Adaptive Neural Network Control for Helicopters in Vertical Flight

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***Abstract*—In this brief, robust adaptive neural network (NN) control is presented for helicopters in vertical flight, with dynamics in single-input–single-output (SISO) nonlinear nonaffine form. Based on the use of the implicit function theorem and the mean value theorem, we propose a constructive approach for adaptive NN control design with guaranteed stability. Considering both full-state and output feedback cases, it is shown that the output tracking error converges to a small neighborhood of the origin, while the remaining closed-loop signals remain bounded. The simulation study demonstrates the effectiveness of the proposed control.**

***Index Terms*—Adaptive control, helicopters, neural networks (NNs), output feedback, uncertain systems.**

I. Introduction

ENSURING stability in helicopter flight is a challenging problem for nonlinear control design and development. Unlike many classes of mechanical systems, which naturally possess desirable structural properties such as passivity or dissipativity, helicopters are inherently unstable without closed-loop control, especially during hover. In addition, the dynamics are highly nonlinear and strongly coupled, such that disturbances along a single degree of freedom can easily propagate to the other degrees of freedom and lead to loss of performance or even destabilization.

Based on various dynamic models of helicopter systems, a myriad of nonlinear control techniques has been applied for stable control of helicopters [1]–[5]. These include feedback linearization [1] and dynamic sliding mode control [2] for basic altitude tracking tasks, as well as internal-model-based control for the challenging problem of landing on an oscillating ship deck [3]. Another interesting technique involves the use of approximate input–output linearization to obtain a dynamic representation possessing the desirable property of differential flatness [4]. Besides these, it was shown that stable dynamic inversion can be performed by modifying the internal dynamics to remove nonhyperbolicity [5].

In the control of helicopters, an important concern is how to deal with unknown perturbations to the nominal model, in the form of parametric and functional uncertainties, unmodeled dynamics, and disturbances from the environment. Helicopter control applications are characterized by aerodynamical disturbances, which are generally difficult to model accurately. Model-based control, such as the aforementioned schemes, tend to be susceptible to uncertainties and disturbances that cause performance degradation. To deal with the presence of model uncertainties, approximation-based techniques using neural networks (NNs) have been proposed. In particular, approximate dynamic inversion with augmented NNs was employed to handle unmodeled dynamics in [6] and [7], while neural dynamic programming was shown to be effective for tracking and trimming control of helicopters in [8].

In this brief, we propose adaptive NN control for helicopters in vertical flight, which can be represented by single-input–single-output (SISO) models to yield useful results, because the coupling between longitudinal and lateral-directional equations in this flight regime is weak [9]. While the proposed controller handles vertical flight, other flight regimes can be handled by other control modules. Motivated by results in NN control of nonlinear systems [10], we utilize Lyapunov-based techniques to design robust adaptive NN control for helicopters with guaranteed stability. Although a nonaffine system can be rendered affine by adding an integrator to the control input, thus allowing many control methods for affine nonlinear system to be used, the disadvantage of this approach is that the dimension of the system is increased, and control efforts are not direct and immediate either [10]. Subsequently, effective control for the system may not be achieved. In this brief, we focus on control design for the nonaffine system directly, without adding any integrators to the input.

Different from the approaches in [6] and [7], which were based on approximate dynamic inversion with augmented NNs, we utilize mean value theorem and implicit function theorem as mathematical tools to handle the nonaffine nonlinearities in the helicopter dynamics, based on the pioneering work of [11]. While the NNs in [6] and [7] compensate for inaccuracy of the inversion model, those in our proposed scheme approximate the ideal feedback control law directly. In cases where reasonably accurate knowledge of the dynamic inversion model is available, the method of [6] and [7] has been shown to provide an effective solution to the problem. However, the construction of the dynamic inversion for a nonaffine system may not be an easy task in general. For such cases, our approach offers a feasible means of tackling the problem, since *a priori* knowledge of the inversion is not required.

II. Problem Formulation and Preliminaries

A. Helicopter Dynamics

The dynamics of helicopters in vertical flight can be represented by the SISO nonlinear nonaffine form

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \tag{1}$$

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where $x \in R^n$ are the states of the system, $u, y \in R$ denote the input and output, respectively, $h : R^n \rightarrow R$ is a partially unknown function, and $f : R^n \times R \rightarrow R^n$ is a partially unknown vector field satisfying certain properties, which will be described shortly.

The control objective is output tracking of a desired reference trajectory $y_d(t)$ such that the tracking error converges to a neighborhood of zero, i.e., $|y(t) - y_d(t)| \leq \delta$, where $\delta > 0$. At the same time, all closed-loop signals are to be kept bounded.

The reference trajectory $y_d(t)$ is generated by the following reference model:

$$\begin{aligned} \dot{\xi}_{di} &= \xi_{di+1}, \qquad 1 \leq i \leq \rho - 1 \\ \dot{\xi}_{d\rho} &= f_d(\xi_d, t) \\ y_d &= \xi_{d1} \end{aligned} \tag{2}$$

where $\rho \geq 2$ is a constant index, $\xi_d = [\xi_{d1}, \xi_{d2}, \ldots, \xi_{d\rho}]^T \in R^\rho$ are the states of the reference system, $y_d \in R$ is the system output, and $f_d : R^\rho \times R_+ \rightarrow R$ is a known function.

Define $\phi_j(x) = L_f^{j-1} h$ for $j = 1, \ldots, \rho$, where $L_f h = (\partial h)/(\partial x) f(x, u)$ denotes the Lie derivative of the function $h(x)$ with respect to the vector field $f(x, u)$, and higher order Lie derivatives are defined recursively as $L_f^k h = L_f(L_f^{k-1} h)$ for $k > 1$.

Definition 1: System (1) is said to have strong relative degree if, for every point (x_0, u_0), there exists a positive integer $1 \leq \rho \leq n$ such that

$$\begin{aligned} &\frac{\partial}{\partial u} L_f^i h = 0, \qquad 1 \leq i < \rho - 1 \\ &\frac{\partial}{\partial u} L_f^\rho h \neq 0 \end{aligned} \tag{3}$$

for all $(x, u) \in B(x_0, u_0)$, a ball centered at (x_0, u_0).

Assumption 1: System (1) is input–output linearizable with strong relative degree $\rho < n$ for all $(x, u) \in R^n \times R$.

