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Adaptive Neural Network Control of Nonlinear MIMO Time-delay Systems with Unknown Bounds on Delay Functionals

Shuzhi Sam Ge and Keng-Peng Tee

***Abstract*— This paper proposes an adaptive neural network (NN) tracking controller for a class of multi-input multi-output (MIMO) nonlinear systems with unknown delays. Using a separation technique, the requirement for special assumptions on the bounds of the delay functionals is removed, thereby extending the applicability of the results to a larger class of systems. To deal with uncertain or unknown functions in the system dynamics, including the unknown bounds of the delay functionals, NNs are utilized to estimate and compensate for them. Using Lyapunov-Krasovskii functionals and adaptive NN backstepping, it is shown that the proposed controller guarantees all signals to be semi-globally uniformly ultimately bounded (SGUUB) while the outputs track specified desired trajectories.**

I. Introduction

Time delay systems constitute an active topic of research, since the systems that they represent have a ubiquitous presence in the real world. Of great concern is the effect of time delay on stability and asymptotic performance [1].

Some of the tools useful in establishing robust stability for time-delay systems are based on Lyapunov's second method and include the Lyapunov-Krasovskii theorem and the Lyapunov-Razumikhin theorem. These have been applied to time-delay systems that are linear (see e.g. [2], [3], [4]), as well those that are nonlinear (see e.g. [5], [6]).

Following its success in stability analysis, the utility of Lyapunov-Krasovskii functionals in control design for time-delay systems is subsequently explored. In [7], linear systems with nonlinear functions of state-delays are considered, and it was assumed that the delay functionals are bounded by linear functions of delayed states. In [8], Lyapunov-Krasovskii functionals are used with backstepping to derive a robust controller for single-input single-output (SISO) nonlinear time-delay systems with known bounds, but it was later commented that the results could not be constructively obtained [9]. In [10], an adaptive NN controller is used to ensure practically stable tracking for unknown nonlinear systems with uncertain time delays but known bounds on the delay functionals and known sign of the control coefficients. A subsequent work extended the problem to the case of completely unknown control coefficients, by using Nussbaum-type functions [11].

As the above-mentioned works are essentially robust approaches, restrictions have been imposed on the structure of the delay functionals in terms of their dynamic bounds. The necessity of such restrictions is apparent in Lyapunov synthesis, where Lyapunov-Krasovskii functionals are used to cancel the delay terms. These restrictive conditions may limit the applicability of the approach to practical systems, where the bounds are not modelled, or where the systems concerned do not fall into the category designed for. In this paper, we show that by using the separation technique in [12], we are able to decompose the norm of a general delay functional into positive bounding functionals of each delayed state. This is obtained free of any assumptions on the structure of the delay functional. Moreover, with the use of NN approximation, the actual functionals are not required to be known. By Lyapunov synthesis, with Lyapunov-Krasovskii functionals and adaptive NN, a stable controller can be obtained. We show the effectiveness of such an approach on the tracking control problem for a class of nonlinear MIMO time-delay systems, which are hardly addressed in the literature.

The extension of results from SISO to MIMO systems is generally non-trivial, due to state and input interconnections found in MIMO systems, which tend to make the analysis much more complex. To simplify analysis, we consider the block triangular realization of the MIMO system, which allows for backstepping to be carried out in a nested fashion across the subsystems, and avoids the need for a decoupling matrix. Such an approach has been used in [13] for continuous-time MIMO systems, in [14] for discrete-time MIMO systems. The above-mentioned difficulty is exacerbated when time delays are present, and there are currently only few results available in the literature directed at MIMO time-delay systems (see [15], [16], [17], [18], which are all for linear systems).

II. Problem Formulation and Preliminaries

A. Plant Dynamics

Consider the following n-input n-output continuous-time MIMO nonlinear plant in block-triangular form with unknown constant delays:

$$\Sigma_j \begin{cases} \dot{x}_{j,i_j} = f_{j,i_j}(\bar{x}_{j,i_j}) + g_{j,i_j}(\bar{x}_{j,i_j})x_{j,i_j+1} \\ \qquad +h_{j,i_j}(\bar{x}_{\tau_{j,i_j}}), \\ \qquad\qquad \text{for} \quad 1 \le i_j \le m_j - 1 \\ \dot{x}_{j,m_j} = f_{j,m_j}(X, \bar{u}_{j-1}) + g_{j,m_j}(X)u_j \\ \qquad +h_{j,m_j}(X_\tau) \end{cases} \tag{1}$$

