

Edge colouring multigraphs

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While edge colouring in graphs is well understood (by Vizing's classical theorem), the chromatic index $\chi'(G)$ of a multigraph G can fall anywhere in the range $[\Delta(G), 3\Delta(G)/2]$ (where $\Delta(G)$ denotes the maximum degree of G), or in terms of the maximum edge multiplicity $\mu(G)$ the range $[\Delta(G), \Delta(G) + \mu(G)]$. We discuss results showing that if $\chi'(G)$ is significantly larger than $\Delta(G)$, then G contains a small subgraph that is very dense. This idea is at the core of the famous open problem of Goldberg and (independently) Seymour, which seeks to identify the properties of G that influence $\chi'(G)$.