ℓ-Connectivity and ℓ-edge-connectivity of random graphs

Ran Gu, Xiaofeng Gu, Yongtang Shi, Hua Wang*

Georgia Southern University, Statesboro, GA 30460-8093, USA
hwang@georgiasouthern.edu

For an integer $\ell \geq 2$, the ℓ-connectivity $\kappa_\ell(G)$ of a graph G is defined to be the minimum number of vertices of G whose removal produces a disconnected graph with at least ℓ components or a graph with fewer than ℓ vertices. The ℓ-edge-connectivity $\lambda_\ell(G)$ of a graph G is the minimum number of edges whose removal leaves a graph with at least ℓ components if $|V(G)| \geq \ell$, and $\lambda_\ell(G) = |E(G)|$ if $|V(G)| < \ell$. In this paper, we establish sharp threshold functions for the ℓ-connectivity and ℓ-edge-connectivity of random graphs, which generalize the result of Erdős and Rényi, and Stepanov. In fact, further strengthening our results, we show that in the random graph process, with high probability the hitting times of minimum degree at least k and of ℓ-connectivity (or ℓ-edge-connectivity) at least $k(\ell-1)$ coincide. This can be seen as a generalization of the results of Bollobás and Thomassen.