$\vdots$

The corresponding author is Prof. Shuzhi Sam Ge.
Shuzhi Sam Ge and Keng-Peng Tee are with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576. Email: elegesz@nus.edu.sg

$$\Sigma_n \begin{cases} \dot{x}_{n,i_n} &= f_{n,i_n}(\bar{x}_{n,i_n}) + g_{n,i_n}(\bar{x}_{n,i_n}(t))x_{n,i_n+1} \\ & \quad +h_{n,i_n}(\bar{x}_{\tau_{n,i_n}}), \\ & \qquad \text{for } \ 1 \le i_n \le m_n - 1 \\ \dot{x}_{n,m_n} &= f_{n,m_n}(X, \bar{u}_{n-1}) + g_{n,m_n}(X)u_n \\ & \quad +h_{n,m_n}(X_\tau) \end{cases}$$

$$y_j = x_{j,1}, \quad 1 \le j \le n$$

where $x_j = [x_{j,1}, \ x_{j,2}, \cdots, \ x_{j,n_j}]^T \in R^{m_j}$ are the state variables of the jth subsystem; $u = [u_1, \cdots, \ u_n]^T \in R^n$ are the system inputs; $y = [y_1, \cdots, y_n]^T \in R^n$ are the outputs; $\bar{u}_{j-1} := [u_1, \cdots, u_{j-1}]^T$ $(j = 2, \cdots, n)$ and $u_0 := 0$; $\bar{x}_{j,i_j} := [x_{j,1}, \cdots, x_{j,i_j}]^T \in R^{i_j}$; $f_{j,i_j}(\cdot)$, $g_{j,i_j}(\cdot)$ and $h_{j,i_j}(\cdot)$ are unknown and smooth nonlinear functions; the vector $X = [x_1^T, x_2^T \cdots, x_n^T]^T$ contains all states; $x_{\tau_{j,i_j}} := x_{j,i_j}(t - \tau_{j,i_j})$ denotes the delayed states; and j, i_j and m_j are positive integers.

It is noted that the term h_{j,i_j} is a function of the previous $(i_j - 1)$th delayed states of the jth subsystem, while h_{j,m_j}, which appears in the last equation of each subsystem, is a function of the delayed states of *all* subsystems. The arguments of these functions are defined as follows

$$\begin{aligned} X_\tau &:= [x_{1,1}(t-\tau_{1,1}), \cdots, x_{j,i_j}(t-\tau_{j,i_j}), \cdots \\ & \quad\ x_{j,m_j}(t-\tau_{j,m_j}), \cdots, x_{n,m_n}(t-\tau_{n,m_n})]^T, \\ \bar{x}_{\tau_{j,i_j}} &:= [x_{j,1}(t-\tau_{j,1}), \cdots, x_{j,i_j}(t-\tau_{j,i_j})]^T. \end{aligned}$$

where $\tau_{j,i_j} > 0$ is the constant unknown time delay for the i_j state of the jth subsystem. For $t \in [-\tau_{j,i_j}, 0]$, we have

$$x_{j,i_j}(t) = \phi_{j,i_j}(t), \qquad 1 \le j \le n, \ 1 \le i_j \le m_j, \quad (2)$$

where the initial function, $\phi_{j,i_j}(t)$, is smooth and bounded. Throughout this paper, for clarity in presentation, we omit the argument t in $x_{j,i_j}(t)$.

Remark 1: In plant model (1), we deal with a system with interconnected states carrying multiple constant delays embedded in a general dynamical structure $h_j(x_1(t - \tau_1), \cdots, x_j(t - \tau_j))$. Each argument x_i, $1 \le i \le j$ is assigned an independent delay τ_i. This is clearly different from the case in [10], [11], where a common delay τ_j was assigned to the argument states in each delay functionals $h_j(x_1(t - \tau_j), \cdots, x_j(t - \tau_j))$. As such, the class of delay functionals considered is more general.

It can be seen that each subsystem of (1) is in strict feedback form, which makes the use of backstepping design technique possible. Furthermore, noting that the control inputs of the whole system are in triangular form, then we may use backstepping in a nested manner to design stable controls for this class of systems.

Now, we present some notions and assumptions that will be used in the remainder of the paper.

Definition 1: [19] The solution of (1) is Semi-Globally Uniformly Ultimately Bounded (SGUUB) if, for any compact set $\Omega_0 \subset R^{m_1+m_2+\cdots+m_n}$, there exists a $S > 0$ and $T(S, X(t_0))$ such that $\|X(t)\| \le S$ for all $X(t_0) \in \Omega_0$ and $t \ge t_0 + T$.

Lemma 1: [20] For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\gamma_1(\|x\|) \le V(x) \le \gamma_2(\|x\|)$, such that $\dot{V}(x) \le -\gamma_3(\|x\|) + c$, where $\gamma_1, \gamma_2, \gamma_3 : R^n \to R$ are class K functions and c is a positive constant, then the solution $x(t)$ is SGUUB.

Lemma 2: Young's Inequality : For any $\lambda > 0$, there exist functions $f(\cdot) \in R$ and $g(\cdot) \in R$, such that

$$f(\cdot)g(\cdot) \le \lambda f^2(\cdot) + \frac{1}{4\lambda} g^2(\cdot) \quad (3)$$

Lemma 3: Separation Lemma [12]: For any continuous function $h(\bar{x}_n) : R^{m_1} \times \cdots \times R^{m_n} \to R$, where $x_j \in R^{m_j}$ $(1 \le j \le n, \ m_j > 0)$, there exist a constant $\varrho_0 \in R \ge 0$ and positive smooth functions $\varrho_j(x_j) : R^{m_j} \to R$ $(1 \le j \le n)$ satisfying $\varrho_j(0) = 0$ such that

$$|h(x_1, \cdots, x_n)| \le \varrho_0 + \sum_{j=1}^{n} \varrho_j(x_j). \quad (4)$$

The condition that $\varrho_j(0) = 0$ is needed to obtain a suitable Lyapunov-Krasovskii functional later.

