_{j=1}^{y} e^{\int_j^y r(u)du} + \sum_{j==0}^{105-y-x-h-1} e^{-\int_0^j r(u)du} \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} \right], \end{aligned} \tag{8}$$

where A_1 is the annual level benefit premium paid by the policyholder annually when he reinstates his lapsed policy, so that

[2] Other interest rate models such as Vasicek (1977), Langetieg(1980), and Yao (1999), assume that the interest rates are normally distributed and there is a positive probability of negative interest rates, which implies arbitrage opportunities. We do not use these models in our paper.

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$$A_1 = \frac{\sum_{j=0}^{105-x-1} e^{-\int_0^{j+1} r(u)du} \cdot \frac{d_{x+j}}{l_x}}{\sum_{j=0}^{105-x-1} e^{-\int_0^{j} r(u)du} \cdot \frac{l_{x+j}}{l_x}}. \quad (9)$$

Similarly, for Alternative 2

$$P_2 = \begin{cases} A_2 \sum_{j=0}^{105-y-h-1} e^{-\int_0^{j} r(u)du} \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} & \text{when } h \le 2 \\ A_2 \sum_{j=0}^{105-y-h-1} e^{-\int_0^{j} r(u)du} \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}} - {}_hCV_x & \text{when } h > 2 \end{cases} \quad (10)$$

where A_2 is the annual level benefit premium for the new policy, i.e.,

$$A_2 = \frac{\sum_{j=0}^{105-y-h-x-1} e^{-\int_0^{j+1} r(u)du} \cdot \frac{d_{x+h+y+j}}{l_{x+h+y}}}{\sum_{j=0}^{105-y-h-x-1} e^{-\int_0^{j} r(u)du} \cdot \frac{l_{x+h+y+j}}{l_{x+h+y}}}, \quad (11)$$

and $${}_hCV_x = K\,{}_hV_x = K\left[1 - \frac{\sum_{j=1}^{105-x-h} e^{-\int_0^{j} r(u)du} \cdot l_{x+h+j}}{\sum_{j=1}^{105-x} e^{-\int_0^{j+1} r(u)du} \cdot l_{x+j}}\right], \quad (12)$$

where K is the ratio of the cash value to the reserve.

In what follows now, we will perform sensitivity analysis. Let $x = 20$, $y = 5$, $K = 0.6$, $\kappa = 0.3$, and $\sigma = 0.12$. Table 4 lists the values of single premiums (P_1 and P_2) for one dollar of benefit when the equilibrium interest rate of long term μ and the holding time of the insurance policy before it lapses h vary.

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Table 4: The Values of Single Premium for One Dollar of Benefit P_1 and P_2 When Interest Rate Is A Stochastic Process

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$\mu = 0.03$								
h	1	2	3	4	5	6	7	8
P_1	0.7989	0.7958	0.7926	0.7892	0.7858	0.7823	0.7787	0.7750
P_2	0.7421	0.7606	0.7797	0.7993	0.8194	0.8401	0.8613	0.8832
$\mu = 0.05$								
h	1	2	3	4	5	6	7	8
P_1	0.2031	0.2028	0.2024	0.2020	0.2016	0.2012	0.2008	0.2003
P_2	0.2380	0.2480	0.2585	0.2695	0.2811	0.2931	0.3057	0.3189
$\mu = 0.10$								
h	1	2	3	4	5	6	7	8
P_1	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014
P_2	0.0221	0.0237	0.0255	0.0275	0.0296	0.032	0.0345	0.0373

Notes: μ is the long run equilibrium rate of interest, h is the holding time of the insurance policy before it lapses, and $P_1(P_2)$ is the single premium for one dollar of benefit of the reinstated policy (a new policy).