Based on Assumption 1, it was shown in [12] that there exist other $n - \rho$ functions $\phi_{\rho+1}, \ldots, \phi_n$ independent of u, such that the mapping $\Phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_n(x)]^T$ has a Jacobian matrix which is nonsingular for all $x \in R^n$. Therefore, $\Phi(x)$ is a diffeomorphism on R^n. By setting $\xi = [\phi_1(x), \phi_2(x), \ldots, \phi_\rho(x)]^T$ and $\eta = [\phi_{\rho+1}(x), \phi_{\rho+2}(x), \ldots, \phi_n(x)]^T$, system (1) can be expressed in the normal form

$$\begin{aligned} \dot{\eta} &= q(\eta, \xi) \\ \dot{\xi}_j &= \xi_{j+1}, \qquad j = 1, \ldots, \rho - 1 \\ \dot{\xi}_\rho &= b(\xi, \eta, u) \\ y &= \xi_1 \end{aligned} \tag{4}$$

where $b(\xi, \eta, u) = L_f^\rho h$, $q(\xi, \eta) = [L_f \phi_{\rho+1}(x), \ldots, L_f \phi_n(x)]^T$, and $x = \Phi^{-1}(\xi, \eta)$, for $(\xi, \eta) \in R^n$ and $u \in R$.

Assumption 2: The reference trajectory $y_d(t)$ and its ρ derivatives remain bounded, i.e., $\xi_d \in \Omega_d \subset R^\rho$, $\forall t \geq 0$, where ρ is the relative degree of (1).

Assumption 3: The zero dynamics of system (4), given by $\dot{\eta} = q(0, \eta)$, are exponentially stable. In addition, the function $q(\xi, \eta)$ is Lipschitz in ξ, i.e., there exist positive constants a_q and a_ξ such that

$$\|q(\xi, \eta) - q(0, \eta)\| \leq a_\xi \|\xi\| + a_q \qquad \forall (\xi, \eta) \in R^n. \tag{5}$$

Under the assumption that the zero dynamics are stable, by the converse Lyapunov theorem, there exists a Lyapunov function $V_0(\eta)$ which satisfies the following inequalities for $(\xi, \eta) \in R^n$:

$$\gamma_1 \|\eta\|^2 \leq V_0(\eta) \leq \gamma_2 \|\eta\|^2 \tag{6}$$

$$\frac{\partial V_0}{\partial \eta} q(0, \eta) \leq -\lambda_a \|\eta\|^2 \tag{7}$$

$$\left\| \frac{\partial V_0}{\partial \eta} \right\| \leq \lambda_b \|\eta\| \tag{8}$$

where γ_1, γ_2, λ_a, and λ_b are positive constants.

For ease of notation, define $g(x, u) := (\partial b(\xi, \eta, u))/(\partial u)$. The following two assumptions specify some conditions on the partially unknown function $g(x, u)$.

Assumption 4: There exist smooth function $\bar{g}(\xi, \eta)$ and a constant $\underline{g}$, such that $\bar{g}(\xi, \eta) \geq |g(\xi, \eta, u)| \geq \underline{g} > 0$. Without loss of generality, it is further assumed that $g(\xi, \eta, u) > 0$.

Assumption 5: For any dynamic feedback control

$$\dot{\omega} = \phi(\xi, \eta, \omega) \quad u = \psi(\xi, \eta, \omega) \tag{9}$$

such that $\psi(\cdot) \in C^1$ and $\omega \in L_\infty$, there exists a positive function $g_0(\xi, \eta)$ such that $|(\dot{g})/(2g)| \leq g_0$.

Remark 1: Assumption 5 is reasonable under appropriate design considerations. The derivative of $g(\xi, \eta, u)$ is given by

$$\dot{g} = \sum_{i=1}^{\rho-1} \frac{\partial g}{\partial \xi_i} \xi_{i+1} + \frac{\partial g}{\partial \xi_\rho} b(\xi, \eta, u) + \frac{\partial g}{\partial \eta} q(\xi, \eta) + \frac{\partial g}{\partial u} \dot{u} \tag{10}$$

which depends on $\dot{u}$. From (9), it can be shown that

$$\begin{aligned} \dot{u} = \frac{\partial \psi}{\partial \omega} \phi(\omega, \xi, \eta) &+ \sum_{i=1}^{\rho-1} \frac{\partial \psi}{\partial \xi_i} \xi_{i+1} \\ &+ \frac{\partial \psi}{\partial \xi_\rho} b(\xi, \eta, u) + \frac{\partial \psi}{\partial \eta} q(\xi, \eta). \end{aligned} \tag{11}$$

Substituting (11) into (10), it is clear that $\dot{g}$ can be expressed in terms of variables ξ, η, and ω, and similarly for g. Given that $(\dot{g})/(2g)$ is continuous, it can be shown that $|(\dot{g})/(2g)| \leq g_f(\xi, \eta) + g_c(\omega)$, where $g_f(\cdot)$ and $g_c(\cdot)$ are positive functions [13]. Because $\omega \in L_\infty$, we know that there exists positive function $g_0(\xi, \eta)$ such that $|(\dot{g})/(2g)| \leq g_0$. In our control design, the state ω comprises $y_\rho, \dot{y}_\rho, \ldots, y_\rho^{(\rho)}$, which are bounded from Assumption 2, as well as NN weights, which will be ensured to be bounded by projection algorithm designed in Section III.

Last, we present the following definition and Lemma, which are important for stability and performance analysis.

Definition 2: For the system $\dot{X} = F(X, t)$, where $F : R^n \times R_+ \rightarrow R^n$ is a smooth vector field, the solution $X(t)$ is semiglobally uniformly bounded (SGUB) if, for any compact set Ω_0, there exists constant $S > 0$ such that $\|X(t)\| \leq S$ for all $X(0) \in \Omega_0$ and $t \geq 0$.

Lemma 1 [14]: Suppose that there exists a C^1 continuous and positive–definite Lyapunov function $V(x)$ satisfying $\gamma_1(\|x\|) \leq V(x) \leq \gamma_2(\|x\|)$, such that its derivative $\dot{V}(x) := dV(x)/dt$ satisfies $\dot{V}(x) \leq -c_1 V(x) + c_2$, where γ_1 and γ_2 are class K_∞ functions and c_1 and c_2 are positive constants, then the solution $x(t)$ is SGUB.

