Extending Fisher's inequality to coverings

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A (v, k, λ)-*design* is a collection of k-element subsets, called *blocks*, of a v-set of *points* such that each pair of points occurs in exactly λ blocks. *Fisher's inequality* is a classical result that states that every nontrivial (v, k, λ)-design has at least v blocks (equivalently, has $v \geq \frac{k(k-1)}{\lambda}+1$). An elegant proof of Fisher's inequality, due to Bose, centres on the observation that if X is the incidence matrix of a nontrivial design, then XX^T is nonsingular. This talk is about extending this proof method to obtain results on coverings.

A (v, k, λ)-*covering* is a collection of k-element blocks of a v-set of points such that each pair of points occurs in at least λ blocks. Bose's proof method can be extended to improve the classical bounds on the number of blocks in a (v, k, λ)-covering when $v < \frac{k(k-1)}{\lambda} + 1$. Specifically, this is accomplished via bounding the rank of XX^T, where X is the incidence matrix of a (v, k, λ)-covering, using arguments involving diagonally dominant matrices and m-independent sets in multigraphs.