The Random Projection Method for Stiff Detonation Capturing

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Outline

- Motivation
- Numerical difficulties & different approaches
- The random projection method
- For detonation waves in 1D
- Extension to 2D
- Extension to multispecies detonation
- Conclusion & Future challenges

Motivation

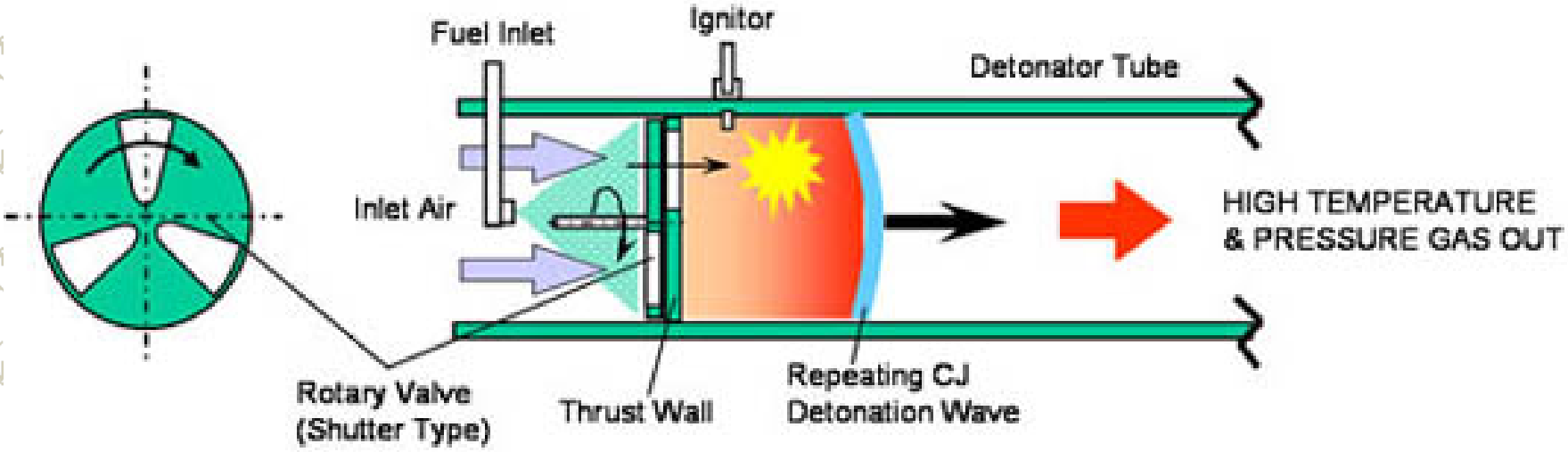
- Consider chemical reaction flows

$$\left.\begin{aligned} \rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u^2 + p)_x &= 0 \\ e_t + (u(e+p))_x &= 0 \\ (\rho z)_t + (\rho u z)_x &= -\frac{1}{\varepsilon}\rho z e^{-T_0/T} \end{aligned}\right\} \Leftrightarrow U_t + F(U)_x = \frac{1}{\varepsilon}\Psi(U) \Rightarrow \left\{\begin{array}{c} \text{Euler system} \\ + \\ \text{Chemical reaction} \end{array}\right.$$

  - State variables (unknowns):
    - Density: ρ ; velocity: u; pressure: p= $(\gamma - 1)(e - u^2/2 - q_0 \rho z)$
    - total energy: e; Temperature: T=p/ρ ; fraction of unreacted fluid: z
  - Parameters:
    - Chemical heat release: q_0 ; ignition temperature: T_0
    - Reaction time: ε ; c_p to c_v ratio: γ

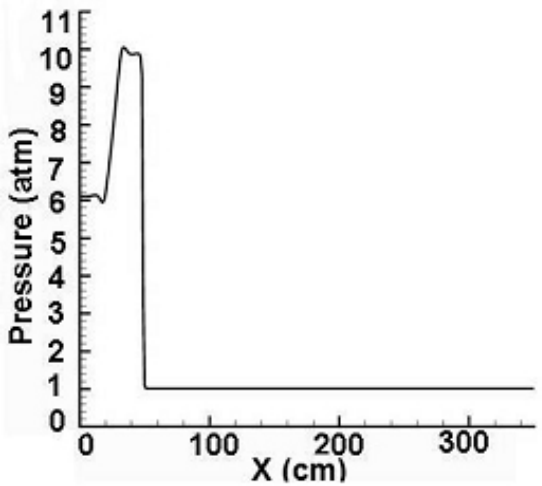
Motivation

- Many applications
  - Ozone decomposition: $2O_3 \rightleftarrows 3O_2$
  - Detonation wave in pulse detonation engine
  - …….



Motivation

- Multiscale problem: when $0 < \varepsilon \ll 1$
  - Time scales
    - Typical fluid dynamical scales: O(1)
    - Typical chemical reaction scale: $O(\varepsilon)$
  - Space scales
    - Typical fluid scale: O(1)
    - Layer of chemical layer (or detonation front): $O(\varepsilon)$



Numerical difficulties

- Numerical difficulties: stiff source terms
  - Stability
  - Spatial-temporal resolution
  - Efficient computation
- Two strategies of computation
  - Resolve the smallest chemical scale: $h = O(\varepsilon) \,\&\, k = O(\varepsilon)$
    - Any methods work well
    - Very expensive: grid points: $O(1/\varepsilon)$; time steps: $O(1/\varepsilon)$
  - Underresolve the chemical scale: $h \gg \varepsilon \,\&\, k \gg \varepsilon$
    - A classic spurious numerical phenomena occur
    - Wrong numerical shock speed !!!!

Numerical difficulties

- Spurious numerical phenomena (Colella, Majda & Roytburd, SISC, 86')
  - Solve Euler system with chemical reaction by time splitting
    - Step one: solve homogeneous system with a TVD scheme

$$U_t + F(U)_x = 0$$

    - Step two: Solve the ODE analytically or numerically in an implicit way

$$U_t = \frac{1}{\varepsilon}\Psi(U) \Leftrightarrow (\rho z)_t = -\frac{1}{\varepsilon}\rho z e^{-T_0/T}$$

  - Numerical results: Initial data (a Chapman-Jouguet (C-J) detonation)
    - Resolve the chemical scale: $h = O(\varepsilon)\ \&\ k = O(\varepsilon)$
    - Underresolve the chemical scale: $h \gg \varepsilon\ \&\ k \gg \varepsilon$
    - Observation: correct result when the chemical is resolved in space and time, wrong detonation speed when it is underresolved!!!

next

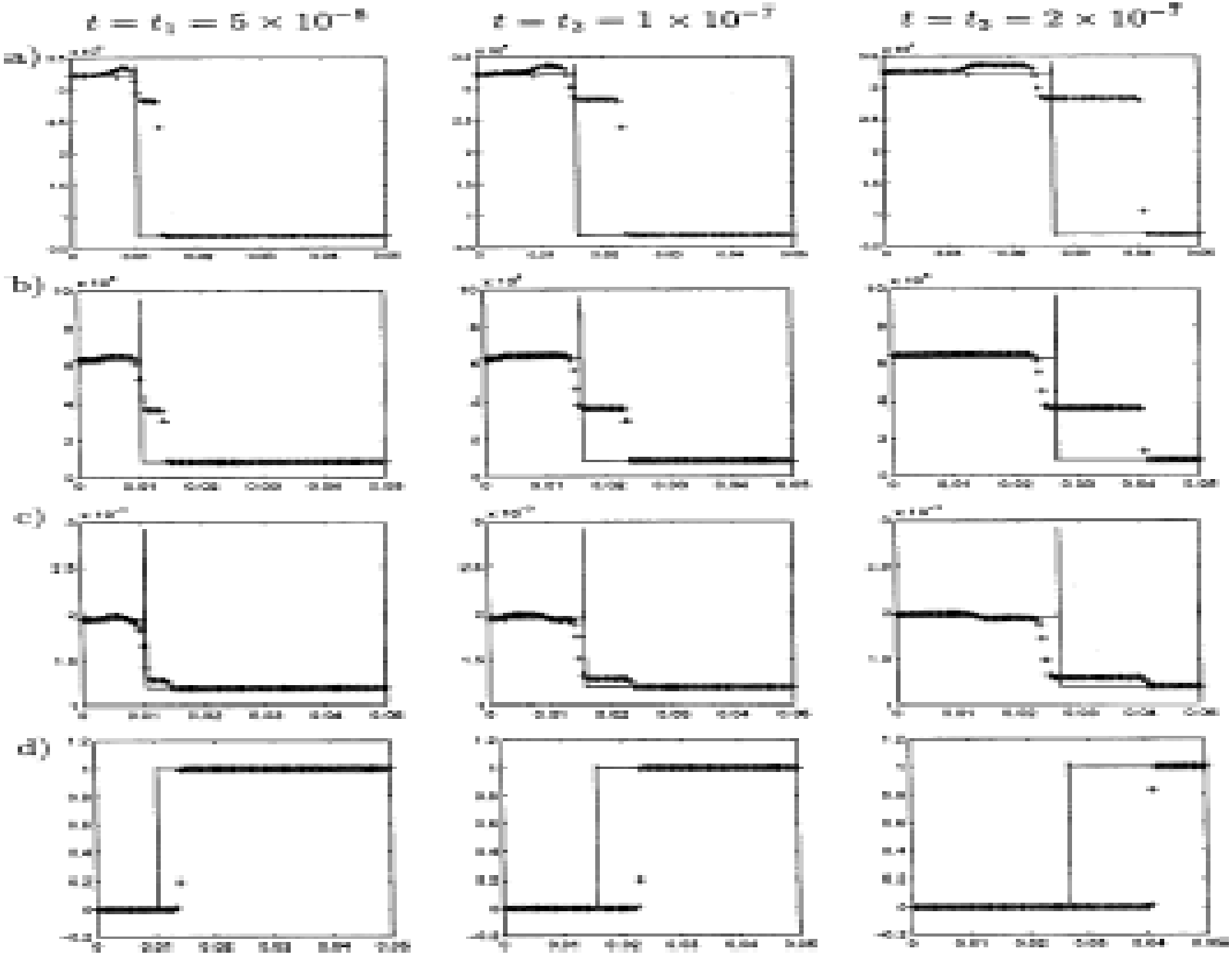


FIG. 3. Numerical results for Example 4.1 by using the deterministic method (4.11). $h = 5 \times 10^{-4}$, $k = 5 \times 10^{-10}$. —: Exact solutions; + +: computed solutions. (a) Temperature T; (b) pressure p; (c) density ρ; and (d) v.

back

Numerical difficulties

- A scalar model problem (LeVeque & Yee, JCP, 90')
  - Hyperbolic conservation law with stiff source term

$$u_t + f(u)_x = \frac{1}{\varepsilon} u(1-u^2) := \frac{1}{\varepsilon} s(u), \qquad -\infty < x < \infty$$

  - With piecewise constant initial data (Fan,Jin & Teng, JDE, 01')

$$u(x,0) = u_0(x) = \begin{cases} 1, & x \le x_0 \\ -1, & x > x_0 \end{cases}$$

  - f(u): convex function & has 3 equilibrium u=0, +1, -1.
  - Stiff source term: $0 < \varepsilon \ll 1$
  - Analytical solutions: shock moves at a constant speed!!

$$u(x,t) = u_0(x - st), \qquad s = \frac{f(1) - f(-1)}{2}: \text{ Rankine-Hugoniot condition}$$

Numerical difficulties

- Solve numerically: splitting in time
  - Step one: solve homogeneous equation with a TVD

$$u_t + f(u)_x = 0 \Rightarrow u^* := S_c(k)u^n \qquad S_c(k): \text{ shock capturing method}$$

  - Step two: solve the ODE analytically

$$u_t = \frac{1}{\varepsilon}u(1-u^2) \Rightarrow u^{n+1} = \frac{u^*}{\sqrt{(u^*)^2 + [1-(u^*)^2]e^{-2k/\varepsilon}}} \overset{k \gg \varepsilon}{\approx} \begin{cases} 1 & u^* > 0 \\ 0 & u^* = 0 \\ -1 & u^* < 0 \end{cases} := S_p(k)u^*$$

- The numerical Method $u^{n+1} := S_p(k)\, S_c(k)u^n$
  - Numerical results

next

- Observation: Shock moves one grid point per time step when CFL >0.5 & doesn't move when CFL<0.5. Numerical shock speed is either h/k or zero which is NOT the exact one s!!!