B. Neural Networks

In this paper, we employ multilayer neural networks (MNN) to approximate unknown functions. Universal approximation results indicate that, given a desired level of accuracy ε, approximation to that level of accuracy can be guaranteed by making l sufficiently large [15]. Thus, the NN can approximate any continuous function $p_{nn}(Z)$ as

$$p_{nn}(Z) = W^T S(V^T Z) + \varepsilon(Z) \tag{12}$$

where the vector $Z = [\zeta_1, \zeta_2, \ldots, \zeta_m, 1]^T \in \Omega_Z \subset R^{m+1}$ are the input variables to the NNs, $W \in R^l$ and $V \in R^{(m+1)\times l}$ are adaptable weights, and $S(V^T Z) = [s_1(v_1^T Z), \ldots, s_l(v_l^T Z)]^T \in R^l$ is basis function vector, with $s_i := s(v_i^T Z)$, $s(z_a) = 1/(1 + e^{-\mu z_a})$, v_i being the column vectors of V, l being the number of NN nodes, μ being a positive number, $i = 1, 2, \ldots, l$, and $\varepsilon(Z)$ being the approximation error which is bounded over the compact set, i.e., $|\varepsilon(Z)| \leq \bar{\varepsilon}$, $\forall Z \in \Omega_Z$, where $\bar{\varepsilon} > 0$ is an unknown constant. For analytical purposes, the ideal weights W^* and V^* are defined as the values of $W \in \Omega_W$ and $V \in R^{(m+1)\times l}$ that minimize $|\varepsilon(Z)|$ for all $Z \in \Omega_Z$, i.e., there exists a constant $M_W > 0$ such that

$$(W^*, V^*) := \text{argmin}_{W \in \Omega_W, V \in R^{(m+1)\times l}} \left\{ \sup_{Z \in \Omega_Z} |p_{nn}(Z) - W^T S(V^T Z)| \right\}.$$

where $\Omega_W := \{W \in R^l : \|W\| \leq M_W\}$.

Lemma 2 [10]: The approximation error can be expressed as

$$\hat{W}^T S(V^T Z) - W^{*T} S(V^{*T} Z) = \tilde{W}^T(\hat{S} - \hat{S}'\hat{V}^T Z) + \hat{W}^T \hat{S}' \tilde{V}^T Z + d_u$$

where $\hat{S} := S(\hat{V}^T Z)$, $\tilde{W} := \hat{W} - W^*$ and $\tilde{V} := \hat{V} - V^*$ are the weights estimation errors, and $\hat{S}' := \text{diag}\{\hat{s}'_1, \ldots, \hat{s}'_l\}$ with $\hat{s}'_i = s'(\hat{v}_i^T Z) = (ds(z_a)/dz_a)|_{z_a = \hat{v}_i^T Z}$, $i = 1, 2, \ldots, l$. The residual term is bounded by

$$d_u \leq \|W^*\| \|\hat{S}'\hat{V}^T Z\| + \|V^*\|_F \|Z\hat{W}^T \hat{S}'\|_F + \|W^*\|$$

where $\|P\|_F$ denotes the Frobenius norm of matrix P.

III. Adaptive NN Control Design

We employ backstepping for the ξ subsystem, and then make use of the exponential stability of the zero dynamics to show that the overall closed-loop system is stable and that output tracking is achieved. The control design is performed first for the full-state case and subsequently for the output feedback case with high gain observers.

A. Full-State Feedback Control

Step 1: Let $z_1(t) = \xi_1(t) - y_d(t)$ and $z_2(t) = \xi_2(t) - \alpha_1(t)$, where $\alpha_1(t)$ is a virtual control function to be determined. Define quadratic function $V_1 = (1/2)z_1^2$. Choosing the virtual control as $\alpha_1 = -k_1 z_1 + \dot{y}_d$, where k_1 is a positive constant, we can show that $\dot{V}_1 = -k_1 z_1^2 + z_1 z_2$, where the term $z_1 z_2$ will be canceled in the subsequent step.

Step i(i=2,…,ρ−1): Let $z_i(t) = \xi_i(t) - \alpha_{i-1}(t)$, where $\bar{\xi}_i := [\xi_1, \ldots, \xi_i]^T$ and $\alpha_i(t)$ is a virtual control function to be determined. Define quadratic function $V_i = V_{i-1} + (1/2)z_i^2$. Choosing the virtual control as $\alpha_i = -k_i z_i - z_{i-1} + \dot{\alpha}_{i-1}$, it can be shown that $\dot{V}_i = -\sum_{j=1}^{i} k_j z_j^2 + z_i z_{i+1}$, where the term $z_i z_{i+1}$ will be canceled in the subsequent step.

Step ρ: This is the final step where the actual control law u will be designed. From Assumption 4, we know that $g(\xi, \eta, u) \geq \underline{g} > 0$ for all $(\xi, \eta) \in R^n$ and $u \in R$. Define $z_\rho(t) = \xi_\rho(t) - \alpha_{\rho-1}(t)$ and $\nu = -\dot{\alpha}_{\rho-1} + g_0(\xi, \eta) z_\rho$. It is clear that ν is a function of ξ, η, y_d, and $y_d^{(1)}, \ldots, y_d^{(\rho)}$. Considering the fact that $(\partial \nu / \partial u) = 0$, we know that $(\partial [b(\xi, \eta, u) + \nu] / \partial u) \geq \underline{g} > 0$.

