SIAM J. MATH. ANAL.
Vol. 25, No. 3, pp. 962–975, May 1994

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HERMITE INTERPOLATION ON THE LATTICE $\mathbb{Z}^d$*

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Abstract. This paper deals with the interpolation of derivative data on the integer lattice $\mathbb{Z}^d$ by means of spaces generated by the lattice translates of several functions. The derivatives to be interpolated can be of a general form, given by a set of linear constant coefficient differential operators induced by a linearly independent set of polynomials on $\mathbb{R}^d$; for example, successive partial derivatives (Hermite interpolation) or powers of the Laplacian. The method used here is to adapt the generating functions of the interpolation space to the derivatives to be interpolated. This is done by introducing oscillations in the space through multiplication by certain shift invariant (1-periodic) trigonometric polynomials. The resulting interpolation schemes do provide convergence to smooth functions as the mesh size is reduced to zero under suitable restrictions.

Key words. cardinal interpolation, Hermite interpolation, multivariate interpolation, splines, radial basis functions

AMS subject classifications. 41A05, 65D05, 41A15, 65D07, 41A29, 41A63

1. Introduction.

In this paper we take another look at the general problem of interpolating derivative data on the lattice $\mathbb{Z}^d$. So far the following situations were considered in the literature:

- interpolation of data on the lattice from spaces generated by lattice translates of either compactly supported functions (cf. the survey [13]) or radial basis functions (cf. [12]);
- hermite interpolation of derivative values by spaces of box splines [14] or by similarly generated spaces (cf. [13]);
- interpolation on periodic meshes [4].

For many spaces generated by lattice translates of compactly supported functions, the exponential decay of the fundamental solutions for the interpolation is lost in passing from interpolation of function values to interpolation of derivative values when $d > 1$. The reason for this may be that the proper space of interpolating functions has not yet been discovered (or that the univariate model is being forced on the multivariate setting). In this paper, we search for an appropriate generating family that will provide fundamental solutions with exponential decay when such solutions exist for interpolation of function values. The essential idea is to introduce oscillations in the approximating family that are appropriate to the problem at hand.

The general cardinal interpolation problem reads as follows: For given functions

$$\phi_1, \ldots, \phi_r : \mathbb{R}^d \to \mathbb{C}$$

and functionals $\lambda_1, \ldots, \lambda_r$, we want to interpolate (real or complex) data

$$d_k := (d_k(\alpha))_{\alpha \in \mathbb{Z}^d}, \qquad k = 1, \ldots, r,$$

*Received by the editors March 2, 1992; accepted for publication (in revised form) April 7, 1993. This research was supported by National Science and Engineering Research Council of Canada grant A7687.

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