On Strongly Chromatic Choosable Graphs with an Application to List Coloring the Cartesian Product of Graphs

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The list chromatic number of the Cartesian product of graphs is not well understood. The best result is by Borowiecki, Jendrol, Kral, and Miskuf (2006) who proved that the list chromatic number of the Cartesian product of two graphs can be bounded in terms of the list chromatic number and the coloring number of the factors, implying a bound exponential in the list chromatic number of the factors. We show how to improve this bound for certain large classes of graphs.

We generalize the notion of strong critical graphs, introduced by Stiebitz, Tuza, and Voigt in 2008, to strong k-chromatic choosable graphs, and we show that it gives a strictly larger family of graphs that includes odd cycles, cliques, join of a clique with any other such graph, and many more families of graphs. Our main result gives a sharp bound on choosability of the Cartesian product of a strong k-chromatic choosable graph satisfying an edge bound and a traceable graph. This result can be applied to find chromatic choosable families of graphs improving the existing bounds on their choosability. The proof uses the notion of unique-choosability as a sufficient condition for list colorability, discovered by Akbari, Mirrokni, and Sadjad in 2006, to set up a loaded inductive statement that guarantees non-unique list colorings.