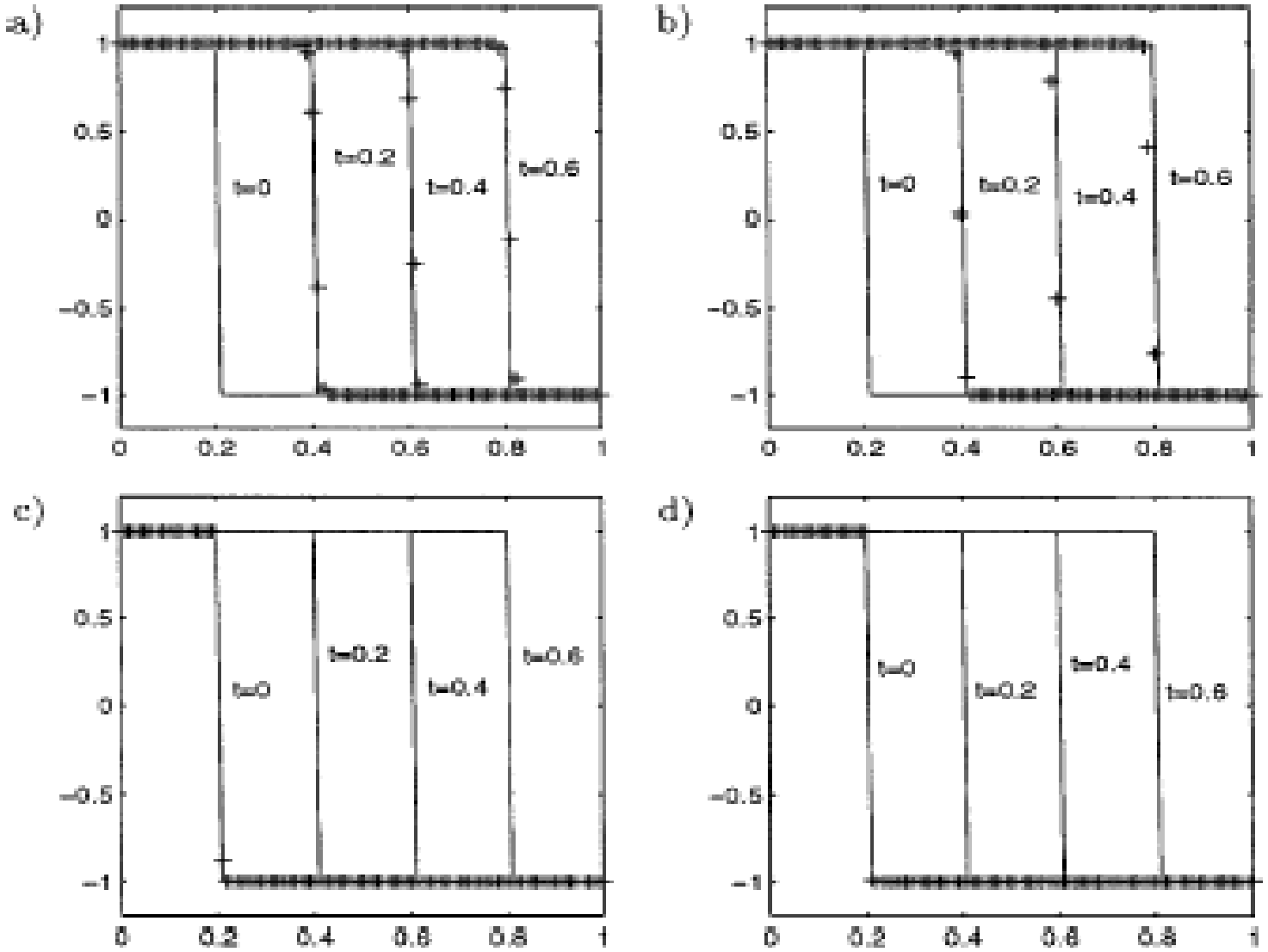


FIG. 2. Numerical results using the explicit method (2.34) for Example 2.1. $h = 0.01$. —: True solution; ++: computed solutions. (a) $\varepsilon = 0.1$, $k = 0.001$; (b) $\varepsilon = 0.01$, $k = 0.0001$; (c) $\varepsilon = 0.001$, $k = 0.00001$; (d) $\varepsilon = 0.0001$, $k = 0.000001$.

back

Numerical difficulties

- What is wrong?
  - In analytical solution: width of shock layer is zero
  - In numerical solution for convection term:
    - width of numerical shock layer is finite !!
    - The smeared points in the shock layer are nonphysical, but essential for high-resolution shock capturing scheme
    - Can NOT be avoided except the random choice method
  - In numerical solution for stiff source term:
    - Any error in previous step is amplified significantly
    - Error is always amplified in one direction,no cancellation!!!

Numerical difficulties

- Simple explanation
  - For first step: using upwind scheme

$$u_j^* = u_j^n - \frac{k}{h}[f(u_j^n) - f(u_{j-1}^n)]$$

$$\Downarrow$$

$$u_j^* = \begin{cases} 1, & u_j^n = 1 \\ -1 - \frac{k}{h}[f(-1) - f(1)] := a, & u_{j_n-1}^n = 1, u_{j_n}^n = -1 \\ -1, & u_{j-1}^n = -1 \end{cases}$$

Numerical difficulties

- For second step:
  - a>0: jump moves one grid per time step

$$u_j^{n+1} = \begin{cases} 1, & u_j^n = 1 \text{ or } u_{j-1}^n = 1, u_j^n = -1 \\ -1, & u_{j-1}^n = -1 \end{cases}$$

  - a<0: jump doesn't move

$$u_j^{n+1} = \begin{cases} 1, & u_j^n = 1 \\ -1, & u_{j-1}^n = -1 \text{ or } u_{j-1}^n = 1, u_j^n = -1 \end{cases}$$

Different approaches

- Observe the spurious numerical phenomena:
  - For chemical reaction flow (Colella, Majda & Roytburd, SISC, 86')
  - For Riemann problem of reactive flows (Ben-Artzi, JCP, 89')
  - Scalar model problem (LeVeque & Yee, JCP, 90')
- Different numerical approaches
  - Temperature extrapolation (Engquist & Sjogreen, preprint, 91')
  - Analysis of the wrong solution (Pember, SIAP, 93')
  - Ghost fluid method (Fedkiw, Aslam & S. Xu, JCP, 99')
  - Random projection method (Bao & Jin, JCP, 00'; SISC, 02')
  - Fractional step method (Helzel, LeVeque, Warnecke, SISC,00')
  - Moving (adaptive) mesh method (Azarenok & Tang, JCP, 05')
  - HMM for interface tracking (Sun & Engquist, MMS, 06')

The random projection method

- Idea: To replace the unstable equilibrium, or the critical value, u=0, with a uniformly distributed random sequence $\theta_n \in (-1,1)$.
  - This is based on the observation that there is no correlation between the center of the shock and the grid.
  - Denote

    $u^* := S_c(k)u^n$ $\qquad$ $S_c(k)$: shock capturing method
  - Replace the deterministic projection $S_p(k)$ by a random projection

$$S_\theta(k): \quad u_j^{n+1} = \begin{cases} 1 & u_j^* > \theta_n \\ -1 & u_j^* \le \theta_n \end{cases} \qquad \text{for all } j$$

The random projection method

- Choice of the random sequence
  - At each time step, only one random number is chosen
  - It doesn't depend on grid points at each time step
  - It is chosen as a random variable in the interval of two stable equilibrium of the source term
  - We require it as an equidistributed sequence
  - An example: van der Corput's sampling sequence on (0,1)

$$\vartheta n = \sum_{k=0}^{m} i_k \, 2^{-(k+1)}, \quad n = 1, 2, ... \qquad 1 \le n = \sum_{k=0}^{m} i_k \, 2^{m}, \quad i_k = 0,1: \text{ binary expansion}$$

  - Pseudo-random sequence $\vartheta_1 = \frac{1}{2}, \vartheta_2 = \frac{1}{4}, \vartheta_3 = \frac{3}{4}, \vartheta_4 = \frac{1}{8}, \vartheta_5 = \frac{5}{8}, \vartheta_6 = \frac{3}{8}, ...$
  - The derivation is minimal among all known uniformly distributed sequences

The random projection method

- The random method (Bao & Jin, JCP, 00')

$$u^{n+1} := S_r(k)\, S_c(k) u^n$$

- Error estimate for scalar problem: (Bao & Jin, Appl. Numer. Math., 02')
  - Theorem: The difference between the shock location of the exact solution and the numerical one as determined by the random projection method in which any conservative monotonicity-preserving scheme is used for the convection part, has the following estimate for any $t_n \in [0, T]$

$$| x_0 + s\, t_n - l(n)h | \le h \left[C_1 + C_2 \ln \frac{2sT}{\lambda h} \right], \qquad \lambda = \frac{sk}{h}: \text{ CFL condition \#}$$

The random projection method

- A simple proof for upwind scheme:

$$u_j^* = u_j^n - \frac{k}{h}[f(u_j^n) - f(u_{j-1}^n)] \Rightarrow u_j^* = \begin{cases} 1, & u_j^n = 1 \\ -1 - \frac{k}{h}[f(-1) - f(1)] := a, & u_{j_n-1}^n = 1, u_{j_n}^n = -1 \\ -1, & u_{j-1}^n = -1 \end{cases}$$

$$S_\theta(k): \quad u_j^{n+1} = \begin{cases} 1 & u_j^* > \theta_n \\ -1 & u_j^* \le \theta_n \end{cases} \qquad \text{for all } j$$

  - Statistical average of the numerical shock speed

$$s_a = P(\theta_n \in (-1, a) \mid \theta_n \in (-1, 1)) \frac{h}{k} = \frac{a - (-1)}{2} \frac{h}{k} = -\frac{k[f(-1) - f(1)]}{2h} \frac{h}{k} = \frac{f(1) - f(-1)}{2} = s$$

Numerical results

- The equation $$u_t + f(u)_x = \frac{1}{\varepsilon} u(1-u^2) := \frac{1}{\varepsilon} s(u), \qquad -\infty < x < \infty$$
- Initial data $$u(x,0) = u_0(x) = \begin{cases} 1, & x \le x_0 \\ -1, & x > x_0 \end{cases}$$
  - Example 1: $f(u) = \frac{u^2}{2} + u, \quad \varepsilon = 10^{-4}, \quad x_0 = 0.2 \Rightarrow s = 1$
    - Result
  - Example 2: $f(u) = e^u, \quad \varepsilon = 10^{-4}, \quad x_0 = 0.2 \Rightarrow s = \frac{e - e^{-1}}{2}$
    - Result
  - Observations: The random projection method can capture speeds of the discontinuities for scalar hyperbolic conservation laws with stiff source terms even when the reaction scale is NOT resolved spatially and temporally.