Remark 2: Lemma 3 is useful for separating the functional $h(\cdot)$ containing combinations of delayed states into individual bounding functionals for each delayed state. The separation property holds for a general continuous $h(\cdot)$, and its use can be seen as an improvement over [7]-[11], where $h(\cdot)$ was assumed to possess bounds with special structures. In contrast to [7]-[11], which assumed the bounds on the delay functionals to be known, we show that we can relax this condition and allow for the bounds to be unknown, and estimated with neural networks.

Assumption 1: The signs of $g_{j,i_j}(\bar{x}_{j,i_j})$ are known, and there exist constants $g_{0_{j,i_j}}$ and known smooth functions $\bar{g}_{j,i_j}(\bar{x}_{j,i_j})$ such that $0 < g_{0_{j,i_j}} \le |g_{j,i_j}(\bar{x}_{j,i_j})| \le \bar{g}_{j,i_j}(\bar{x}_{j,i_j})$. Without loss of generality, we further assume that the signs of $g_{j,i_j}(\bar{x}_{j,i_j})$ are all positive.

Remark 3: Assumption 1 is made to simplify the technical derivation and to make the presentation as concise as possible. In the case when the input coefficient $g_{j,i_j}(\bar{x}_{j,i_j})$ are unknown, Nussbaum-type approaches [11] may be used to probe the control direction and ensure SGUUB.

Assumption 2: The first-order derivatives of all the states are available.

B. Tracking

The control objective is to ensure that all signals are bounded while tracking the desired trajectories y_{dj}, $1 \le j \le n$ such that the tracking errors converge to a small neighbourhood of the origin, i.e.,

$$\lim_{t\to\infty} |y_j(t) - y_{dj}(t)| \le \delta \tag{5}$$

for some $\delta > 0$. The desired trajectories can be generally defined by

$$\begin{aligned} \dot{x}_{d_{j,i_j}} &= f_{d_{j,i_j}}(x_{dj}),\ 1 \le i_j \le m_j \\ y_{dj} &= x_{d_{j,1}} \end{aligned} \tag{6}$$

where, for the jth subsystem, $f_{d_{j,i_j}}(\cdot)$, $i_j \in [1, 2, \cdots, m_j]$ is a known smooth nonlinear function, $x_{dj} = [x_{d_{j,1}}, x_{d_{j,2}}, \cdots, x_{d_{j,m_j}}]^T \in R^m$ are the states generated by the known exosystem, and $y_{dj} \in R$ is the output.

C. Neural Networks

In this paper, we shall use Radial Basis Function (RBF) NN, which are linearly parametrized, to approximate the continuous function $p(Z) : R^q \to R$ as

$$p(Z) = W^T S(Z) \tag{7}$$

where the input vector $Z \in \Omega_Z \subset R^q$, weight vector $W \in R^l$, and basis function vector $S(Z) = [s_1(Z), s_2(Z), ..., s_l(Z)]^T \in R^l$, with l being the NN node number and $s_i(Z)$ chosen as the commonly used Gaussian functions, which have the form $s_i(Z) = \exp[-(Z - \mu_i)^T (Z - \mu_i)/\eta_i^2]$, $i = 1, ..., l$ where $\mu_i = [\mu_{i1}, \mu_{i2}, ..., \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. Universal approximation results in [21], [22] indicate that, if l is chosen sufficiently large, $W^T S(Z)$ can approximate any continuous function to any desired accuracy over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy as

$$p(Z) = W^{*T} S(Z) + \varepsilon(Z), \forall Z \in \Omega_Z \subset R^q \tag{8}$$

where W^* is the ideal constant weight vector, and $\epsilon(Z)$ is the approximation error which is bounded over the compact set, i.e., $|\varepsilon(Z)| \le \varepsilon^*$, $\forall Z \in \Omega_Z$ where $\varepsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an "artificial" quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon|$ for all $Z \in \Omega_Z \subset R^q$, i.e., $W^* := \arg\min_{W \in R^l} \{\sup_{Z \in \Omega_Z} |p(Z) - W^T S(Z)|\}$.

III. Control Design

The controller is based on a robust control approach, using *memoryless* affine controls to dominate the delay functionals, thus rendering the closed loop system stable and achieving a desired level of tracking performance.