According to the implicit function theorem [16], for every value of ξ, η, and ν, there exists a smooth ideal control input $u \in R$ such that $b(\xi, \eta, u^*) + \nu = 0$. Using the mean value theorem [17], there exists $(0 < \lambda < 1)$ such that

$$b(\xi, \eta, u) = b(\xi, \eta, u^*) + g_\lambda (u - u^*) \tag{13}$$

where $g_\lambda := g(\xi, \eta, u_\lambda)$ and $u_\lambda := \lambda u + (1 - \lambda) u^*$. Then, the derivative of z_ρ can be written as

$$\dot{z}_\rho = -g_0(\xi, \eta) z_\rho + g_\lambda (u - u^*). \tag{14}$$

We employ a robust MNN controller of the form

$$u = \hat{W}^T S(\hat{V}^T Z) - k_\rho z_\rho - z_{\rho-1} - k_b \times \left(\|Z\hat{W}^T \hat{S}'\|_F^2 + \|\hat{S}'\hat{V}^T Z\|^2 \right) z_\rho. \tag{15}$$

The component $\hat{W}^T S(\hat{V}^T Z)$ is an MNN that approximates $u^*(\xi, \eta)$, which can be expressed as $u^* = W^{*T} S(V^{*T} Z) + \varepsilon$, where $Z = [\xi, \eta, z_\rho, \dot{\alpha}_{\rho-1}, 1]^T \in \Omega_Z \subset R^{n+3}$, W^* denotes the vector of ideal constant weights, and $|\varepsilon| \leq \bar{\varepsilon}$ is the approximation error with constant $\bar{\varepsilon} > 0$.

The projection-based adaptation laws are designed as follows:

$$\dot{\hat{W}} = -\Gamma_W \left[\left(I - \chi_W \frac{\hat{W}\hat{W}^T}{\|\hat{W}\|^2} \right) (\hat{S} - \hat{S}'\hat{V}^T Z) z_\rho + \sigma_W \hat{W} \right] \tag{16}$$

$$\dot{\hat{V}} = -\Gamma_V \left[Z\hat{W}^T \hat{S}' z_\rho + \sigma_V \hat{V} \right] \tag{17}$$

where $\Gamma_W = \Gamma_W^T > 0$, $\Gamma_V = \Gamma_V^T > 0$, $\sigma_W > 0$, $\sigma_V > 0$, and

$$\chi_W = \begin{cases} 0, & \text{if } \|\hat{W}\| < M_W, \text{ or if } \begin{cases} \|\hat{W}\| = M_W \text{ and} \\ \hat{W}^T(\hat{S} - \hat{S}'\hat{V}^T Z) z_\rho \geq 0 \end{cases} \\ I, & \text{otherwise.} \end{cases} \tag{18}$$

Remark 2: The previous projection algorithm is used to ensure $\hat{W} \in \Omega_W$, which is important for the practicality of Assumption 5. Note that no projection is needed for $\hat{V}$ because u in (15) does not grow with $\hat{V}$, since $\|S(\hat{V}^T Z)\| \leq \sqrt{l}$, $\|\hat{S}'\|_F \leq 0.25\sqrt{l}$, and $\|\hat{S}'\hat{V}^T Z\| \leq 0.2239\sqrt{l}$, $\forall \hat{V} \in R^{(m+1)\times l}$ [10].

Lemma 3 [18]: From (16) and (18), whenever $\|\hat{W}\| = M_W$, we have that $\tilde{W}^T\hat{W} \geq M_W(M_W - \|W^*\|) \geq 0$. If, additionally, $\hat{W}^T(\hat{S} - \hat{S}'\hat{V}^T Z)z_\rho < 0$, then $\mathrm{sgn}((\tilde{W}^T\hat{W}\hat{W}^T(\hat{S} - \hat{S}'\hat{V}^T Z)z_\rho)/(\|\hat{W}\|^2)) = \mathrm{sgn}(\tilde{W}^T\hat{W})$.

Consider the Lyapunov function candidate

$$V_\rho = V_{\rho-1} + \frac{1}{2g_\lambda}z_\rho^2 + \frac{1}{2}\tilde{W}^T\Gamma_W^{-1}\tilde{W} + \frac{1}{2}\mathrm{tr}\left\{\tilde{V}^T\Gamma_V^{-1}\tilde{V}\right\} \quad (19)$$

where $\tilde{W} := \hat{W} - W^*$, $\tilde{V} := \hat{V} - V^*$, and $\Gamma_W = \Gamma_W^T > 0$ and $\Gamma_V = \Gamma_V^T > 0$ are the design parameters. Using Lemma 3, it can be shown that the derivative of V_ρ, along (15)–(17), satisfies the following:

$$\begin{aligned}\dot{V}_\rho \leq &-\sum_{j=1}^{\rho-1} k_j z_j^2 - \left(g_\lambda(k_\rho - 1) + g_0 + \frac{\dot{g}_\lambda}{2g_\lambda}\right)\frac{z_\rho^2}{g_\lambda} \\ &- \left(k_b - \frac{1}{2}\right)\left(\|\hat{S}'\hat{V}^T Z\|^2 + \|Z\hat{W}^T\hat{S}'\|_F^2\right)z_\rho^2 \\ &- \frac{\sigma_W}{2}\|\tilde{W}\|^2 - \frac{\sigma_V}{2}\|\tilde{V}\|_F^2 + \frac{1}{2}\bar{\varepsilon}^2 \\ &+ \frac{\sigma_W + 2}{2}\|W^*\|^2 + \frac{\sigma_V + 1}{2}\|V^*\|_F^2. \end{aligned} \quad (20)$$

From Assumption 5, we know that $(g_0 + (\dot{g}_\lambda)/(2g_\lambda)) \geq 0$. Hence, by choosing the control parameters $k_\rho > 1$ and $k_b > 0.5$, the second and third right-hand side (RHS) terms of (20) are strictly negative, thus leading to the following simplification:

$$\dot{V}_\rho \leq -c_1 V_\rho + c_2 \quad (21)$$

$$c_1 = \min\left\{2k_1, 2k_2, \ldots, 2k_{\rho-1}, 2\underline{g}(k_\rho - 1), \times \frac{\sigma_W}{\lambda_{\max}\left(\Gamma_W^{-1}\right)}, \frac{\sigma_V}{\lambda_{\max}\left(\Gamma_V^{-1}\right)}\right\} \quad (22)$$

$$c_2 = \frac{1}{2}\bar{\varepsilon}^2 + \frac{\sigma_W + 2}{2}\|W^*\|^2 + \frac{\sigma_V + 1}{2}\|V^*\|_F^2. \quad (23)$$

The following lemma is useful for stability analysis of the internal dynamics.

Lemma 4 [19]: Denote positive constants $a_1 = (\lambda_b a_\xi)/(\lambda_a)$ and $a_2 = (\lambda_b a_q)/(\lambda_a)$. If Assumptions 2–3 are satisfied, there exists positive constant T_0 such that the trajectories $\eta(t)$ of the internal dynamics satisfy

$$\|\eta(t)\| \leq a_1\|\xi(t)\| + a_2 \qquad \forall t > T_0. \quad (24)$$

We summarize our results for the full-state feedback case in the following theorem.