next

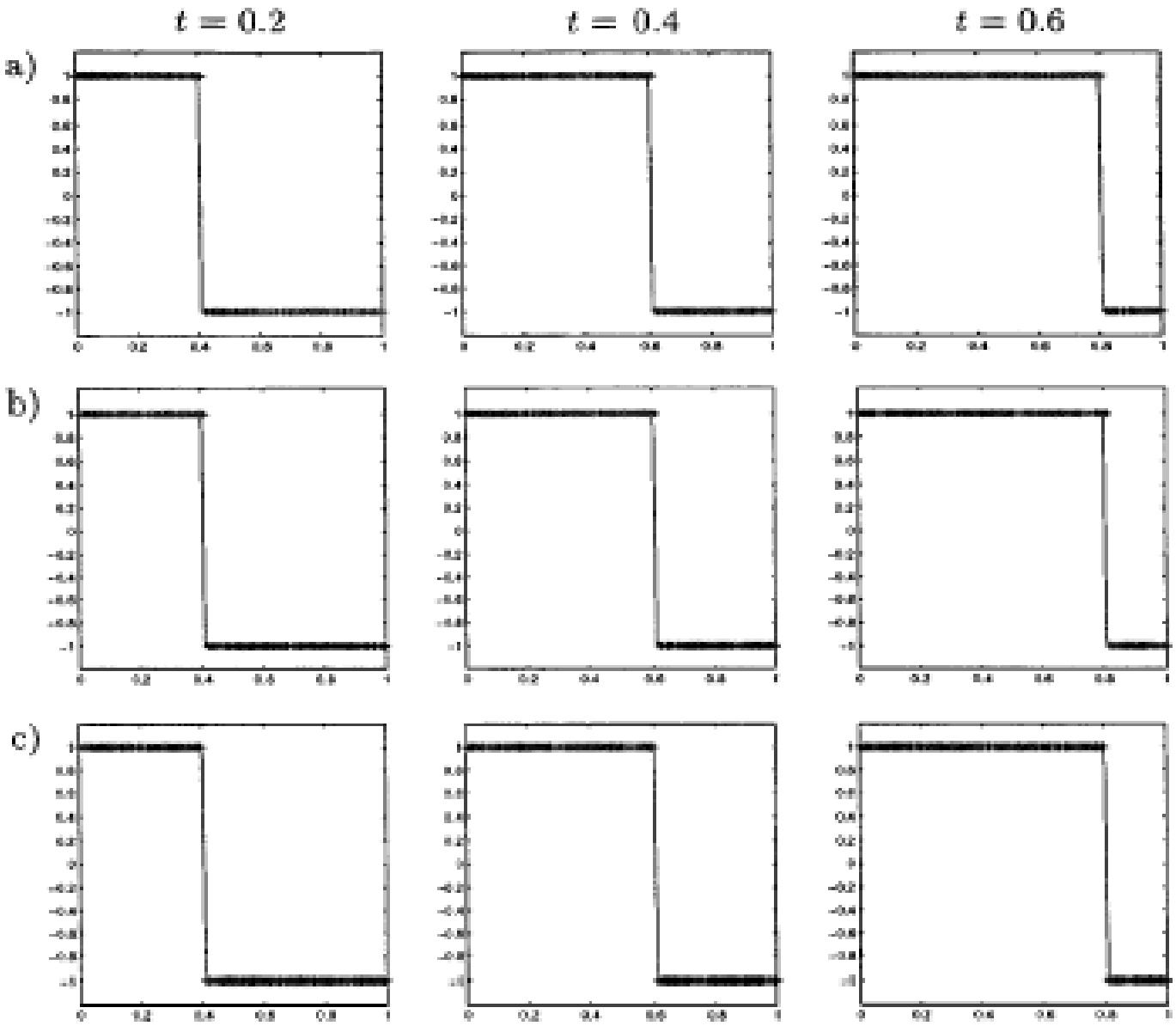


FIG. 1. Numerical results using the random projection method for example 2.1. $\varepsilon = 10^{-4}$, $h = 0.01$. —: True solution; ++: computed solutions. (a) $k = 0.004$; (b) $k = 0.001$; and (c) $k = 0.0001$.

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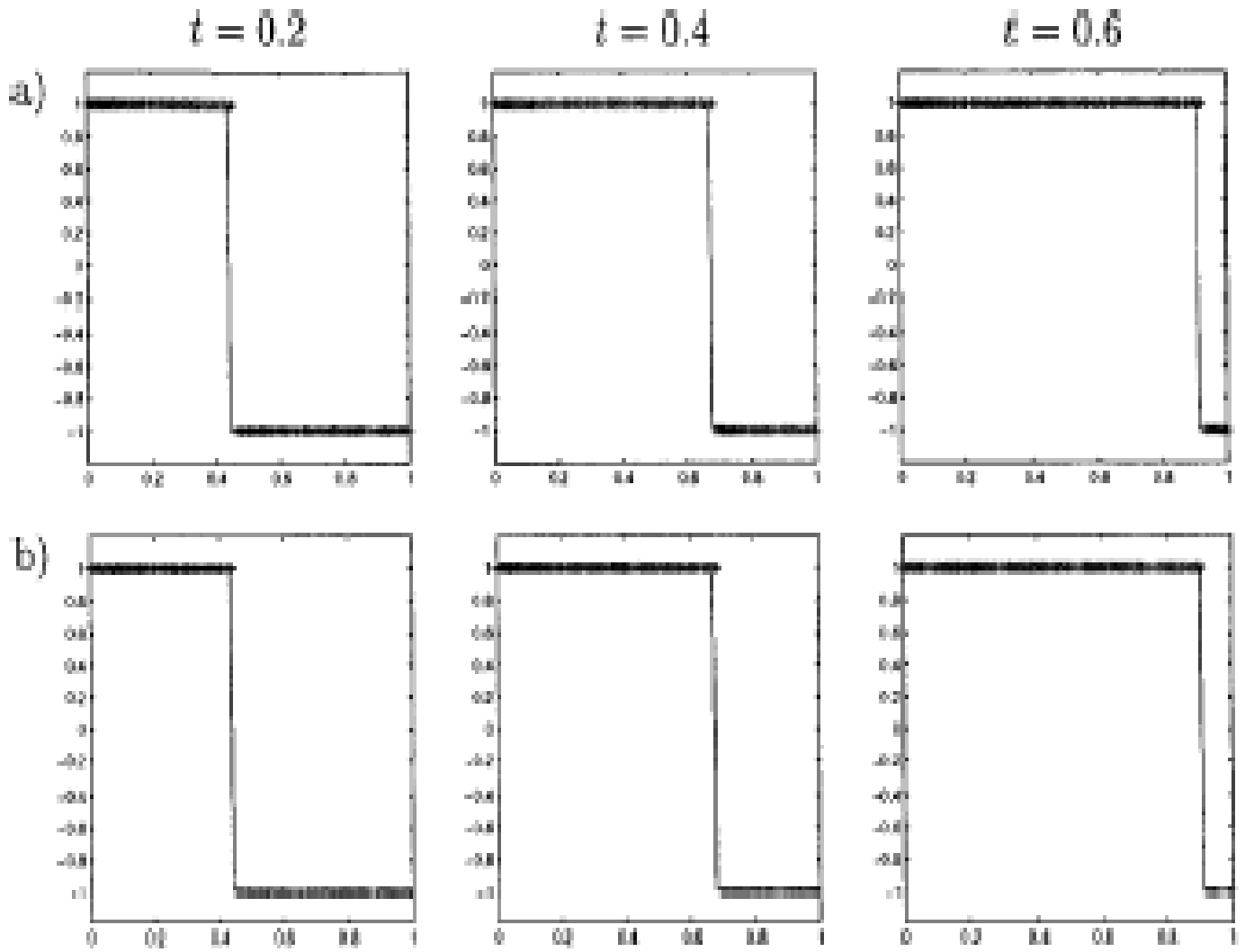

FIG. 3. Numerical results by using the random projection method for Example 2.2. $\varepsilon = 10^{-4}$, $h = 0.01$. —: True solution; ++: computed solutions. (a) $k = 0.001$; (b) $k = 0.0001$.

back

Extension to detonation wave in 1D

- Consider stiff detonation waves in 1D (Bao & Jin, JCP, 00')

$$\left.\begin{aligned} \rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u^2 + p)_x &= 0 \\ e_t + (u(e+p))_x &= 0 \\ (\rho z)_t + (\rho u z)_x &= -\frac{1}{\varepsilon}\rho z e^{-T_0/T} \end{aligned}\right\} \Leftrightarrow U_t + F(U)_x = \frac{1}{\varepsilon}\Psi(U) \Rightarrow \left\{\begin{array}{c} \text{Euler system} \\ + \\ \text{Chemical reaction} \end{array}\right.$$

- Initial data:

$$(\rho(x,0), u(x,0), p(x,0), z(x,0)) = \begin{cases} (\rho_l, u_l, p_l, 0), & x \le x_0 \\ (\rho_r, u_r, p_r, 1), & x > x_0 \end{cases}$$

- Key idea: Make the ignition temperature random

The random projection method

- The random projection method: a fraction method
  - Step one: solve the Euler system by a standard shock capturing method

$$U_t + F(U)_x = 0 \Rightarrow U^* = S_F(k)\, U^n$$

  - Step two: use random projection for the stiff source term

$$S_\theta(k): \qquad \rho_j^{n+1} = \rho_j^*, \quad m_j^{n+1} = m_j^*, \quad e_j^{n+1} = e_j^*, \quad z_j^{n+1} = \begin{cases} 0 & T_j^* > \theta_n \\ 1 & T_j^* \le \theta_n \end{cases}$$

    - Choice of the random variable

$$\theta_n = (T_l - T_r)\vartheta_n + T_r, \quad T_l = p_l / \rho_l, \quad T_r = p_r / \rho_r$$

- The global random projection method:

$$S_1(k): \qquad U^{n+1} = S_\theta(k)\, S_F(k)\, U^n$$

The random projection method

- Reason why the method works: The speed of the detonation front does NOT depend on the specific value of the critical temperature T_0, as long as it is in the range between the left and right states.
- Reasons why classical methods don't work: Since any shock capturing method will have a few grid points in the shock profile, the corresponding temperature values, once above the critical temperature T_0, may trigger a too early chemical reaction, causing non-physical values.

Numerical results

- Parameters:

$$\gamma = 1.4, \quad q_0 = 0.5196 \times 10^{10}, \quad \frac{1}{\varepsilon} = K = 0.5825 \times 10^{10}, \quad T_0 = 0.1155 \times 10^{10}$$

  - Example 1: a Chapman-Jouguet (C-J) detonation

$$p_l = 8.321 \times 10^5,\ \rho_l = 1.201 \times 10^{-3},\ u_l = 0,\ p_r = 6.270 \times 10^6,\ \rho_r = 1.945 \times 10^{-3},\ u_r = 4.162 \times 10^4$$

    - Result

  - Example 2: a strong detonation

$$p_l = 8.321 \times 10^5,\ \rho_l = 1.201 \times 10^{-3},\ u_l = 0,\ p_r = 6.270 \times 10^6,\ \rho_r = 1.945 \times 10^{-3},\ u_r = 9.162 \times 10^4$$

    - Result

  - Example 3: (weak) Detonation and other waves

$$p_l = 8.321 \times 10^5,\ \rho_l = 1.201 \times 10^{-3},\ u_l = 0,\ p_r = 8.270 \times 10^6,\ \rho_r = 1.945 \times 10^{-3},\ u_r = 4.162 \times 10^4$$

    - Result

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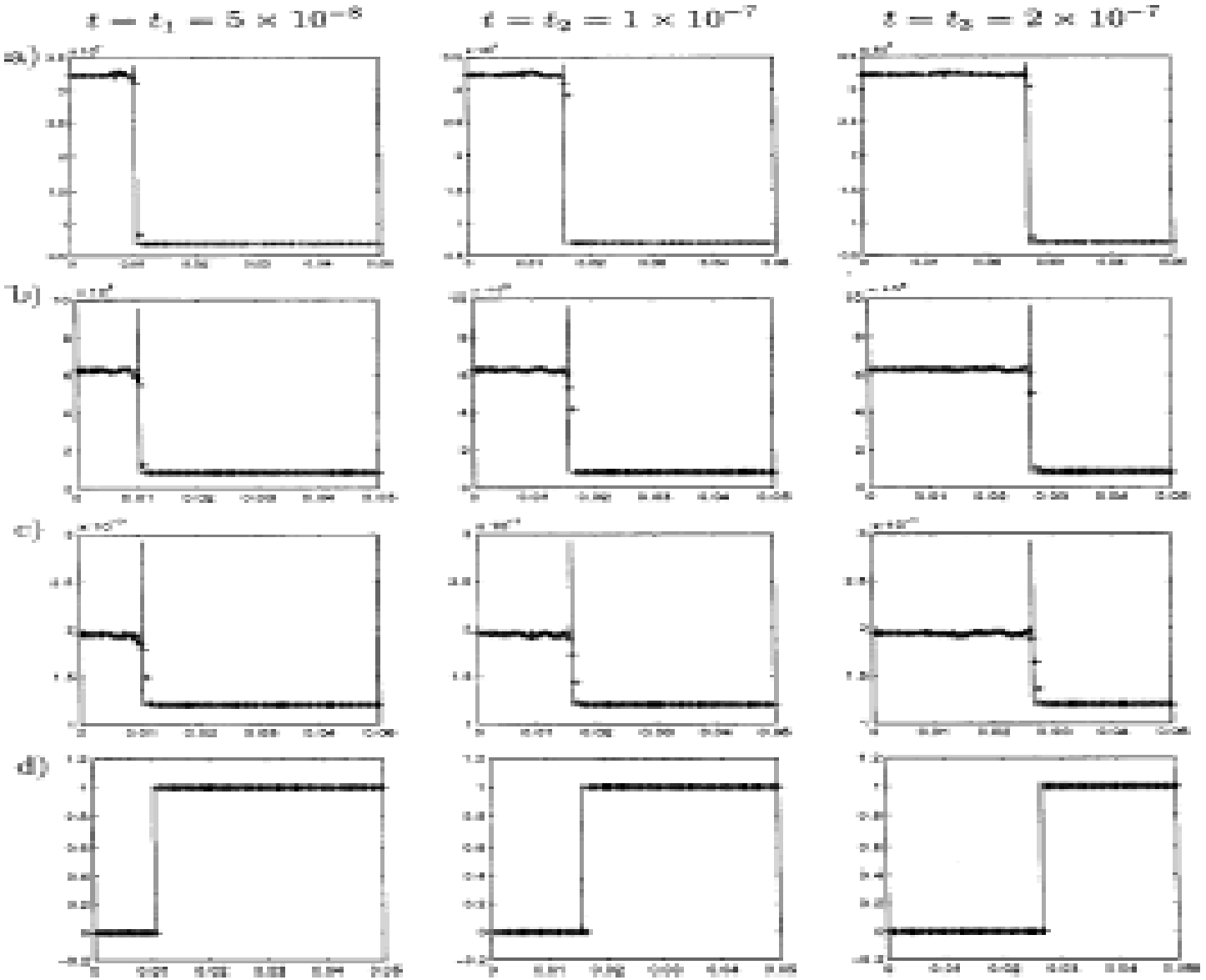

FIG. 6. Numerical results for a C–J detonation in Example 4.1 by using the global random projection method (4.6). $h = 5 \times 10^{-4}$, $k = 5 \times 10^{-10}$. —: Exact solutions; + +: computed solutions. (a) Temperature T; (b) pressure p; (c) density ρ; and (d) z.