Noting that each subsystem is in strict-feedback form, our control design adopts embedded backstepping. We choose the intermediate and practical control laws, α_{j,i_j} and u_j respectively, as follows:

$$\begin{aligned} \alpha_{j,i_j} &= -z_{j,i_j-1} - \kappa_{j,i_j} z_{j,i_j} + \hat{W}_{j,i_j}^T S(Z_{j,i_j}), \\ u_j &= -z_{j,m_j-1} - \kappa_{j,m_j} z_{j,m_j} + \hat{W}_{j,m_j}^T S(Z_{j,m_j}), \end{aligned} \tag{9}$$

for $1 \le j \le n$, $1 \le i_j \le m_j - 1$, where $\kappa_{j,i_j}, \kappa_{j,m_j} > \frac{1}{4\lambda}$, $\lambda > 0$ are design parameters, $\hat{W}_{j,i_j}$ are the NN weights estimates, and $S(\cdot)$ is the basis function vector. The error of each step, z_{j,i_j}, is defined as

$$\begin{aligned} z_{j,1} &= x_{j,1} - y_{dj} \\ z_{j,i_j} &= x_{j,i_j} - \alpha_{j,i_j-1},\ 2 \le i_j \le m_j, \end{aligned} \tag{10}$$

and the inputs to the neural networks as

$$\begin{aligned} Z_{j,1} &= [x_{j,1}, y_{dj}, \dot{y}_{dj}]^T, \\ Z_{j,i_j} &= [\bar{x}_{j,i_j}, \alpha_{j,i_j-1}, \dot{\alpha}_{j,i_j-1}]^T,\ \ 1 \le i_j \le m_j - 1 \\ Z_{j,m_j} &= [X, \dot{x}_{1,m_1}, \dot{x}_{2,m_2}, \cdots, \dot{x}_{j,m_{j-1}}, \dot{x}_{j,m_{j+1}}, \\ &\quad \cdots, \dot{x}_{n,m_n}, \alpha_{j,m_j-1}, \dot{\alpha}_{j,m_j-1}, \bar{u}_{j-1}]^T \end{aligned} \tag{11}$$

The neural network weights adaptation law is given by

$$\dot{\hat{W}}_{j,i_j} = -\Gamma_{j,i_j} \left[S(Z_{j,i_j}) z_{j,i_j} + \sigma_{j,i_j} (\hat{W}_{j,i_j} - W_{j,i_j}^0) \right], \tag{12}$$

where $\Gamma_{j,i_j} = \Gamma_{j,i_j}^T > 0$ denotes the adaptation gain, $\sigma_{j,i_j} > 0$ denotes the growth restriction parameter, and W_{j,i_j}^0 is a design constant.

Theorem 1: The closed-loop system consisting of the plant (1) under Assumptions 1 and 2, control law (9) and the NN adaptation law (12) is SGUUB.

Proof: The proof will be presented by an inductive approach. Within the jth ($1 \le j \le n$) subsystem in strict feedback form, virtual controls are designed via backstepping up to the ($m_j - 1$)th step. For the m_jth ($1 \le j \le n$) equation of each subsystem, the interconnections with the states, delayed states, and inputs of all other subsystems are present, but the block triangular structure allows backstepping to be used across the subsystems, thereby guaranteeing stablility of the entire interconnected MIMO system.

Step j, i_j Consider the i_jth equation of the jth subsystem. Let $z_{j,i_j+1} = x_{j,i_j+1} - \alpha_{j,i_j}$, with $\alpha_{j,0} := y_{dj}$. Define integral Lyapunov function [23] as follows:

$$\begin{aligned} V_{z_{j,i_j}} &= \int_0^{z_{j,i_j}} \beta g_{\lambda_{j,i_j}}^{-1} (\bar{x}_{j,i_j-1}, \beta + \alpha_{j,i_j-1}) d\beta \\ &= z_{j,i_j}^2 \int_0^1 \theta g_{\lambda_{j,i_j}}^{-1} (\bar{x}_{j,i_j-1}, \theta z_{j,i_j} + \alpha_{j,i_j-1}) d\theta. \end{aligned}$$

where $g_{\lambda_{j,i_j}}^{-1}(\cdot) := \bar{g}_{j,i_j}(\cdot)/g_{j,i_j}(\cdot)$. Differentiating $V_{z_{j,i_j}}$ along (1) and (6) yields

$$\begin{aligned} \dot{V}_{z_{j,i_j}} \le\ & z_{j,i_j} \Big[g_{\lambda_{j,i_j}}^{-1} (f_{j,i_j} + \varrho_{0_{j,i_j}}) + z_{j,i_j+1} \\ & + \alpha_{j,i_j} - \dot{\bar{x}}_{j,i_j-1}^T \int_0^1 \theta \frac{\partial g_{\lambda_{j,i_j}}^{-1}}{\partial \bar{x}_{j,i_j-1}} d\theta \\ & - \dot{\alpha}_{j,i_j-1} \int_0^1 g_{\lambda_{j,i_j}}^{-1} d\theta \Big] + \frac{1}{2} z_{j,i_j}^2 g_{\lambda_{j,i_j}}^{-2} \\ & + \frac{1}{2} \sum_{k=1}^{i_j} \varrho_{j,k}^2 (x_{\tau_{j,k}}), \end{aligned}$$

where $\varrho_{0_{j,i_j}} \geq 0$ and $\varrho_{j,k}(0) = 0$.