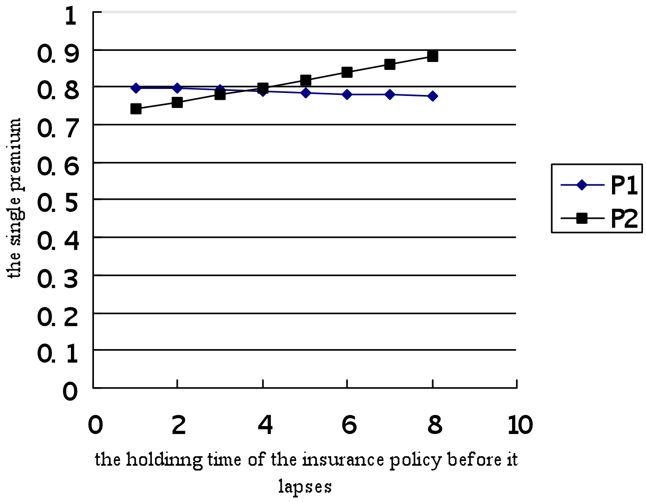

Fig. 4. The values of single premium of P_1 **and** P_2 **when** $\mu = 0.03$

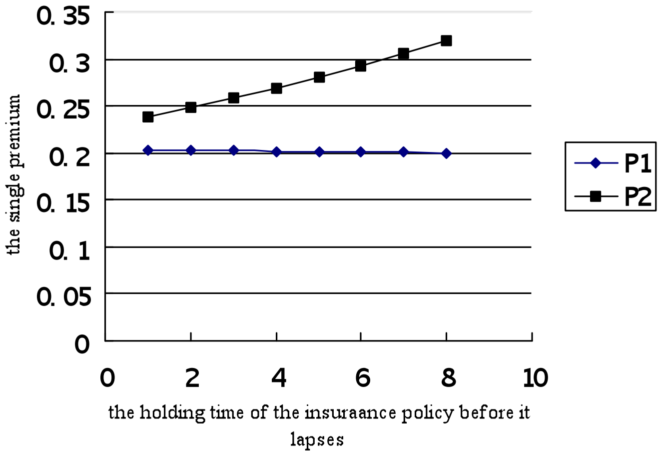

Fig. 5. The values of single premium of P_1 **and** P_2 **when** $\mu = 0.05$

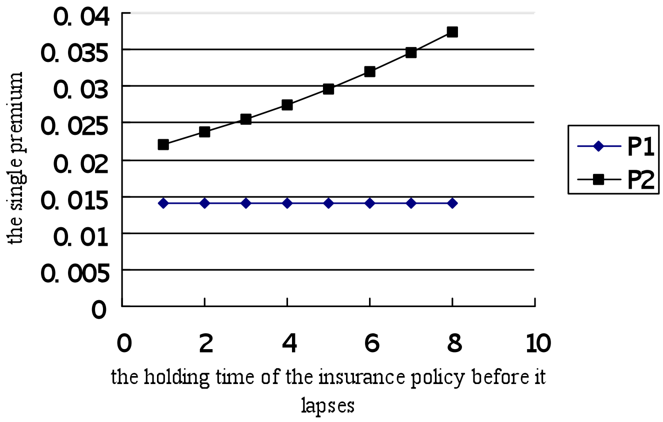

Fig. 6. The values of single premium of P_1 and P_2 when $\mu = 0.10$

From Figure 4, Figure 5 and Figure 6 and Table 4, we can see that there exists break-even point when $\mu = 0.03$. In this case, purchasing a new policy is better than reinstatement when $h < 4$, and worse when $h > 4$. In addition, we find that when $\mu = 0.04$, there also exists the break-even point ($h^* = 2$). But when $\mu > 0.04$, there are no break-even points, and reinstatement is always better than purchasing a new policy in these cases. We can see that this situation is similar to the cases of constant interest rates, as the lines of the single premium of Alternative 1 in that case are also flatter than the lines of the single premium of Alternative 2, which illustrates the fact that the single premium for a new policy is more sensitive to the changes of the holding time of the insurance policy before it lapses than that of reinstated policy.