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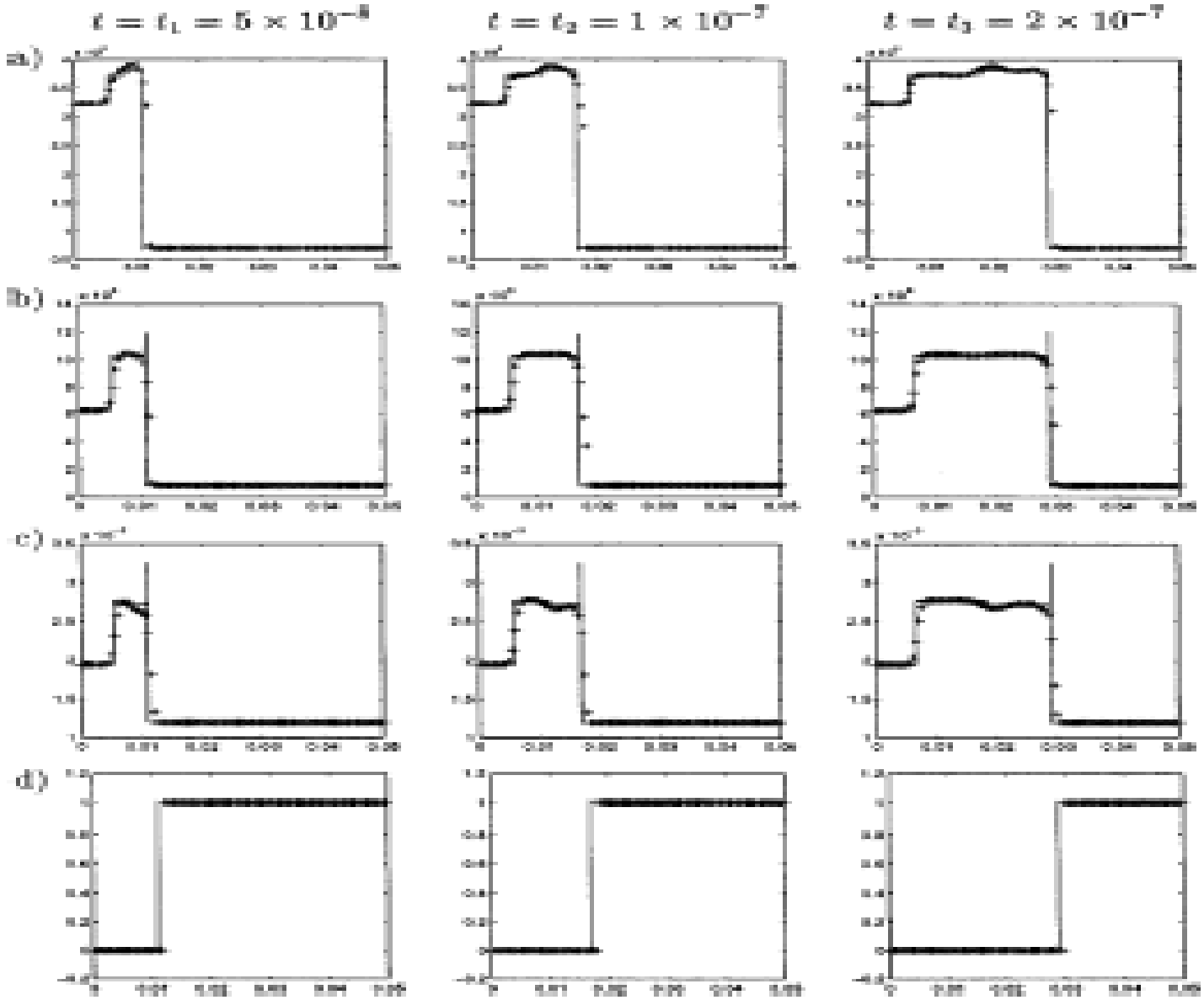

FIG. 9. Numerical results for Example 4.2 by using the local random projection method (4.9). $h = 5 \times 10^{-4}$, $\delta = 5 \times 10^{-15}$. —: Exact solutions; +–+: computed solutions. (a) Temperature T; (b) pressure p; (c) density ρ; and (d) u.

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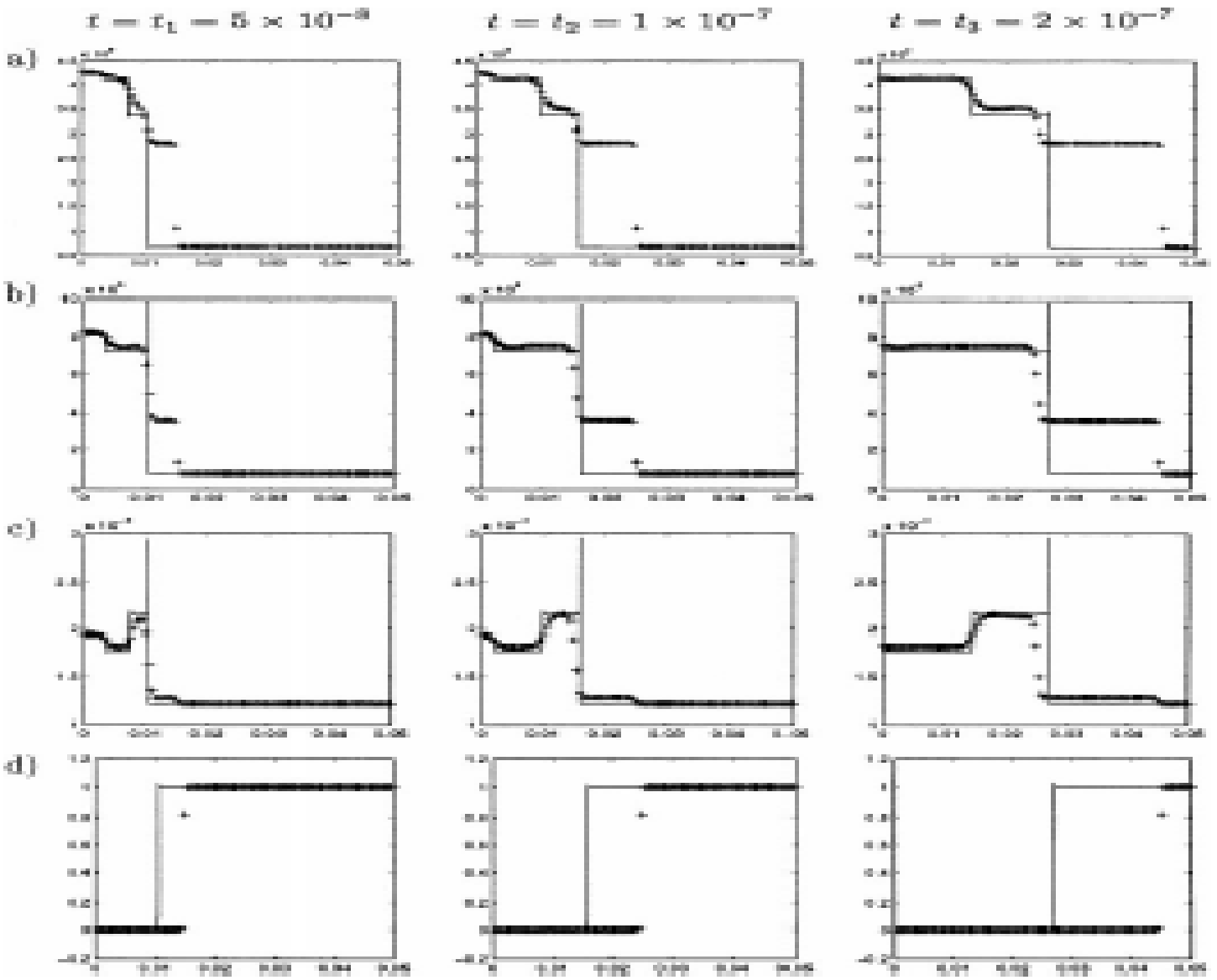

FIG. 14. Numerical results for Example 4.4 using the deterministic method (4.11). $h = 5 \times 10^{-4}$, $k = 5 \times 10^{-10}$. —: Exact solutions; +++ computed solutions. (a) Temperature T; (b) pressure p; (c) density ρ; and (d) z.

back

Numerical results

- Observations
  - For C-J detonation and strong detonation, the random projection method always gives correct results
  - For weak detonation, the result is wrong!!!
- Explaination
  - For C-J detonation, only two temperature states, i.e. $T_l > T_r$
  - For strong and weak detonations, there are at least three temperature states, i.e., T_l, T_r & T_m ! We don't know T_m at beginning!!
    - When $T_m >= T_l$, the global random projection treats the state of T_m as burnt and the results will be correct.
    - When $T_r < T_m < T_l$, once the random number is above T_m, the state of T_m will be treated as an unburnt state and chemical reaction will take place in that state, which yields the wrong solution !!!

Local random projection method

- Idea:
  - Since the reaction zone is local, it makes more sense to do the random projection around the reaction zone.
  - We perform the random projection only near the detonation front.This guarantees that a state, once burnt, remains burnt !
  - The location of the front can be easily determined from the value of the mass fraction variable z since the projection step always makes z either 0 or 1. At any time step t_n, there is an l(n) = j_0, such that

$$z_j^n = \begin{cases} 0 & j \le l(n) \\ 1 & j > l(n) \end{cases}$$

Local random projection method

– For step two:

$$\tilde{S}_\theta(k): \quad \rho_j^{n+1} = \rho_j^*, \quad m_j^{n+1} = m_j^*, \quad e_j^{n+1} = e_j^*,$$

$$\text{set } \; l(n+1) := l(n) - 1$$

$$\text{For } \; l = l(n)-1, l(n), ..., l(n)+d \; \text{ do: } \; l(n+1) = l \; \text{ if } \; T_j^* > \theta_n$$

$$z_j^{n+1} = \begin{cases} 0 & j \le l(n+1) \\ 1 & j > l(n+1) \end{cases}$$

- d is the number of smeared points in the shock layer
- Only d+2 points of temperature will be scanned!!

The local random projection method

$$S_2(k): \qquad U^{n+1} = \tilde{S}_\theta(k)\, S_F(k)\, U^n$$

Numerical results

- Give the same results for
  - Example 1: a Chapman-Jouguet (C-J) detonation
  - Example 2: a strong detonation
- Give the correct results for
  - Example 3: (weak) Detonation and other waves
    - Result
- Observations
  - The local random projection captures all wave propagations in all cases
  - It is a little more efficient!
  - We recommend the local random projection method for stiff detonation capturing if it is only interested in detonation front & fluid structure!
  - Of course, the method cannot give detonation front layer structure!!!