Consider the Lyapunov-Krasovskii functional

$$V_{U_{j,i_j}} = \frac{1}{2}\sum_{k=1}^{i_j}\int_{t-\tau_{j,k}}^{t} \varrho_{j,k}^2(x_{j,k}(\tau))d\tau. \quad (13)$$

The time-derivative is

$$\dot{V}_{U_{j,i_j}} = \frac{1}{2}\sum_{k=1}^{i_j}(\varrho_{j,k}^2(x_{j,k}) - \varrho_{j,k}^2(x_{\tau_{j,k}})). \quad (14)$$

It can be seen that when $\dot{V}_{z_{j,i_j}}$ and $\dot{V}_{U_{j,i_j}}$ are summed, the time delay terms are cancelled exactly, yielding

$$\begin{aligned}\dot{V}_{z_{j,i_j}} + \dot{V}_{U_{j,i_j}} \leq\ & z_{j,i_j}\Big[g_{\lambda_{j,i_j}}^{-1}(f_{j,i_j} + \varrho_{0_{j,i_j}}) + z_{j,i_j+1} \\ & +\alpha_{j,i_j} - \dot{\bar{x}}_{j,i_j-1}^T\int_0^1 \theta\frac{\partial g_{\lambda_{j,i_j}}^{-1}}{\partial \bar{x}_{j,i_j-1}}d\theta \\ & -\dot{\alpha}_{j,i_j-1}\int_0^1 g_{\lambda_{j,i_j}}^{-1}d\theta\Big] \\ & +\frac{1}{2}z_{j,i_j}^2 g_{\lambda_{j,i_j}}^{-2}) + \frac{1}{2}\sum_{k=1}^{i_j}\varrho_{j,k}^2(x_{j,k}).\end{aligned}$$

The virtual control law is

$$\alpha_{j,i_j} = -z_{j,i_j-1} - \kappa_{j,i_j}z_{j,i_j} + \hat{W}_{j,i_j}^T S(Z_{j,i_j}), \quad (15)$$

where $\hat{W}_{j,i_j}^T S(Z_{j,i_j})$ approximates $W_{j,i_j}^{*T} S(Z_{j,i_j})$ defined in the following

$$\begin{aligned}F(Z_{j,i_j}) =\ & -g_{\lambda_{j,i_j}}^{-1}(f_{j,i_j} + \varrho_{0_{j,i_j}}) \\ & -\frac{1}{2}g_{\lambda_{j,i_j}}^{-2}z_{j,i_j} + \dot{\alpha}_{j,i_j-1}\int_0^1 g_{\lambda_{j,i_j}}^{-1}d\theta \\ & +\dot{\bar{x}}_{j,i_j-1}^T\int_0^1 \theta\frac{\partial g_{\lambda_{j,i_j}}^{-1}}{\partial \bar{x}_{j,i_j-1}}d\theta - \frac{1}{2z_{j,i_j}}\sum_{k=1}^{i_j}\varrho_{j,k}^2 \\ =\ & W_{j,i_j}^{*T} S(Z_{j,i_j}) + \varepsilon_{j,i_j}. \quad (16)\end{aligned}$$

The neural network inputs are given by

$$Z_{j,i_j} = [\bar{x}_{j,i_j}, \alpha_{j,i_j-1}, \dot{\alpha}_{j,i_j-1}]^T, \quad (17)$$

where $\dot{\alpha}_{j,i_j-1}(\bar{x}_{j,i_j}, \hat{W}_{j,1}, \cdots, \hat{W}_{j,i_j}, y_{dj}, \cdots, y_{dj}^{(i_j)})$ is computable as

$$\begin{aligned}\dot{\alpha}_{j,i_j-1} =\ & \sum_{k=1}^{i_j-1}\left(\frac{\partial \alpha_{j,i_j-1}}{\partial x_{1,k}}\dot{x}_{j,k} + \frac{\partial \alpha_{j,i_j-1}}{\partial \hat{W}_{j,k}}\dot{\hat{W}}_{j,k}\right) \\ & +\sum_{k=0}^{i_j-1}\frac{\partial \alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}}y_{dj}^{(k+1)}, \quad (18)\end{aligned}$$

with $y_{dj}^{(k)}$ denoting $\frac{d^k}{dt^k}[y_{dj}]$.

Choosing the Lyapunov functional as

$$V_{j,i_j} = V_{j,i_j-1} + V_{z_{j,i_j}} + V_{U_{j,i_j}} + \frac{1}{2}\tilde{W}_{j,i_j}^T\Gamma_{j,i_j}^{-1}\tilde{W}_{j,i_j}, \quad (19)$$

with adaptation law

$$\dot{\hat{W}}_{j,i_j} = -\Gamma_{j,i_j}[S(Z_{j,i_j})z_{j,i_j} + \sigma_{j,i_j}(\hat{W}_{j,i_j} - W_{j,i_j}^0)], \quad (20)$$

and noting that $z_{j,i_j}\varepsilon_{j,i_j} \leq \frac{1}{4\lambda}z_{j,i_j}^2 + \lambda\varepsilon_{j,i_j}^2$, $\lambda > 0$, we have

$$\begin{aligned}\dot{V}_{j,i_j} \leq\ & -\sum_{k=1}^{i_j}\left(\kappa_{j,k} - \frac{1}{4\lambda}\right)z_{j,k}^2 + z_{j,i_j}z_{j,i_j+1} \\ & -\sum_{k=1}^{i_j}\frac{\sigma_{j,k}}{2}\|\tilde{W}_{j,k}\|^2 \\ & +\sum_{k=1}^{i_j}\left(\lambda\varepsilon_{j,k}^2 + \frac{\sigma_{j,k}}{2}\|W_{j,k}^* - W_{j,k}^0\|^2\right) \quad (21)\end{aligned}$$

where $\kappa_{j,i_j} > \frac{1}{4\lambda}$, and the parameters λ, $\sigma_{j,k}$ and $W_{j,k}^0$ can be designed to make the positive residual term small. The $z_{j,i_j}z_{j,i_j+1}$ terms will be cancelled in the subsequent recursive step.

