

A New Approach to the Total Chromatic Number Conjecture

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Total Chromatic Number Conjecture (TCNC) is an open problem in graph theory, which after more than 50 years still challenges many researchers. This classic conjecture provides an upper bound $\Delta(G) + 2$ for the total chromatic number of an arbitrary simple graph G with maximum degree $\Delta(G)$, where total chromatic number $\chi''(G)$ is defined as the minimum number of colors required to color both vertices and edges of G with the three natural coloring conditions. Thus far the best upper bound provided in the literature for $\chi''(G)$, under a preassigned condition, is $\Delta(G) + 10^{26}$. In this paper, we introduce an infinite class of graphs for each of which 10^{26} is reduced to 5. The technique presented here is also used to prove that for an arbitrary graph G , $\chi''(G) \leq 2\Delta(G) + 5$. Further investigation regarding adaptation of this new approach to other classes of graphs is highly advisable for more research.