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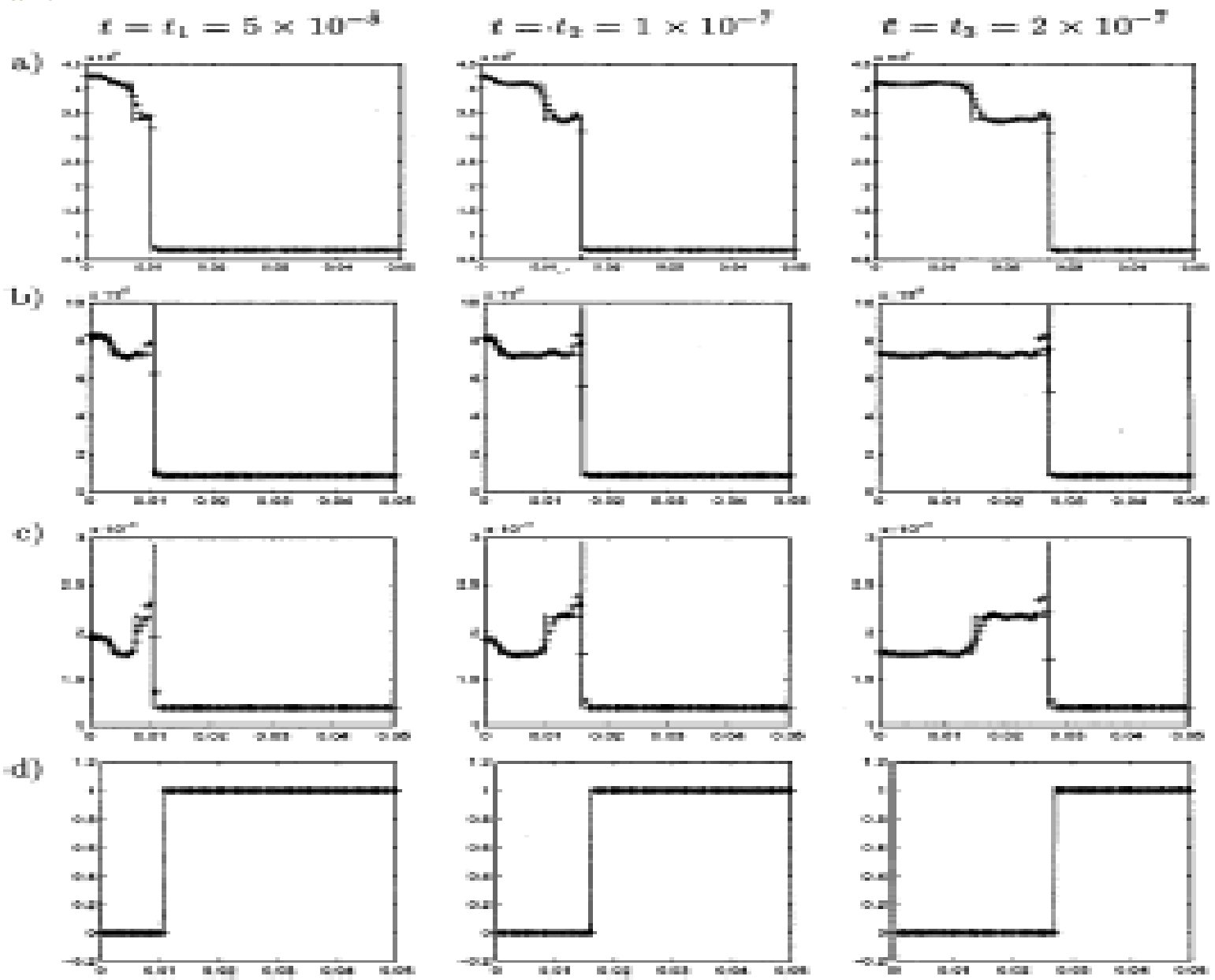

FIG. 13. Numerical results for Example 6.4 by using the local random projection method (4.9). $h = 5 \times 10^{-4}$, $\delta t = 5 \times 10^{-10}$. —: Exact solutions; +++: computed solutions. (a) Temperature T, (b) pressure p, (c) density ρ, and (d) z.

back

Applications

- Collision between detonation & waves
  - Case 1: Detonation & rarefaction wave
    - Before collision & after collision
  - Case 2: Detonation & sine-waves
    - before collision & after collision
  - Case 3: Detonation, shock & rarefaction wave
    - Before collision, during collision & after collision
  - Case 4: Detonation & detonation
    - Before collision & after collision

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- Observations: The local random projection gives correct results of detonation front & fluid structures in underresolved regime!!

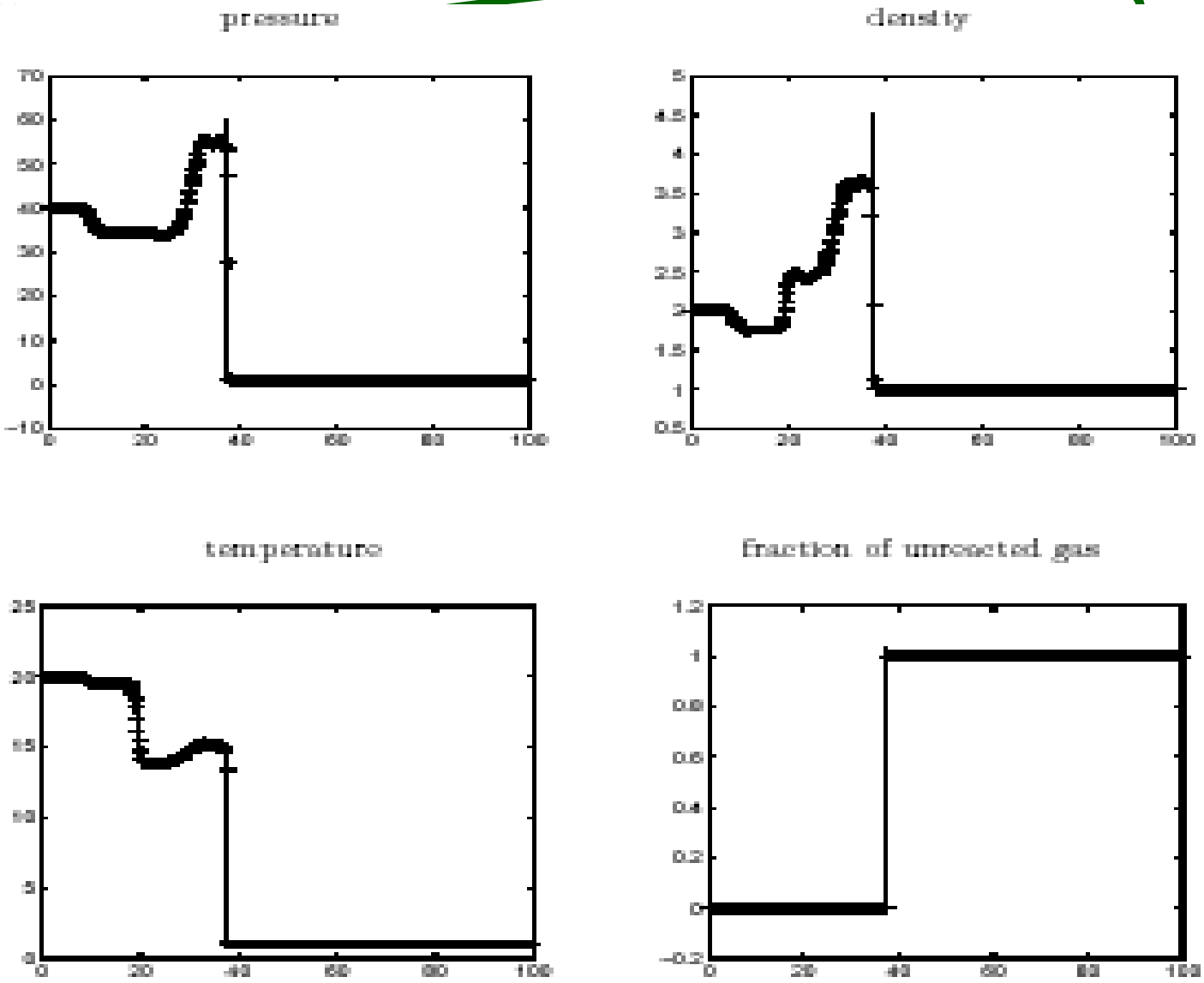

FIG. 4.3. *Numerical results of Example 4.3 involving the collision of a detonation with a rarefaction wave using the random projection method (2.10). $h = 0.25$, $k = 0.01$. —: "exact" solutions; ++: computed solutions. (a) $t = 2$ (before collision).*

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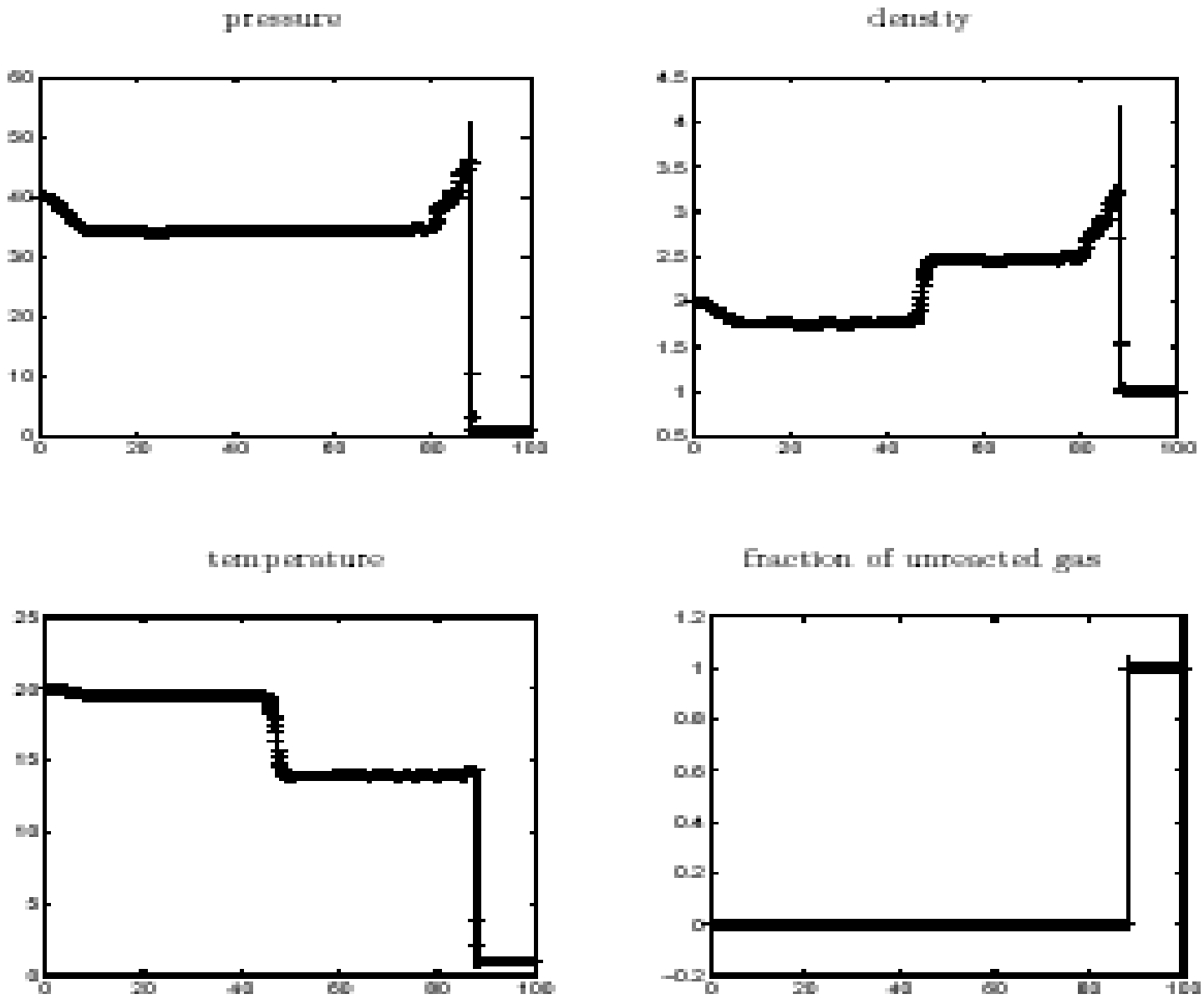

FIG. 4.3 (cont.). (b) $t = 8$ (after collision).

