A Bijection between Schröder Paths and Permutations

Michael W. Schroeder*, Rebecca Smith

Department of Mathematics, Marshall University, Huntington, WV 25755
schroederm@marshall.edu

A permutation (as a word) is *stack sortable* if the entries can be rearranged in increasing order through the use of pop and push operations with a specified number of stacks. Knuth showed that permutations that avoid the pattern 231 are those that can be sorted with a single stack, which are in correspondence to Dyck paths. We consider the following variation: let $\mathcal{P}_n$ be the set of permutations that avoid the patterns 3142 and 3241 and $\mathcal{S}_n$ be the set of Schröder paths of order n, which are the lattice paths from $(0,0)$ to (n,n) composed of East steps $(1,0)$, North steps $(0,1)$, and Diagonal steps $(1,1)$ that travel weakly below the line $y = x$. The permutations of $\mathcal{P}_n$ are precisely those that are sortable by a DI-sorting machine, that is, two stacks in series where at all times entries in the first stack must be in decreasing order from top to bottom. In this talk, we present a bijection between $\mathcal{P}_n$ and $\mathcal{S}_{n-1}$, which arises "naturally" from the process by which a permutation in $\mathcal{P}_n$ is sorted by the DI-machine, where symbols are moved through the two stacks according to a specified algorithm.