On disjoint cycles in a graph and a theorem of Dirac and Erdős

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For an integer $k \geq 2$ and a graph G, let $H_k(G)$ be the set of vertices with degree at least $2k$ and $L_k(G)$ be the set of vertices of degree at most $2k-2$. A seminal result of Corrádi and Hajnal from 1963 proves that if $|V(G)| \geq 3k$ and $H_k(G) = V(G)$ (i.e. $\delta(G) \geq 2k$), then G contains k disjoint cycles. In the same year, Dirac and Erdős proved that G contains k disjoint cycles whenever $|H_k(G)| - |L_k(G)| \geq k^2 + 2k - 4$. We prove a stronger version of this theorem, showing that a graph has k disjoint cycles whenever the difference is at least $3k$ and that this bound is sharp. We also show that a difference of $2k$ is sufficient when G is large, G is planar, or G contains at most two disjoint triangles.