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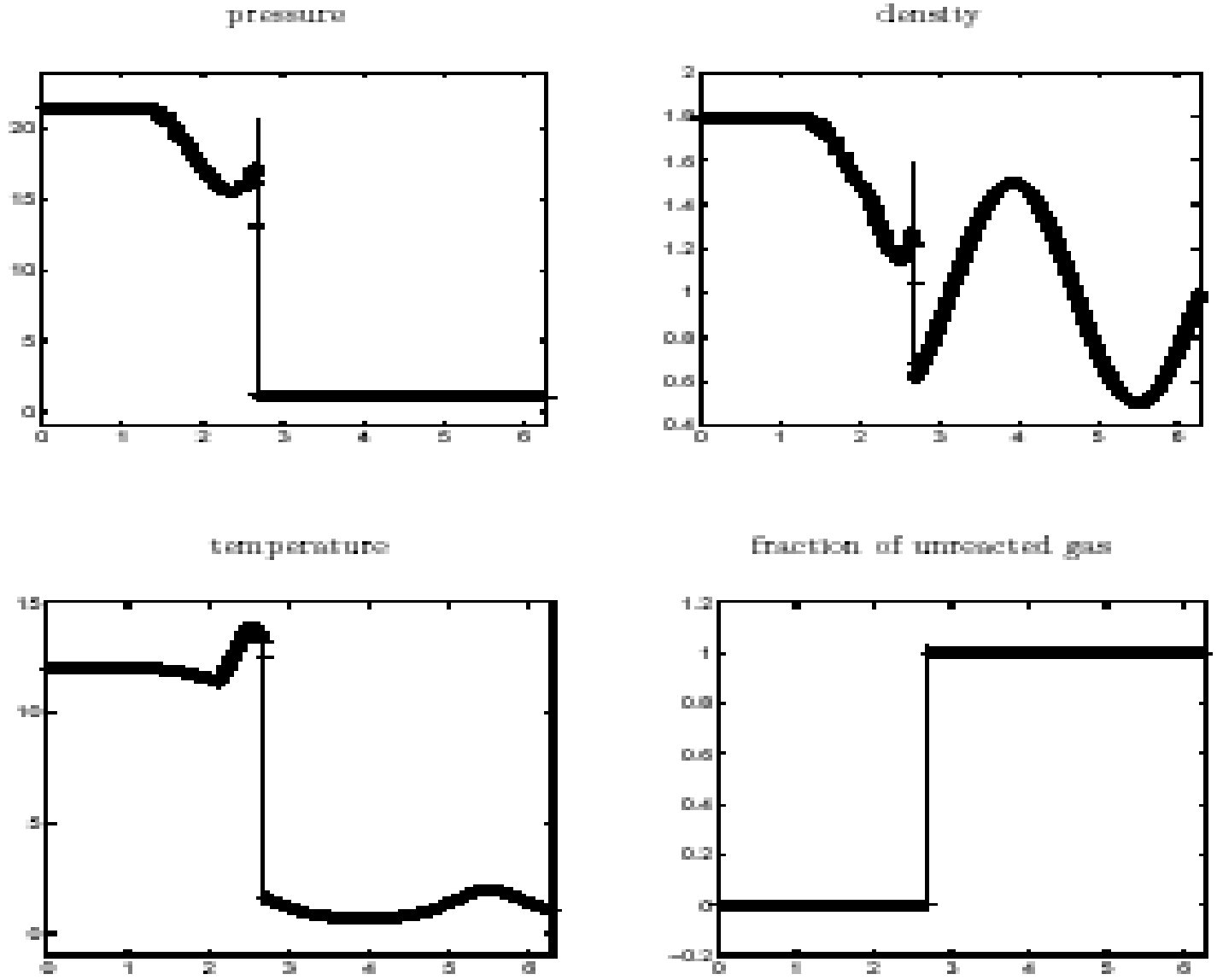

FIG. 4.4. *Numerical results of Example 4.4 by the random projection method (2.10).* $h = \frac{1}{200}$, $k = \frac{1}{20}$. —: "exact" solutions; ++: computed solutions. (a) $t = \frac{1}{20}$.

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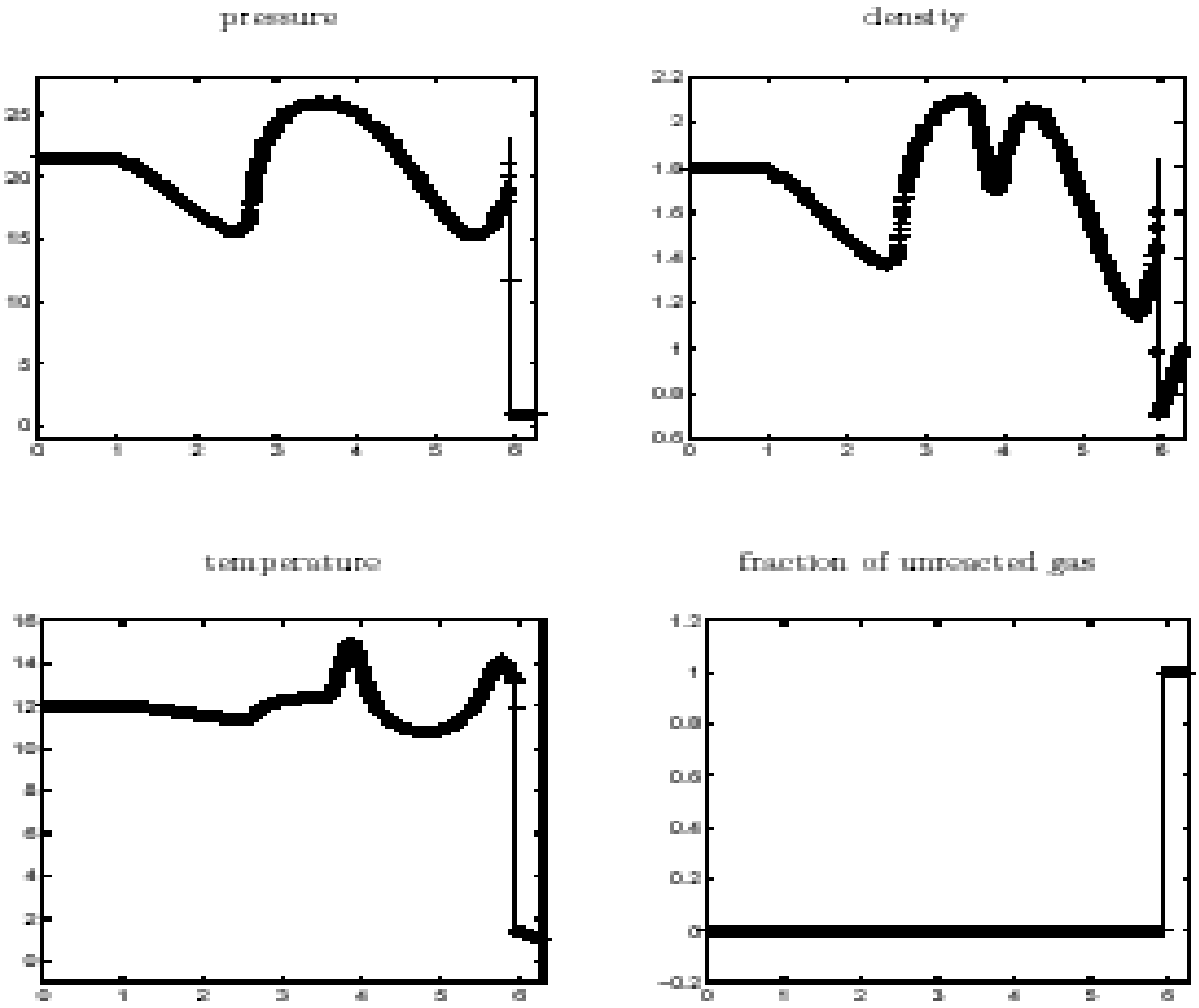

FIG. 4.4 (cont.). (b) $t = \frac{4}{3}$.

back

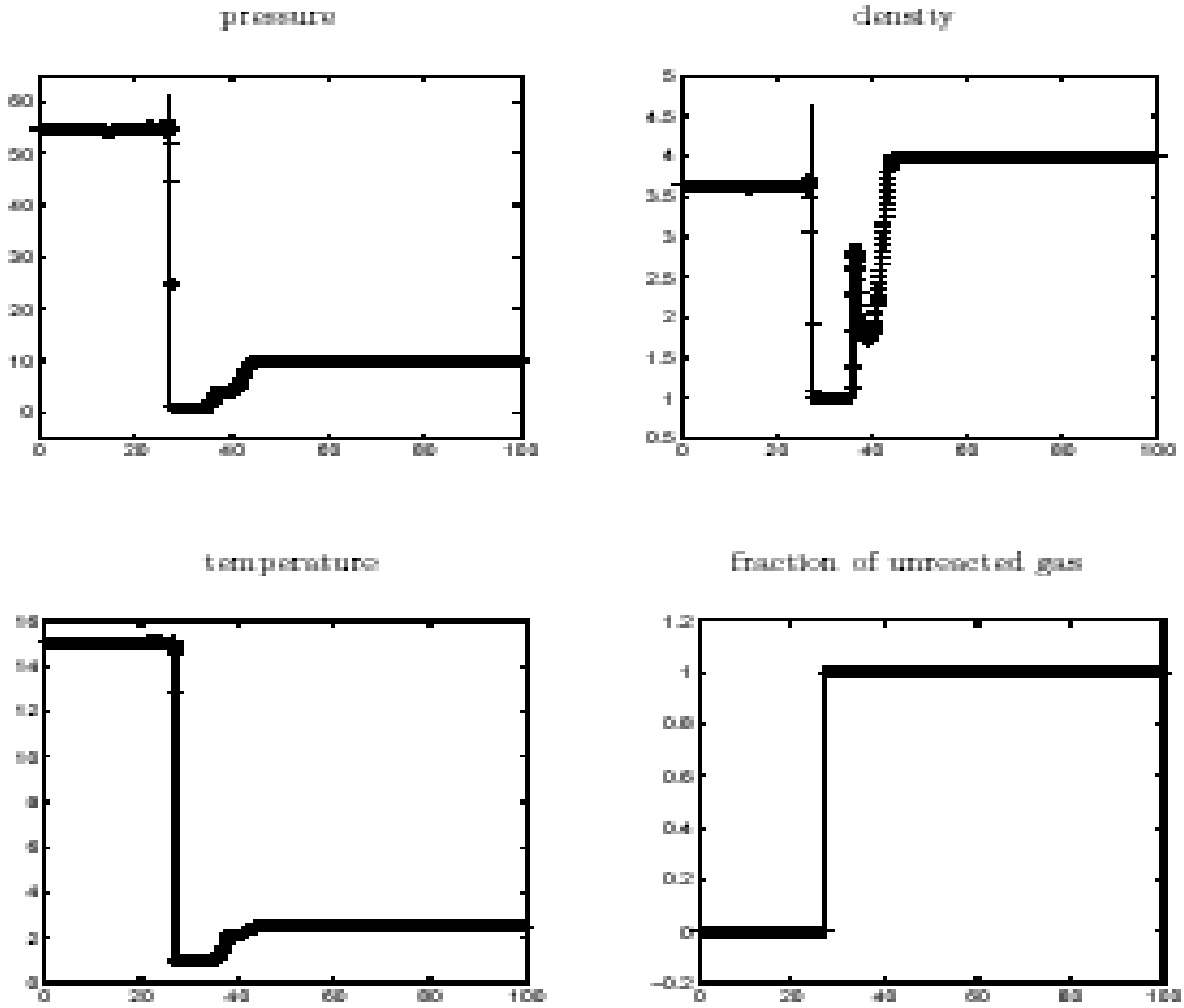

FIG. 4.5. *Numerical results of Example 4.5 modeling the collision of a detonation with a shock and then a rarefaction wave by the random projection method (2.10).* $h = 0.125$, $k = 0.005$. $-$: "exact" solutions; $++$: computed solutions. (a) $t = 2$ *(before collision).*

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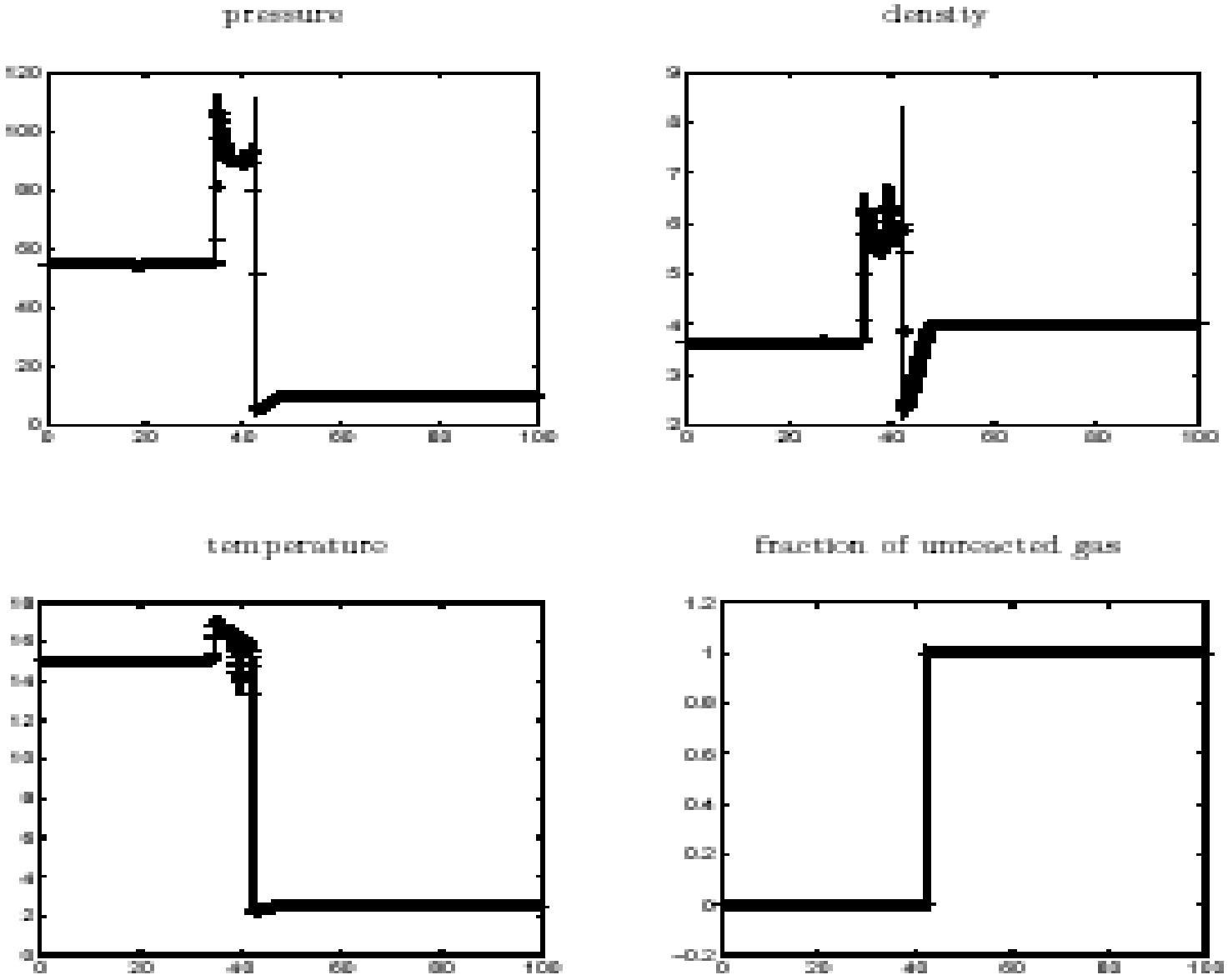