Theorem 1: Consider the SISO helicopter dynamics (1) satisfying Assumptions 1–5, with control law (15) and adaptation laws (16)–(18). For initial conditions $\xi(0)$, $\eta(0)$, $\hat{W}(0)$, and $\hat{V}(0)$ belonging to any compact set Ω_0, all closed-loop signals are SGUB, and the tracking error $z_1 = y - y_d$ converges to the compact set

$$\Omega_{z_1} := \left\{ z_1 \in R \,\middle|\, \|z_1\| \leq \sqrt{\frac{2c_2}{c_1}} \right\} \quad (25)$$

where c_1 and c_2 are constants defined in (22) and (23), respectively.

Proof: Denote $D = 2(V_\rho(0) + (c_2)/(c_1))$. According to [14, Lemma 1.1–1.2], we know from (21) that z, $\tilde{W}$, and $\tilde{V}$ are bounded within the compact sets for all $t > 0$

$$\Omega_z = \{z \in R^\rho \,|\, \|z\| \leq \sqrt{D}\} \quad (26)$$

$$\Omega_{\tilde{W}} = \left\{ \tilde{W} \in R^l \,\middle|\, \|\tilde{W}\| \leq \sqrt{D/\lambda_{\min}\left(\Gamma_W^{-1}\right)} \right\} \quad (27)$$

$$\Omega_{\tilde{V}} = \left\{ \tilde{V} \in R^{(m+1)\times l} \,\middle|\, \|\tilde{V}\|_F \leq \sqrt{D/\lambda_{\min}\left(\Gamma_V^{-1}\right)} \right\}. \quad (28)$$

From projection algorithm (16) and (18), we have $\hat{W} \in \Omega_W$. Because V^* is bounded and $\tilde{V} \in \Omega_{\tilde{V}}$, it is clear that $\hat{V}$ is also bounded. From the fact that $z, y_d, y_d^{(1)}, \ldots, y_d^{(\rho)}$ are bounded, we know that the virtual controls α_i, $i = 1, 2, \ldots, \rho$ are bounded. Hence, there exists a constant $a_3 > 0$ such that $\|\xi_d\| \leq a_3$, where $\xi_d := [y_d, \alpha_1, \alpha_2, \ldots, \alpha_{\rho-1}]^T$.

From Lemma 4, it can be seen that η is bounded if ξ is bounded. As a result, the states of the internal dynamics will converge to the compact set $\Omega_\eta := \{\eta \in R^{n-\rho} \,|\, \|\eta\| \leq a_1(\sqrt{(2c_2)/(c_1)} + a_3) + a_2\}$, where $a_1 = (\lambda_b a_\xi/\lambda_a)$ and $a_2 = (\lambda_b a_q)/(\lambda_a)$ are positive constants. Because the control signal $u(t)$ is a function of the weights $\hat{W}(t)$ and $\hat{V}(t)$ and the states $\xi(t)$ and $\eta(t)$, we know that it is also bounded. Therefore, we have shown that all the closed-loop signals are SGUB. To show that the tracking error $z_1 = y - y_d$ converges to the compact set Ω_{z_1}, we multiply (21) by $e^{c_1 t}$ and integrate over $[0, t]$ to obtain that $|z(t)| \leq \sqrt{2(V_\rho(0) + (c_2/c_1))e^{-c_1 t} + 2(c_2/c_1)}$, from which it is easy to see that $|z_1(t)| \leq \sqrt{(2c_2/c_1)}$ as $t \to \infty$. ■

B. Output Feedback Control

In Section III-A, we have considered the case where full-state measurement is possible, that is, η and ξ are all available. In this section, we tackle the output feedback problem, where only η and ξ_1 are available, by utilizing high-gain observers.

Lemma 5 [19]: Consider the following linear system:

$$\begin{aligned} \epsilon\dot{\pi}_i &= \pi_{i+1}, \qquad i = 1, 2, \ldots, \rho - 1 \\ \epsilon\dot{\pi}_\rho &= -\gamma_1\pi_\rho - \gamma_2\pi_{\rho-1} - \cdots - \gamma_{\rho-1}\pi_2 - \pi_1 + \xi_1(t) \end{aligned} \quad (29)$$

where ϵ is a small positive constant and the parameters γ_1 to $\gamma_{\rho-1}$ are chosen such that the polynomial $s^\rho + \gamma_1 s^{\rho-1} + \cdots + \gamma_{\rho-1}s + 1$ is Hurwitz. Suppose the states ξ belong to a compact set, so that $|\xi_k| < Y_k$, then the following property holds:

$$\tilde{\xi}_k := \frac{\pi_k}{\epsilon^{k-1}} - \xi_k = -\epsilon\zeta^{(k)}, \qquad k = 1, 2, \ldots, \rho \quad (30)$$

where $\zeta := \pi_\rho + \gamma_1\pi_{\rho-1} + \cdots + \gamma_{\rho-1}\pi_1$ and $\zeta^{(k)}$ denotes the kth derivative of ζ. Furthermore, there exist positive constants h_k and t^* such that $|\tilde{\xi}_k| \leq \epsilon h_k$ is satisfied for $t > t^*$.

To prevent peaking [20], the observer signals are saturated whenever they are outside the domain of interest $\Omega := \Omega_z \times \Omega_{\tilde{W}} \times \Omega_{\tilde{V}}$, where Ω_z, $\Omega_{\tilde{W}}$, and $\Omega_{\tilde{V}}$ are defined in (26), (27), and (28), respectively

$$\pi_i^s = \bar{\pi}_i \mathrm{sat}\left(\frac{\pi_i}{\bar{\pi}_i}\right), \qquad \bar{\pi}_i \geq \max_{(z,\tilde{W},\tilde{V})\in\Omega}(\pi_i)$$

$$\mathrm{sat}(a) = \begin{cases} -1, & \text{for } a < -1 \\ a, & \text{for } |a| \leq 1 \\ 1, & \text{for } a > 1 \end{cases} \quad (31)$$

for $i = 1, 2, \ldots, \rho$, where $\tilde{\xi} = [\tilde{\xi}_1, \ldots, \tilde{\xi}_\rho]^T$.

