A stability version for a theorem of Erdős on nonhamiltonian graphs

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In 1962, Erdős proved an upper bound for the number of edges in a nonhamiltonian graph with n vertices and minimum degree d. For each n and d, he also provided an extremal example that meets the bound. In particular, for $d < n/6$, the extremal example is a graph $H_{n,d}$. We show that for $d < n/6$, every nonhamiltonian graph on n vertices with minimum degree at least d with "close" to the maximum number of edges is either a subgraph of $H_{n,d}$ or a subgraph of $K'_{n,d}$ where $K'_{n,d}$ is composed of edge-disjoint copies of K_{n-d} and K_{d+1} sharing one vertex. Thus we give a classification of all nonhamiltonian graphs with minimum degree d and "a lot" of edges. Furthermore, this yields a polynomial-time algorithm that determines if a graph with minimum degree d and sufficiently many edges is hamiltonian.