FIG. 4.5 (cont.). (b) $t = 4$ (between the collisions with the shock and the rarefaction).

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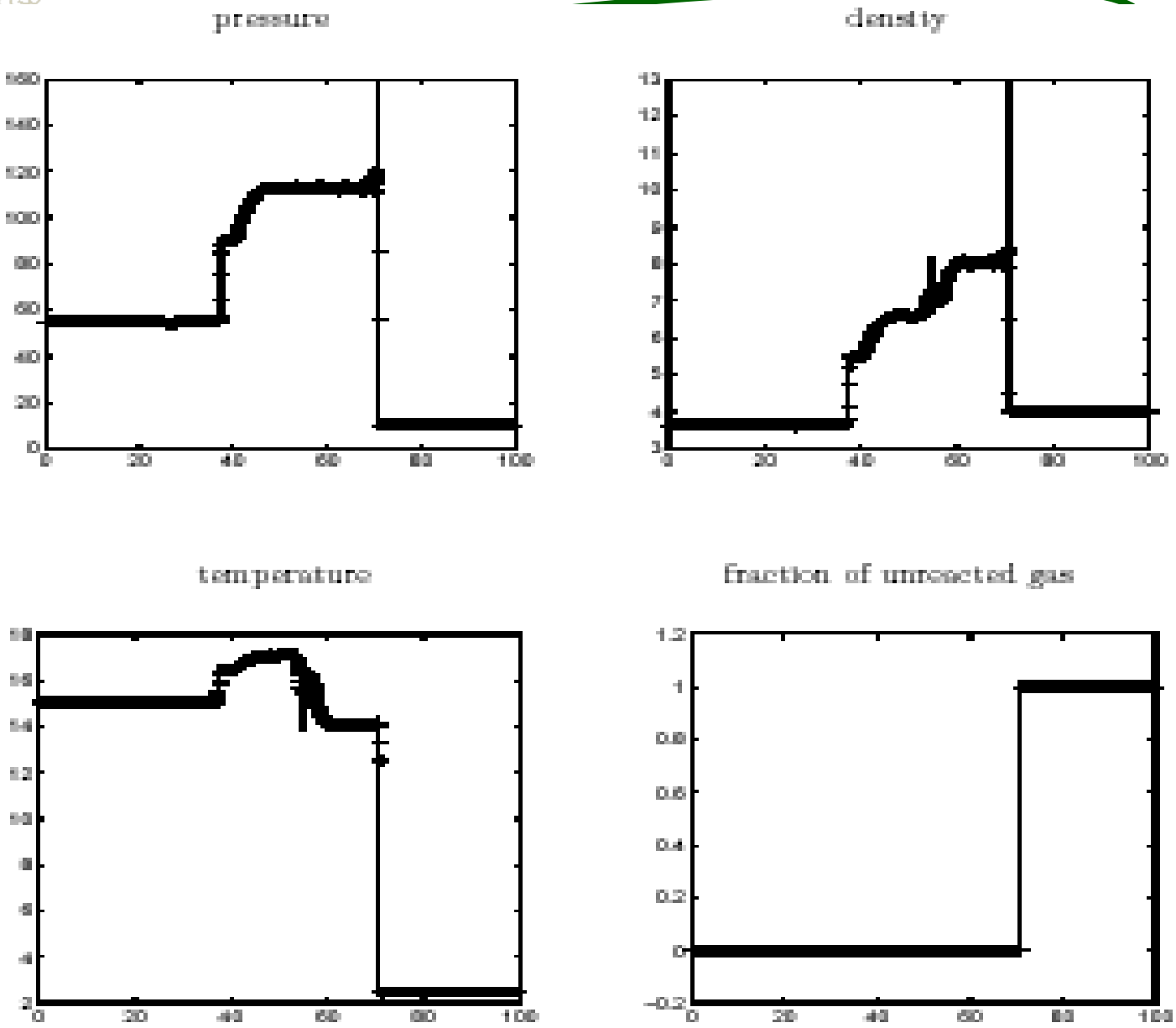

FIG. 4.5 (cont.). (c) $t = 8$ (after all collisions).

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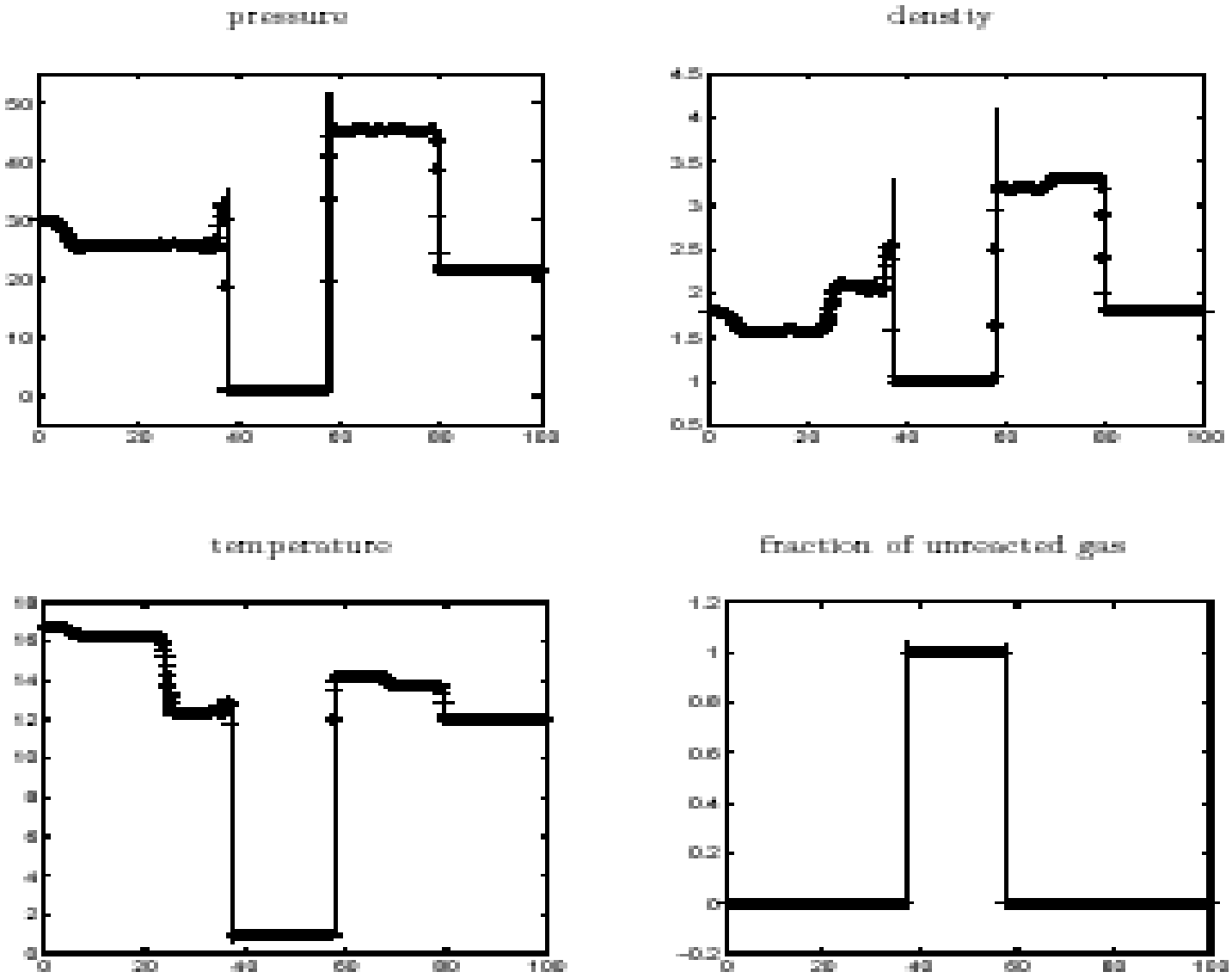

FIG. 4.6. *Numerical results of Example 4.6 involving the collision of two detonations by the random projection method (2.15). $h = 0.25$, $k = 0.01$. —: "exact" solutions; ++: computed solutions. (a) at $t = 4$ (before collision).*

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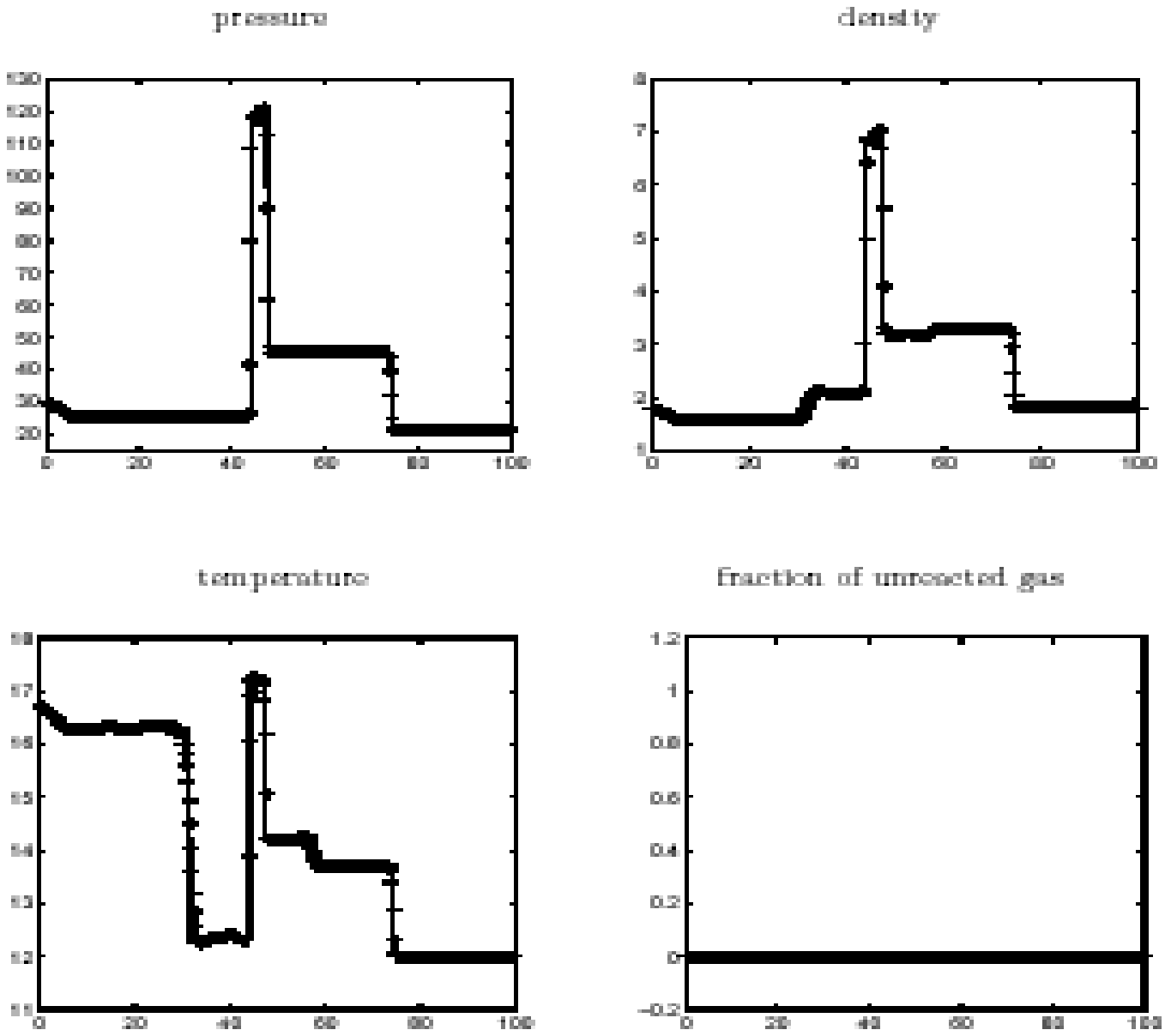