Now, we revisit the control law (15) and adaptation laws (16) and (17) for the full-state feedback case. Via the certainty equivalence approach, we modify them by replacing the partially available quantities z_i, $\dot{\alpha}_{i-1}$, and Z with their estimates, $\hat{z}_i := (\pi_i^s)/(\epsilon^{i-1}) - \alpha_{i-1}$, $\hat{\dot{\alpha}}_{i-1} := (d\alpha_{i-1}/dt)|_{z=\hat{z}}$, and $\hat{Z} := [\xi_1, (\pi_2^s/\epsilon), \dots, (\pi_\rho^s/\epsilon^{\rho-1}), \eta, \hat{z}_\rho, \hat{\dot{\alpha}}_{\rho-1}, 1]^T$, respectively, for $i = 2, \dots, \rho$. Therefore, the control laws are given by

$$\begin{aligned}
\alpha_1 &= -k_1 z_1 + \dot{y}_d \\
\alpha_2 &= -k_2 \hat{z}_i - z_1 + \hat{\dot{\alpha}}_1 \\
\alpha_i &= -k_i \hat{z}_i - \hat{z}_{i-1} + \hat{\dot{\alpha}}_{i-1,}, \qquad i = 3, \dots, \rho - 1 \\
u &= \hat{W}^T S(\hat{V}^T \hat{Z}) - k_\rho \hat{z}_\rho - \hat{z}_{\rho-1} - k_b \\
&\quad \times \left(\|\hat{Z}\hat{W}^T \hat{S}_o'\|_F^2 + \left(\|\hat{S}_o\| + \|\hat{S}_o' \hat{V}^T \hat{Z}\| \right)^2 \right) \hat{z}_\rho \quad (32)
\end{aligned}$$

where $k_1, \dots, k_\rho$, and k_b are positive constants, $\hat{S}_o := S(\hat{V}^T\hat{Z})$, and $\hat{S}_o' := \mathrm{diag}\{\hat{s}_{o1}', \dots, \hat{s}_{ol}'\}$ with $\hat{s}_{oi}' = s'(\hat{v}_i^T \hat{Z}) = (ds(z_a)/dz_a)|_{z_a = \hat{v}_i^T \hat{Z}}$, $i = 1, 2, \dots, l$. Due to the fact that the actual NN is in terms of $\hat{Z}$ while the ideal NN is in terms of Z, Lemma 2 needs to be modified as follows.

Lemma 6 [19]: The error between the actual and ideal NN output can be written as

$$\begin{aligned}
&\hat{W}^T S(\hat{V}^T\hat{Z}) - W^{*T} S(V^{*T} Z) \\
&\qquad = \tilde{W}^T (\hat{S}_o - \hat{S}_o' \hat{V}^T \hat{Z}) + \hat{W}^T \hat{S}_o' \tilde{V}^T \hat{Z} + d_u
\end{aligned}$$

where the residual term d_u is bounded by $d_u \leq \|W^*\|(\|\hat{S}_o'\hat{V}^T\hat{Z}\| + \|\hat{S}_o\|) + \|V^*\|_F \|\hat{Z}\hat{W}^T\hat{S}_o'\|_F$.

Accordingly, the adaptation laws are designed as

$$\dot{\hat{W}} = -\Gamma_W \left[\left(I - \chi_W \frac{\hat{W}\hat{W}^T}{\|\hat{W}\|^2} \right) (\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{Z})\hat{z}_\rho + \sigma_W \hat{W} \right] \quad (33)$$

$$\dot{\hat{V}} = -\Gamma_V [\hat{Z}\hat{W}^T \hat{S}_o' \hat{z}_\rho + \sigma_V \hat{V}] \quad (34)$$

where $\Gamma_W = \Gamma_W^T > 0$, $\Gamma_V = \Gamma_V^T > 0$, $\sigma_W > 0$, $\sigma_V > 0$, and

$$\chi_W = \begin{cases} 0, & \text{if } \|\hat{W}\| < M_W, \text{or if } \begin{cases} \|\hat{W}\| = M_W \text{ and} \\ \hat{W}^T(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{Z})\hat{z}_\rho \geq 0 \end{cases} \\ I, & \text{otherwise.} \end{cases} \quad (35)$$

Using the backstepping procedure similar to Section III-A, and substituting (32) –(35) into the derivative of V_ρ, it can be shown that

$$\begin{aligned}
\dot{V}_\rho \leq& -\sum_{j=1}^{\rho-1} k_j z_j^2 - \left(k_\rho - \frac{1}{2}\right) z_\rho^2 - \left(g_0 + \frac{\dot{g}_\lambda}{2g_\lambda} \right) \frac{z_\rho^2}{g_\lambda} - \frac{\sigma_W}{2}\|\tilde{W}\|^2 \\
&- \frac{\sigma_V}{2}\|\tilde{V}\|_F^2 - \hat{W}^T\hat{S}_o'\tilde{V}^T\hat{Z}\tilde{z}_\rho - \tilde{W}^T(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{Z})\tilde{z}_\rho \\
&- \sum_{j=2}^{\rho} k_j z_j \tilde{z}_j + \sum_{j=3}^{\rho} z_j (\hat{\dot{\alpha}}_{j-1} - \dot{\alpha}_{j-1}) + \sum_{j=3}^{\rho} z_j \tilde{z}_{j-1} \\
&+ \frac{\sigma_W + 1}{2}\|W^*\|^2 + \frac{\sigma_V + 1}{2}\|V^*\|_F^2 + \frac{1}{2}\bar{\varepsilon}^2 - \left(k_b - \frac{1}{2}\right) \\
&\times \left[(\|\hat{S}_o'\hat{V}^T\hat{Z}\| + \|\hat{S}_o\|)^2 + \left\|\hat{Z}\hat{W}^T\hat{S}_o'\right\|_F^2 \right] (z_\rho^2 + z_\rho \tilde{z}_\rho).
\end{aligned} \quad (36)$$

The following lemma is useful for handling the terms containing the estimation errors $\tilde{z}_j$, for $j = 1, 2, \dots, \rho$.

