

CHEN-BO ZHU'S RESEARCH IN PICTURES

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Key words:

- Classical groups, invariant distributions, local theta correspondence.
- Unitary representations, models of representations.

FIGURE 1. Conservation relations for local theta correspondence

[Reference: Jour. Amer. Math. Soc. 28 (2015); with B. Sun]

$$\forall \pi \in \text{Irr}(\bar{G}(U)) :$$

$$\boxed{n_{\mathbf{t}_1}(\pi) + n_{\mathbf{t}_2}(\pi) = 2 \dim U + d_{D,\epsilon},}$$

where $d_{D,\epsilon} = 0, 1, 2, 3, 4$, depending on D and ϵ .

FIGURE 2. Theta lifting of generalized Whittaker models

[Reference: GAFA 24 (2014); with R. Gomez]

$$\begin{array}{ccc} & \text{Hom}(V, \tilde{V}) & \\ \varphi \swarrow & & \searrow \tilde{\varphi} \\ \mathcal{O} \subset \mathfrak{g} & & \tilde{\mathfrak{g}} \supset \tilde{\mathcal{O}} \end{array}$$

$$\boxed{\text{Wh}_{\tilde{\mathcal{O}}}(\Theta(\pi)) \cong \text{Wh}_{\mathcal{O}}(\tilde{\pi}), \quad \forall \pi \in \text{Irr}(G).}$$

FIGURE 3. **Multiplicity one theorems** (for classical spherical pairs)
 [Reference: Annals of Math. 175 (2012); with B. Sun]

$$\forall \pi \in \text{Irr}(G) \quad \text{and} \quad \pi' \in \text{Irr}(G') :$$

$$\boxed{\dim \text{Hom}_{G'}(\pi, \pi') \leq 1.}$$

FIGURE 4. **Uniqueness of Bessel models**
 [Reference: GAFA 20 (2010); with D. Jiang and B. Sun]

$$\forall \pi \in \text{Irr}(G) \quad \text{and} \quad \pi_0 \in \text{Irr}(G_0) :$$

$$\boxed{\dim \text{Hom}_{S_r}(\widehat{\pi} \otimes \pi_0, \chi_{S_r}) \leq 1,}$$

where $S_r = G_0 \times N_{S_r}$ is the r -th Bessel subgroup of G .

FIGURE 5. **Theta lifting and associated cycles**
 [Reference: Duke Math. J. 125 (2004); with K. Nishiyama]

$$\begin{array}{ccccc}
 V & \xleftarrow{\//K'_c} & \Xi \times_Y Z & \xrightarrow{\//K_c} & Z \\
 \downarrow \//H & & \downarrow \//H & & \downarrow \//H \\
 X & \xleftarrow{\//K'_c} & \Xi & \xrightarrow{\//K_c} & Y
 \end{array}$$

FIGURE 6. **Transfer of unitary representations** (between real forms)
 [Reference: Asian J. Math. 8 (2004); with N. Wallach]

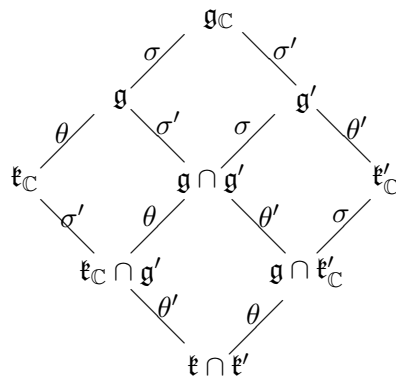


FIGURE 7. **Degenerate principal series and local theta correspondence**
 [Reference: Trans. Amer. Math. Soc. 350 (1998); with S. T. Lee]

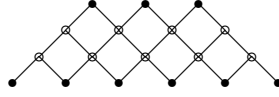


FIGURE 8. **Small representations of indefinite orthogonal groups**
 [Reference: Represent. Theory 1 (1997); with J.-S. Huang]

$$O(p, q) \curvearrowright \mathcal{N}^{\text{reg}} \quad (\mathcal{N} = \text{Null cone} \subset M_{p+q, n}(\mathbb{R})) \rightsquigarrow \pi_n$$

π_n quantizes certain quadratic nilpotent orbit \mathcal{O}_n , for $n \leq \frac{1}{2} \min(p, q)$.

FIGURE 9. **Invariant distributions of classical groups**
 [Reference: Duke Math. J. 65 (1992)]