FIG. 4.6 (cont.). (b) at $t = 6$. After collision the detonation becomes extinct and the gas is completely burned.

back

Extension to 2D

- For scalar conservation problem (Bao & Jin, JCP, 00')

$$u_t + f(u)_x + g(u)_y = \frac{1}{\varepsilon} u(1-u^2) := \frac{1}{\varepsilon} s(u),$$

  - Step one: solve

$$u_t + f(u)_x + g(u)_y = 0 \Rightarrow u^* := S_c(k) u^n \qquad S_c(k): \text{ shock capturing method}$$

  - Step two: random projection

$$S_\theta(k): \quad u_{ij}^{n+1} = \begin{cases} 1 & u_{ij}^* > \theta_n \\ -1 & u_{ij}^* \le \theta_n \end{cases} \qquad \text{for all } i, j$$

- The method

$$u^{n+1} := S_\theta(k)\, S_c(k) u^n$$

Numerical results

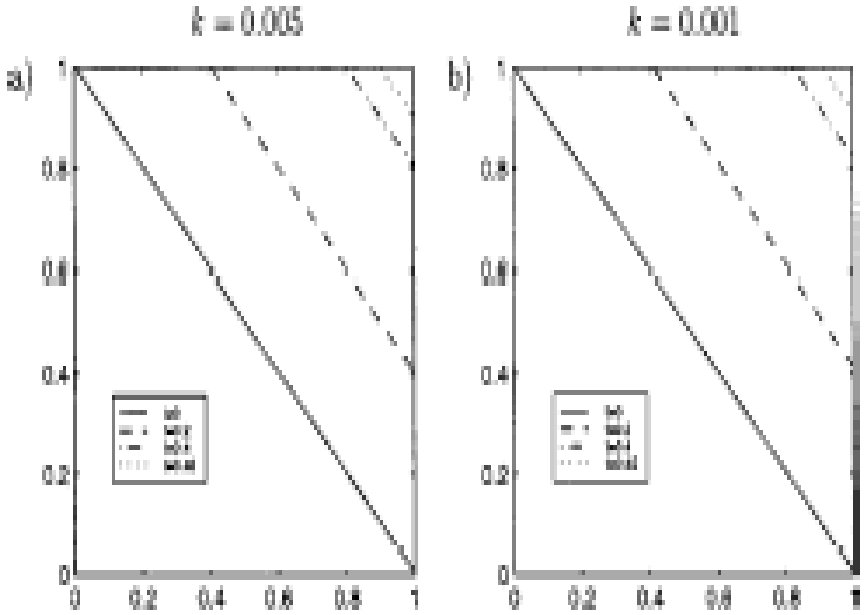

FIG. 4. Numerical results for Example 3.1 using the 2D random projection method (3.6). $\varepsilon = 10^{-4}$, $h = 0.01$. (a) $k = 0.005$; (b) $k = 0.001$.

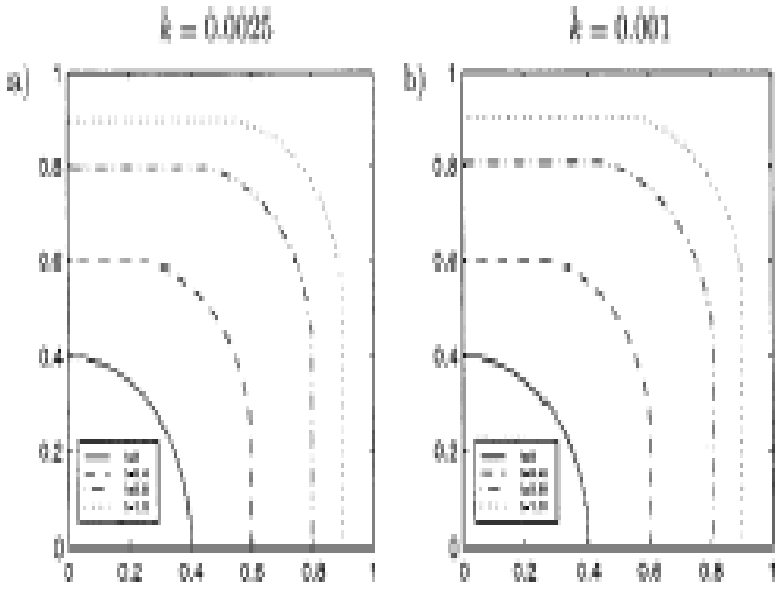

FIG. 5. Numerical results for Example 3.2 by using the 2D random projection method (3.6). $\varepsilon = 10^{-4}$, $h = 0.005$. (a) $k = 0.0025$; (b) $k = 0.001$.

Extension to 2D

- For chemical reaction flows

$$\left.\begin{aligned}
&\rho_t + (\rho u)_x + (\rho v)_y = 0 \\
&(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0 \\
&(\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0 \\
&e_t + (u(e+p))_x + (v(e+p))_y = 0 \\
&(\rho z)_t + (\rho uz)_x + (\rho vz)_x = -\frac{1}{\varepsilon}\rho z e^{-T_0/T}
\end{aligned}\right\}$$

$$U_t + F(U)_x + G(U)_y = \frac{1}{\varepsilon}\Psi(U) \Rightarrow \left\{\begin{array}{c} \text{Euler system} \\ + \\ \text{Chemical reaction} \end{array}\right.$$

Numerical method & results

- The method
  - Step one: solve

$$U_t + F(U)_x + G(U)_y = 0$$

  - Step two: do local random projection for the chemical reaction

$$\hat{S}_\theta(k): \quad \rho_{ij}^{n+1} = \rho_{ij}^*, \quad m_{ij}^{n+1} = m_{ij}^*, \quad n_{ij}^{n+1} = n_{ij}^*, \quad e_{ij}^{n+1} = e_{ij}^*,$$

For j do

$$\text{Set } l_j(n+1) := l_j(n) - 1, \tag{5.5}$$

$$\text{For } l = l_j(n) - 1,\ l_j(n), \ldots, l_j(n) + d \quad \text{do}: l_j(n+1) = l, \quad \text{if } T_{lj}^* > \theta_n;$$

$$z_{ij}^{n+1} = \begin{cases} 0, & \text{if } i \le l_j(n+1), \\ 1, & \text{if } i > l_j(n+1). \end{cases} \tag{5.6}$$

- The results

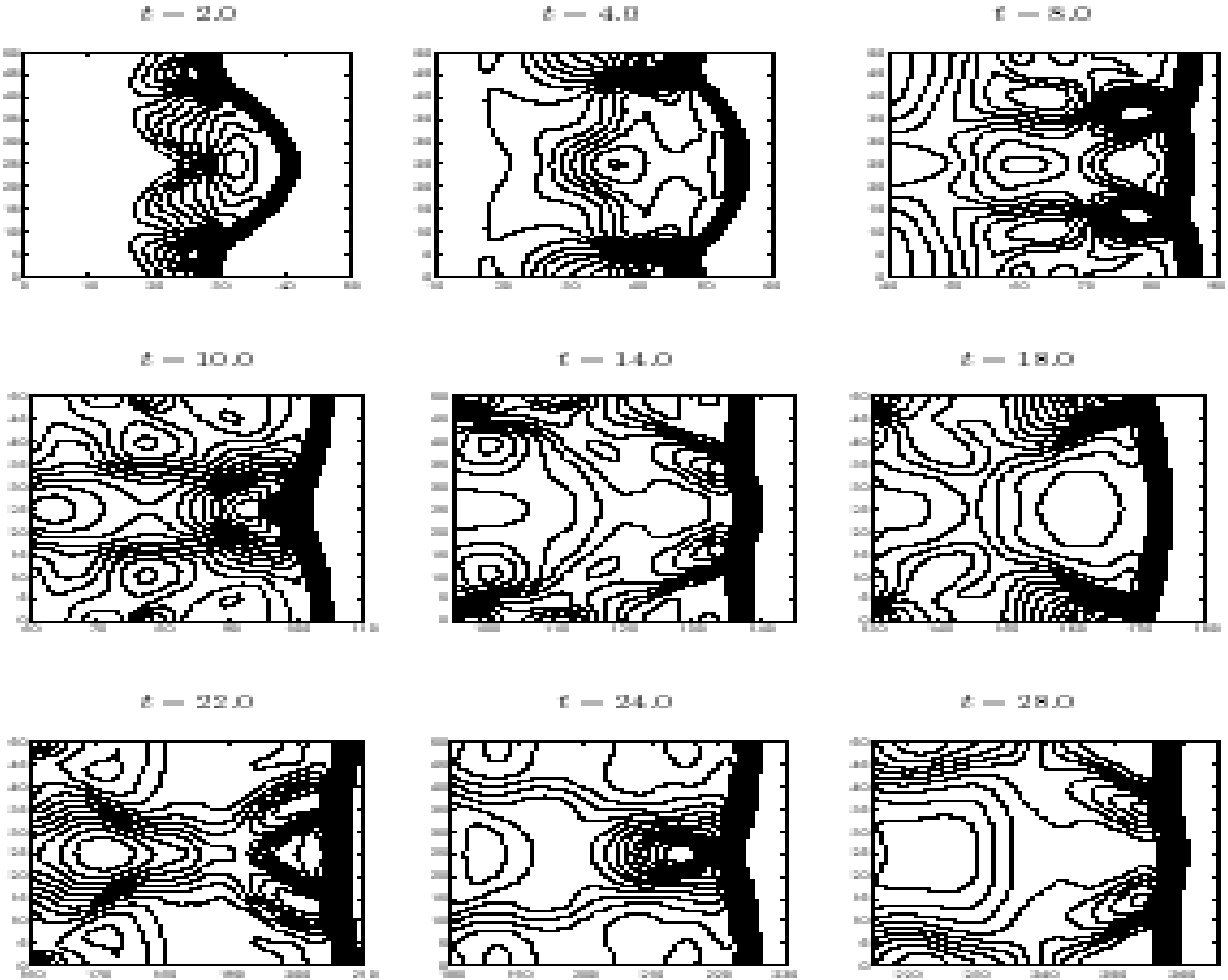

FIG. 4.7. *Numerical density contours for Example 4.7 by the two-dimensional random projection method (3.4). $h = 1.0$, $k = 0.01$.*

Extension to multi-species detonation

- For stiff multi-species detonation (Bao & Jin, JCP, 02')

$$U_t + F(U)_x + G(U)_y = S(U), \tag{1.1}$$

where

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \\ \rho z_1 \\ \rho z_2 \\ \cdots \\ \rho z_N \end{pmatrix}, \quad F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e+p)u \\ \rho u z_1 \\ \rho u z_2 \\ \cdots \\ \rho u z_N \end{pmatrix}, \quad G(U) = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e+p)v \\ \rho v z_1 \\ \rho v z_2 \\ \cdots \\ \rho v z_N \end{pmatrix}, \quad S(U) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ w_1 \\ w_2 \\ \cdots \\ w_N \end{pmatrix}, \tag{1.2}$$

with

$$w_m = W_m \sum_{k=1}^{M} (\nu''_{mk} - \nu'_{mk}) B_k T^{\alpha_k} e^{-T_k/T} \prod_{j=1}^{N} \left(\frac{\rho z_j}{W_j} \right)^{\nu'_{jk}}, \quad 1 \le m \le N, \tag{1.3}$$

Conclusions & Future challenges

- Conclusions:
  - A random projection method
    - For conservation laws with stiff source terms
      - Analytical proof in 1D
      - Numerical results
    - For chemical reactive flows
      - Numerical results
      - Applications for collision between detonation & waves
  - Give correct result of detonation front & fluid structures
    - In spatial & temporal underresolved of chemical reaction scale
    - More efficient for detonation travels in a long distance