Step j,m_j Consider the last equation of subsystem Σ_j, where the control input u_j appears. Let $x_{j,m_j}^c \subset X$ such that $x_{j,m_j}^c \bigcup x_{j,m_j} = X$ and $x_{j,m_j}^c \bigcap x_{j,m_j} = 0$. Define integral Lyapunov function

$$\begin{aligned}V_{z_{j,m_j}} =\ & \int_0^{z_{j,m_j}} \beta g_{\lambda_{j,m_j}}^{-1}(X)d\beta \\ =\ & \int_0^{z_{j,m_j}} \beta g_{\lambda_{j,m_j}}^{-1}(x_{j,m_j}^c, x_{j,m_j})d\beta \quad (22) \\ =\ & z_{j,m_j}^2\int_0^1 \theta g_{\lambda_{j,m_j}}^{-1}(x_{j,m_j}^c, \theta z_{j,m_j} + \alpha_{j,m_j-1})d\theta.\end{aligned}$$

The time-derivative along (1) and (6) is

$$\begin{aligned}\dot{V}_{z_{j,m_j}} \leq\ & z_{j,m_j}\Big[g_{\lambda_{j,m_j}}^{-1}(X)(f_{j,m_j}(X, \bar{u}_{j-1}) + \varrho_{0_{j,m_j}}) \\ & +u_j - \int_0^1 \theta\frac{\partial g_{\lambda_{j,m_j}}^{-1}(X)}{\partial x_{j,m_j}^c}d\theta\ \dot{x}_{j,m_j}^c \\ & -\dot{\alpha}_{j,m_j-1}\int_0^1 g_{\lambda_{j,m_j}}^{-1}(X)d\theta\Big] \\ & +\frac{1}{2}z_{j,m_j}^2 g_{\lambda_{j,m_j}}^{-2}(X) + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{m_j}\varrho_{j,k}^2(x_{\tau_{j,k}}),\end{aligned}$$

where $x_{\tau_{j,k}} := x_{j,k}(t - \tau_{j,k})$.

In view of the interconnections between the different subsystems in the last equation, and according to Lemma 3, we consider the following Lyapunov-Krasovskii functional, which has a form slightly different from that of the previous $m_j - 1$ equations.

$$V_{U_{j,m_j}} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{m_j}\int_{t-\tau_{j,k}}^{t}\varrho_{j,k}^2(x_{j,k}(\tau))d\tau, \quad (23)$$

and the time-derivative is as follows

$$\dot{V}_{U_{j,m_j}} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{m_j}\left(\varrho_{j,k}^2(x_{j,k}) - \varrho_{j,k}^2(x_{\tau_{j,k}})\right). \quad (24)$$

Summing $\dot{V}_{z_{j,m_j}}$ and $\dot{V}_{U_{j,m_j}}$ eliminates the delayed states from the analysis, yielding

$$\begin{aligned}\dot{V}_{z_{j,m_j}} + \dot{V}_{U_{j,m_j}} \leq\ & z_{1,m_1}\Big[g^{-1}_{\lambda_{j,m_j}}(f_{1,m_1} + \varrho_{0_{j,m_j}}) \\ & + u_j - \int_0^1 \theta \frac{\partial g^{-1}_{\lambda_{j,m_j}}}{\partial x^c_{j,m_j}} d\theta\ \dot{x}^c_{j,m_j} \\ & - \dot{\alpha}_{j,m_j-1}\int_0^1 g^{-1}_{\lambda_{j,m_j}} d\theta\Big] \\ & + \frac{1}{2} z^2_{j,m_j} g^{-2}_{\lambda_{j,m_j}} + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{m_j} \varrho^2_{j,k}(x_{j,k}).\end{aligned}$$

The practical control law for subsystem Σ_j is

$$u_j = -z_{j,m_j-1} - \kappa_{j,m_j} z_{j,m_j} + \hat{W}^T_{j,m_j} S(Z_{j,m_j}), \quad (25)$$

where κ_{j,m_j} will be defined later and $\hat{W}^T_{j,m_j} S(Z_{j,m_j})$ approximates $W^{*T}_{j,m_j} S(Z_{j,m_j})$, defined by

$$\begin{aligned} F(Z_{j,m_j}) =\ & -g^{-1}_{\lambda_{j,m_j}}(f_{j,m_j} + \varrho_{0_{j,m_j}}) - \frac{1}{2} g^{-2}_{\lambda_{j,m_j}} z_{j,m_j} \\ & + \dot{\alpha}_{j,m_j-1}\int_0^1 g^{-1}_{\lambda_{j,m_j}} d\theta + \int_0^1 \theta \frac{\partial g^{-1}_{\lambda_{j,m_j}}}{\partial x^c_{j,m_j}} d\theta\ \dot{x}^c_{j,m_j} \\ & - \frac{1}{2z_{j,m_j}}\sum_{j=1}^{n}\sum_{k=1}^{m_j} \varrho^2_{j,k}(x_{j,k}) \\ =\ & W^{*T}_{j,m_j} S(Z_{j,m_j}) + \varepsilon_{j,m_j}. \end{aligned} \quad (26)$$