Lemma 7: There exist positive constants F_i and G_i, which are independent of ϵ, such that, for $t > t^*$, $i = 1, 2, \dots, \rho - 1$, the estimates $\hat{\dot{\alpha}}_i$ and $\hat{z}_i$ satisfy the following inequalities:

$$|\hat{\dot{\alpha}}_i - \dot{\alpha}_i| \leq \epsilon F_i \quad |\tilde{z}_i| := |\hat{z}_i - z_i| \leq \epsilon G_i. \quad (37)$$

Using Lemma 7, the following inequalities hold for $t > t^*$:

$$\begin{aligned}
-\tilde{W}^T \left(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{Z} \right) \tilde{z}_\rho &\leq \frac{\epsilon}{2}\|\tilde{W}\|^2 \\
&\quad + \frac{\epsilon}{2}(\|\hat{S}_o\| + \|\hat{S}_o'\hat{V}^T\hat{Z}\|)^2 G_\rho^2 \\
-\hat{W}^T\hat{S}_o'\tilde{V}^T\hat{Z}\tilde{z}_\rho &\leq \frac{\epsilon}{2}\|\tilde{V}\|_F^2 + \frac{\epsilon}{2}\left\|\hat{Z}\hat{W}^T\hat{S}_o'\right\|_F^2 G_\rho^2 \\
-\sum_{j=2}^{\rho} k_j z_j \tilde{z}_j &\leq \sum_{j=2}^{\rho} \frac{k_j}{2}\left(z_j^2 + \epsilon^2 G_j^2\right) \\
\sum_{j=2}^{\rho-1} z_j (\hat{\dot{\alpha}}_{j-1} - \dot{\alpha}_{j-1}) &\leq \sum_{j=2}^{\rho-1} \frac{1}{2}\left(z_j^2 + \epsilon^2 F_j^2\right) \\
\sum_{j=3}^{\rho} z_j \tilde{z}_{j-1} &\leq \sum_{j=3}^{\rho} \frac{1}{2}\left(z_j^2 + \epsilon^2 G_{j-1}^2\right).
\end{aligned} \quad (38)$$

By substitution of the inequalities (38) into (36), it is straightforward to obtain the following expression:

$$\begin{aligned}
\dot{V}_\rho \leq& -k_1 z_1^2 - \frac{1}{2}(k_2 - 1)z_2^2 - \sum_{j=3}^{\rho-1}\frac{1}{2}(k_j - 2)z_j^2 - \frac{1}{2}(k_\rho - 3)z_\rho^2 \\
&- \left(g_0 + \frac{\dot{g}_\lambda}{2g_\lambda} \right)\frac{z_\rho^2}{g_\lambda} - \frac{(\sigma_W - \epsilon)}{2}\|\tilde{W}\|^2 \\
&- \frac{(\sigma_V - \epsilon)}{2}\|\tilde{V}\|_F^2 + \sum_{j=2}^{\rho}\frac{k_j}{2}\epsilon^2 G_j^2 + \sum_{j=2}^{\rho}\frac{1}{2}\epsilon^2 F_j^2 \\
&+ \sum_{j=3}^{\rho}\frac{1}{2}\epsilon^2 G_{j-1}^2 + \frac{\sigma_W + 1}{2}\|W^*\|^2 \\
&+ \frac{\sigma_V + 1}{2}\|V^*\|_F^2 + \frac{1}{2}\bar{\varepsilon}^2 - \frac{1}{2}\left(k_b - \frac{1}{2}\right) \\
&\times \left[(\|\hat{S}_o'\hat{V}^T\hat{Z}\| + \|\hat{S}_o\|)^2 + \|\hat{Z}\hat{W}^T\hat{S}_o'\|_F^2 \right] \\
&\times \left(z_\rho^2 - \left(\epsilon^2 + \frac{2\epsilon}{2k_b - 1} \right) G_\rho^2 \right).
\end{aligned} \quad (39)$$

Finally, by appropriately choosing the control parameters as follows:

$$\begin{aligned}
&k_1 > 0 \quad k_2 > 1 \quad k_3, \dots, k_{\rho-1} > 2 \\
&k_\rho > 3 \quad k_b > \frac{1}{2} \quad \sigma_W \quad \sigma_V > \epsilon
\end{aligned} \quad (40)$$

it can be shown that

$$\dot{V}_\rho \leq -c_1 V_\rho + c_2 - K\left(z_\rho^2 - \epsilon c_3\right) \quad (41)$$

$$\begin{aligned}
c_1 := \min\Bigg\{ & 2k_1, (k_2 - 1), (k_3 - 2), \dots, (k_{\rho-1} - 2), \\
& \underline{g}(k_\rho - 3), \frac{(\sigma_W - \epsilon)}{\lambda_{\max}\left(\Gamma_W^{-1}\right)}, \frac{(\sigma_V - \epsilon)}{\lambda_{\max}\left(\Gamma_V^{-1}\right)} \Bigg\}
\end{aligned} \quad (42)$$

$$c_2 := \frac{1}{2}\bar{\varepsilon}^2 + \frac{\sigma_W+1}{2}\|W^*\|^2 + \frac{\sigma_V+1}{2}\|V^*\|_F^2 + \frac{1}{2}\epsilon^2\left(\sum_{j=2}^{\rho}F_j^2 + \sum_{j=2}^{\rho-1}(k_j+1)G_j^2 + G_\rho^2\right) \quad (43)$$

$$c_3 := \left(\epsilon + \frac{2}{2k_b-1}\right)G_\rho^2 \quad (44)$$

$$K := \frac{1}{2}\left(k_b - \frac{1}{2}\right)\left[(\|\hat{S}_o'\hat{V}^T\hat{Z}\| + \|\hat{S}_o\|)^2 + \|\hat{Z}\hat{W}^T\hat{S}_o'\|_F^2\right]. \quad (45)$$

We are ready to summarize our results for the output feedback case under the following theorem.

Theorem 2: Consider the helicopter dynamics (1) under Assumptions 1–5, with output feedback control law (32), adaptation laws (33)–(35), and high-gain observer (29), which is turned on at time t^* in advance. For initial conditions $\xi(0)$, $\eta(0)$, $\tilde{W}(0)$, and $\tilde{V}(0)$ starting in any compact set Ω_0, all closed-loop signals are SGUB, and the tracking error z_1 converges to the steady-state compact set $\Omega_{z_1} := \{z_1 \in R \| z_1| \leq \sqrt{(2\bar{c}_2)/(c_1)}\}$, where $\bar{c}_2 := c_2 + c_3\bar{K}$, and c_1 is as defined in (42).

