On a Family of the Super Edge-Graceful Trees

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A (p, q)-graph G is said to be edge graceful if the edges can be labeled by $1, 2, \ldots, q$ so that the vertex sums are distinct, modulo p. It is shown that if a tree T is edge-graceful then its order must be odd. Lee conjectured that all trees of odd orders are edge-graceful. J. Mitchem and A. Simoson introduced the concept of super edge-graceful graphs that is a stronger concept than edge-graceful for some classes of graphs. A graph $G = (V, E)$ of order p and size q is said to be super edge-graceful if there exists a bijection

$$f\colon E \to \begin{cases} \left\{0, +1, -1, +2, -2, \ldots, \frac{q-1}{2}, -\frac{q-1}{2}\right\} \text{ if } q \text{ is odd;} \\ \left\{+1, -1, +2, -2, \ldots, \frac{q}{2}, -\frac{q}{2}\right\} \text{ if } q \text{ is even.} \end{cases}$$

such that the induced vertex labeling f^* defined by $f^*(u) = \sum f(u, v) \mid (u, v) \in E$ has the property:

$$f^*\colon V \to \begin{cases} \left\{0, +1, -1, +2, -2, \ldots, \frac{p-1}{2}, -\frac{p-1}{2}\right\} \text{ if } p \text{ is odd;} \\ \left\{+1, -1, +2, -2, \ldots, \frac{p}{2}, -\frac{p}{2}\right\} \text{ if } p \text{ is even.} \end{cases}$$

is a bijection. Lee conjectured that all odd trees are super edge-graceful. The conjecture is still unsettled. In this paper we exhibit a family of trees of odd orders which are super edge-graceful.