Remark 4: Note that f_{j,m_j} is a function of $\bar{u}_{j-1}$ as defined in (1). However, this does not pose a problem since the recursive approach has already derived expressions for u_k, $1 \leq k \leq j-1$. This is feasible due to the fact that the last equations of all subsystems form is in strict feedback with respect to the multiple inputs.

The neural network inputs Z_{j,m_j} are given by (11), wherein $\dot{\alpha}_{j,m-1}$, which is a function of $\bar{x}_{j,m_j}, \hat{W}_{j,1}, \cdots, \hat{W}_{j,m_j}, y_{dj}$, $\cdots, y^{(m_j)}_{dj}$, can be computed as

$$\begin{aligned}\dot{\alpha}_{j,m_j-1} =\ & \sum_{k=1}^{m_j-1}\left(\frac{\partial \alpha_{j,m_j-1}}{\partial x_{j,k}}\dot{x}_{j,k} + \frac{\partial \alpha_{j,m_j-1}}{\partial \hat{W}_{j,k}}\dot{\hat{W}}_{j,k}\right) \\ & + \sum_{k=0}^{m_j-1}\frac{\partial \alpha_{j,m_j-1}}{\partial y^{(k)}_{dj}} y^{(k+1)}_{dj}.\end{aligned} \quad (27)$$

We consider the Lyapunov functional as

$$\begin{aligned}V_{j,m_j} =\ & V_{j-1,m_{(j-1)}} + V_{j,m_j-1} + V_{z_{j,m_j}} + V_{U_{j,m_j}} \\ & + \frac{1}{2}\tilde{W}^T_{j,m_j}\Gamma^{-1}_{j,m_j}\tilde{W}_{j,m_j}.\end{aligned} \quad (28)$$

The adaptation law is chosen as

$$\dot{\hat{W}}_{j,m_j} = -\Gamma_{j,m_j}[S(Z_{j,m_j})z_{j,m_j} + \sigma_{j,m_j}(\hat{W}_{j,m_j} - W^0_{j,m_j})]. \quad (29)$$

Taking the derivative of V_{j,m_j} along the trajectories of (1), (6), (25), (29), and noting that $z_{j,m_j}\varepsilon_{j,m_j} \leq \frac{1}{4\lambda} z^2_{j,m_j} + \lambda\varepsilon^2_{j,m_j}$, $\lambda > 0$, we obtain

$$\begin{aligned}\dot{V}_{j,m_j} \leq\ & \dot{V}_{j-1,m_{(j)}} + \dot{V}_{j,m_j-1} - \kappa_{j,m_j} z^2_{j,m_j} \\ & - z_{j,m_j-1} z_{j,m_j} + \frac{1}{4\lambda} z^2_{j,m_j} + \lambda\varepsilon^2_{j,m_j} \\ & + \frac{\sigma_{j,k}}{2}\|\tilde{W}_{j,k}\|^2 + \frac{\sigma_{j,k}}{2}\|W^*_{j,k} - W^0_{j,k}\|^2 \\ \leq\ & -\sum_{i=1}^{j}\sum_{k=1}^{m_i}\left[\left(\kappa_{i,k} - \frac{1}{4\lambda}\right) z^2_{i,k} + \frac{\sigma_{i,k}}{2}\|\tilde{W}_{i,k}\|^2\right] \\ & + \sum_{i=1}^{j}\sum_{k=1}^{m_i}\left(\lambda\varepsilon^2_{i,k} + \frac{\sigma_{i,k}}{2}\|W^*_{i,k} - W^0_{i,k}\|^2\right),\end{aligned} \quad (30)$$

where $\kappa_{i,k} > \frac{1}{4\lambda}$. Now we are in position to derive the last step.

<u>*Step (n, m_n)*</u> This is the final step, where the nth input will be designed to ensure the stability of the entire plant. Let $z_{n,m_n} = x_{n,m_n} - \alpha_{n,m_n-1}$. Consider the following Lyapunov functional

$$\begin{aligned}V_{n,m_n} =\ & V_{n-1,m_{(n-1)}} + V_{n,m_n-1} + V_{z_{n,m_n}} + V_{U_{n,m_n}} \\ & + \frac{1}{2}\tilde{W}^T_{n,m_n}\Gamma^{-1}_{n,m_n}\tilde{W}_{n,m_n},\end{aligned} \quad (31)$$

