

# Embedding factorizations in complete uniform hypergraphs

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We look at when it is possible to embed an  $r$ -factorization of a complete  $h$ -uniform hypergraph on  $m$  vertices into an  $s$ -factorization of a complete  $h$ -uniform hypergraph on  $n$  vertices. There are some “obvious” necessary conditions. Häggkvist and Hellgren showed that for  $r = s = 1$  these are sufficient. We prove that when  $r = s$  the obvious necessary conditions together with  $\gcd(n, h) \mid m$  are sufficient (which includes the result for  $r = s = 1$ ). Furthermore when  $r < s$  the obvious necessary conditions together with  $\gcd(n, h) \mid m$  and  $n \geq 2m$  and  $\frac{s}{r} \leq \frac{m}{k} [1 - \binom{m-k}{h} / \binom{m}{h}]$  are sufficient. Neither of these is an exact characterization, but there is scope for improvement. From another point of view, one can attempt to find an exact characterization for small values of  $h$ . We do this for  $h = 3$  and already discover that the exact characterization has one “less-obvious” necessary condition, namely that if  $s \binom{m-1}{2} = r \binom{n-1}{2}$  and  $m(s - r)$  is odd then  $rm \binom{n-m}{2} \geq \binom{m-1}{2}$ .