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In the following, we will give formulas for calculating cash values of these two alternatives when the interest rate is assumed as a stochastic process, and show the process of finding the crossover points. The cash values of two alternatives can be expressed as the following:

$$ {}_tCV_x^1 = K\left[1-\frac{\sum_{j=1}^{105-x-t} e^{-\int_0^j r(u)du}\cdot l_{x+t+j}}{\sum_{j=1}^{105-x} e^{-\int_0^j r(u)du}\cdot l_{x+j}}\right], \tag{13} $$

and

$$ {}_tCV_x^2 = K_{new}(1-\frac{\sum_{j=1}^{105-x-t} e^{-\int_0^j r(u)du}\cdot l_{x+t+j}/l_{x+t}}{\sum_{j=1}^{105-x-h-y} e^{-\int_0^j r(u)du}\cdot l_{x+h+y+j}/l_{x+h+y}})+{}_hCV_x e^{\int_t^{t+h+y} r(u)du}, \quad t>h+y, \tag{14} $$

where K_{new} is the ratio of the cash value to the reserve of new policy.

Table 5 illustrates the patterns of cash values of the two alternatives with the changes of surrender time t when the ratios of the cash values to the reserves take on different values.

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Table 5: Cash Values of Two Alternatives

($h = 10, y = 5, \mu = 0.05, \sigma = 0.12, \kappa = 0.3$)

K=0.3, $K_{new} = 0.2$								
t	23	24	25	26	27	28	29	30
${}_tCV_x^1$	0.0406	0.0429	0.0453	0.0478	0.0505	0.0532	0.0560	0.0590
${}_tCV_x^2$	0.0385	0.0412	0.0440	0.0470	0.0501	0.0534	0.0568	0.0603
K=0.5, $K_{new} = 0.4$								
t	15	16	17	18	19	20	21	22
${}_tCV_x^1$	0.0426	0.0453	0.0481	0.0510	0.0540	0.0572	0.0606	0.0641
${}_tCV_x^2$	0.0380	0.0415	0.0452	0.0491	0.0532	0.0574	0.0619	0.0665
K=0.7, $K_{new} = 0.6$								
t	13	14	15	16	17	18	19	20
${}_tCV_x^1$	0.0526	0.0560	0.0596	0.0634	0.0673	0.0714	0.0757	0.0801
${}_tCV_x^2$	0.0458	0.0505	0.0554	0.0606	0.0661	0.0717	0.0777	0.0839

Notes: t is the time, $K(K_{new})$ is the ratio of cash value to reserve used in reinstated policy (new policy), and ${}_tCV_x^1({}_tCV_x^2)$ is the cash value of the reinstated policy (a new policy).

Table 5 lists three cases with different K and K_{new}. The results are similar to the cases of determinant interest rates, except that the value of $t*$ in break-even point of the first case here is much smaller.

Example

We will now describe an example of this decision process based on the real world data. We use the data from a big Chinese insurance company called China Life. The

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interest rate $i = 5\%$, the ratio of cash value to reserve $K = 60\%$, and the data of mortality rate is from The Life Table of China (2000). A woman aged 20 bought a whole life policy with a benefit of RMB 50000. The holding time of her policy before it lapsed was $h = 4$. The time of applying for reinstating or buying a new policy in the company is $y = 5$. Using the equations 1-5, we get the single premium for reinstating RMB 1 benefit is 0.1817 (Please see Table 1.) and $0.1817 \times 50000 = 9085$ (RMB) for 50000 (RMB) of benefit. Similarly, we get the single premium for buying a new policy of RMB 50000 benefit is RMB 9745. Therefore, reinstatement is a better decision for this woman.

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CONCLUSIONS

The attractiveness of reinstatement for a lapsed life insurance policy has been pointed out in Rejda (2004). Policyholders decide whether to reinstate a lapsed policy or purchase a new one. This article discusses and compares these two alternatives by developing quantitative models. Our calculations indicate that reinstating a lapsed policy is not always a good thing. Policyholders should carefully compare the premiums and cash values between these two alternatives and then make rational decisions. Our findings show that, under the assumption of a constant interest rate, when holding time of the insurance policy before lapse is longer, reinstatement of the original policy is better than buying a new one if the interest rate is constant. However, when the interest rate is a stochastic process, buying a new policy is better than reinstating the original one if the equilibrium interest rate μ and the holding time of insurance policy before if lapses h assume small values, otherwise, reinstating the original policy is better.

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