where

$$\begin{aligned}V_{z_{n,m_n}} =\ & z^2_{n,m_n}\int_0^1 \theta g^{-1}_{\lambda_{n,m_n}}(X) d\theta, \\ V_{U_{n,m_n}} =\ & \sum_{j=1}^{n}\sum_{k=1}^{m_n}\int_{t-\tau_{j,k}}^{t} \varrho^2_{j,k}(x_{j,k}(\tau)) d\tau,\end{aligned} \quad (32)$$

with practical control law

$$u_n = -z_{n,m_n-1} - \kappa_{n,m_n} z_{n,m_n} + \hat{W}^T_{n,m_n} S(Z_{n,m_n}), \quad (33)$$

where $\hat{W}_{n,m_n}$ is a neural network approximating

$$\begin{aligned}F(Z_{n,m_n}) =\ & -g^{-1}_{\lambda_{n,m_n}}(f_{n,m_n} + \varrho_{0_{n,m_n}}) - \frac{1}{2} g^{-2}_{\lambda_{n,m_n}} z_{n,m_n} \\ & + \dot{\alpha}_{n,m_n-1}\int_0^1 g^{-1}_{\lambda_{n,m_n}} d\theta - \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{m_j}\varrho^2_{j,k}(x_{j,k}) \\ & + \int_0^1 \theta \frac{\partial g^{-1}_{\lambda_{n,m_n}}}{\partial x^c_{n,m_n}} d\theta\ \dot{x}^c_{n,m_n} \\ =\ & W^{*T}_{n,m_n} S(Z_{n,m_n}) + \varepsilon_{n,m_n}.\end{aligned} \quad (34)$$

The adaptation law is given by

$$\begin{aligned}\dot{\hat{W}}_{n,m_n} =\ & -\Gamma_{n,m_n}[S(Z_{n,m_n}) z_{n,m_n} \\ & + \sigma_{n,m_n}(W^*_{n,m_n} - W^0_{n,m_n})],\end{aligned} \quad (35)$$

with neural network inputs

$$Z_{n,m_n} = [X, \dot{x}_{1,m_1}, \cdots, \dot{x}_{n-1,m_{(n-1)}}, \alpha_{n,m_n-1}, \dot{\alpha}_{n,m_n-1}]^T$$

where $\dot{\alpha}_{n,m_n-1}(\bar{x}_{n,m_n},\hat{W}_{n,1},\cdots,\hat{W}_{n,m_n-1},y_{dj},\cdots,y_{dj}^{(m_n)})$ can be computed as

$$\dot{\alpha}_{n,m_n-1} = \sum_{k=1}^{m_n-1}\left(\frac{\partial\alpha_{n,m_n-1}}{\partial x_{n,k}}\dot{x}_{n,k}+\frac{\partial\alpha_{n,m_n-1}}{\partial\hat{W}_{n,k}}\dot{\hat{W}}_{n,k}\right) + \sum_{k=0}^{m_n-1}\frac{\partial\alpha_{n,m_n-1}}{\partial y_{dn}^{(k)}}y_{dn}^{(k+1)}. \quad (36)$$

It can be shown that the time derivative of V_{n,m_n} satisfies the following inequality

$$\dot{V}_{n,m_n} \le -\sum_{j=1}^{n}\sum_{k=1}^{m_j}\left[\left(\kappa_{j,k}-\frac{1}{4\lambda}\right)z_{j,k}^2+\frac{\sigma_{j,k}}{2}\|\tilde{W}_{j,k}\|^2\right]+C,$$

where

$$C=\sum_{j=1}^{n}\sum_{k=1}^{m_j}\left(\lambda\varepsilon_{j,k}^2+\frac{\sigma_{j,k}}{2}\|W_{j,k}^*-W_{j,k}^0\|^2\right) \quad (37)$$

is a positive constant, and the parameters λ, σ_{n,m_n}, and W_{n,m_n}^0 can be designed to make the convergent set arbitrarily small, and $\kappa_{n,m_n} > \frac{1}{4\lambda}$.

The first term of the RHS is negative definite while the second term is a positive constant. From Lemma 1, it can be seen that the signals $z_{1,1},\cdots,z_{j,m_j}$, and $\tilde{W}_{1,1},\cdots,\tilde{W}_{j,m_j}$ are SGUUB. This concludes the proof. ∎

IV. Conclusion

This paper proposed an adaptive neural network controller for a class of block-triangular MIMO nonlinear systems with interconnected states carrying multiple constant delays embedded in a general structure. Through the use of Lyapunov-Krasovskii functionals, the control design, stability analysis and performance analysis of nonlinear MIMO time-delay systems are performed. With the use of a separation technique, more general forms of delay functionals (including complex interconnections of state-delays found in MIMO systems) can be handled, such that no assumptions regarding the bounds of the delay functionals are required. Controller singularity is avoided with the use of integral Lyapunov functions. By employing NN, unknown functions in the system dynamics, as well as the multiple unknown time delays, in states that are interconnected between the subsystems, can be estimated and compensated for. The adaptive NN controller guarantees that the tracking error remains bounded within a neighbourhood of the origin, and can be made arbitrarily small through appropriate choices of parameters. At the same time, all other signals in the closed loop are semi-globally uniformly ultimately bounded.

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