# 3-Flows with Large Support

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Tutte's 3-Flow Conjecture says that every 4-edge-connected graph should have a nowhere-zero 3-flow. The 4-edge-connectivity assumption cannot be weakened—$K_4$ is an example of a 3-edge-connected graph that does not have a nowhere-zero 3-flow. However, $K_4$ is minimal in the sense that $K_4^-$ has a nowhere-zero 3-flow. Since $K_4$ has 6 edges in total, this means that we are able to give $K_4$ a 3-flow in which 5/6 of the edges are nonzero. With DeVos, Pivotto, Rollova, and Samal, we can show that this is the worst case in general—that is, if $G$ is any 3-edge-connected graph, then $G$ has a 3-flow with support size at least $\frac{5}{6}|E(G)|$. As a corollary, this implies that every planar graph has an assignment of three colours to its vertices so that at most a sixth of its edges join vertices of the same colour.