Proof: Note that $K \geq 0$ from (45). According to (41), we see that the term $-K(z_\rho^2 - \epsilon c_3)$ may be positive or negative, depending on z_ρ. We consider the following two cases for the stability analysis:

Case 1) $|z_\rho| > \sqrt{\epsilon c_3}$. We have that $z_\rho^2 - \epsilon c_3 > 0$, and thus $-K(z_\rho^2 - \epsilon c_3) < 0$.

Case 2) $|z_\rho| \leq \sqrt{\epsilon c_3}$. We want to show that, as a result of z_ρ being bounded, the function K in (45) is also bounded. To this end, note that the derivative of $V_{\rho-1}$ is given by

$$\dot{V}_{\rho-1} \leq -\sum_{i=1}^{\rho-1}k_i z_i^2 - \sum_{i=2}^{\rho-1}k_i z_i\tilde{z}_i + \sum_{i=2}^{\rho-1}z_i(\dot{\hat{\alpha}}_{i-1} - \hat{\alpha}_{i-1}) + \sum_{i=3}^{\rho-1}z_i\tilde{z}_{i-1} + z_{\rho-1}z_\rho. \quad (46)$$

According to Lemma 7, we can show that

$$\dot{V}_{\rho-1} \leq -c_4V_{\rho-1} + c_5$$
$$c_4 := \min\{2k_1, k_2-1, k_3-2, \ldots, k_{\rho-1}-2, k_\rho-3\}$$
$$c_5 := \frac{\epsilon}{2}\left[c_3 + \epsilon\left(\sum_{i=2}^{\rho-1}F_i^2 + \sum_{i=2}^{\rho-2}(1+k_i)G_i^2 + k_{\rho-1}G_{\rho-1}^2\right)\right].$$

This implies that $z(t)$ satisfies the inequality $\|z(t)\| \leq \sqrt{2(V_{\rho-1}(0) + (c_5)/(c_4)) + \epsilon c_3} =: \bar{z}$. According to Lemma 4, it follows from the boundedness of $z(t)$ that the internal states $\eta(t)$ are also bounded, i.e., $\|\eta(t)\| \leq a_1(\bar{z} + \bar{\xi}_d) + a_2 =: \bar{\eta}$, where $\xi_d := [y_d, \alpha_1, \alpha_2, \ldots, \alpha_{\rho-1}]^T$, and $\|\xi_d(t)\| \leq \bar{\xi}_d$ for constant $\bar{\xi}_d > 0$, based on Assumption 2. Thus, the vector of NN inputs $\hat{Z}$ satisfies $\|\hat{Z}\| \leq \bar{\hat{Z}}$, where $\bar{\hat{Z}}$ is the 2-norm of constant vector $[\xi_{1d} + \bar{z}, (\bar{\pi}_2/\epsilon), \ldots, (\bar{\pi}_\rho)/(\epsilon^{\rho-1}), \bar{\eta}, \sqrt{c_3} + \epsilon G_\rho, \bar{\dot{\alpha}}_{\rho-1} + \epsilon F_{\rho-1}, 1]^T$ and the positive constant $\bar{\dot{\alpha}}_{\rho-1}$ is an upper bound for $\dot{\alpha}_{\rho-1}(z, \xi_{1d}, \xi_{1d}^{(1)}, \ldots, \xi_{1d}^{(\rho)})$. Exploiting the properties of sigmoidal neurons [10], it can be shown that $(\|\hat{S}_o'\hat{V}^T\hat{Z}\| + \|\hat{S}_o\|) \leq 1.224\sqrt{l}$. From the adaptation law (33) and (35), we know that $\|\hat{W}(t)\| \leq M_W$. Accordingly, from (45), and the fact that $\|S_o'\|_F \leq 0.25\sqrt{l}$, it can be shown that $K \leq (l/2)(k_b - (1)/(2))[1.498 + 0.0625(\bar{\hat{Z}}M_W)^2] =: \bar{K}$, where $\bar{K}$ is a positive constant. Then, we have that $-K(z_\rho^2 - \epsilon c_3) \leq \bar{K}\epsilon c_3$.

From both cases, we conclude that $-K(z_\rho^2 - \epsilon c_3) \leq \bar{K}\epsilon c_3$ for all $z_\rho \in R$, and thus, from (41), that $\dot{V}_\rho \leq -c_1V_\rho + \bar{c}_2$, where $\bar{c}_2 := c_2 + \bar{K}\epsilon c_3$ is a positive constant. Thus, we can directly invoke Lemma 1 to conclude SGUB of all closed-loop signals. A nice property is that as ϵ diminishes to zero, we have $\bar{c}_2 \to c_2$, and the performance can be analyzed from $\dot{V}_\rho \leq -c_1V_\rho + c_2$ instead. By following the steps outlined in the proof of Theorem 1, it is straightforward to prove that the tracking error $z_1 = \xi_1 - \xi_{1d}$ converges to the compact set Ω_{z_1}. ■

Remark 3: It follows from Theorems 1 and 2 that the size of the steady-state compact set Ω_{z1}, to which the tracking error converges, can be reduced by appropriately choosing the design parameters $k_1, \ldots, k_\rho, \sigma_W, \sigma_V, \Gamma_W$, and Γ_V.

IV. Simulation Study

In Section II, we have considered a general representation of helicopters in vertical flight as SISO nonaffine nonlinear systems. Although it would be most realistic to perform simulations on nonaffine helicopter models, an accurate model is difficult to obtain. We will apply our proposed adaptive NN control for general nonlinear systems to linear and nonlinear affine helicopter models, which are available in the literature, and which have been used with success. As affine systems are a special subclass of nonlinear nonaffine systems, the proposed NN control can be applied on them without any complications.

A. Linear Model of Yamaha R50

Consider the linearized model of the Yamaha R50 helicopter as detailed in [6] with the longitudinal cyclic input δ set to zero

$$\begin{bmatrix}\dot{u}\\ \dot{q}\\ \dot{\theta}\\ \dot{\beta}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}X_u & X_q & X_\theta & X_\beta & X_\omega\\ M_u & M_q & 0 & M_\beta & M_\omega\\ 0 & 1 & 0 & 0 & 0\\ B_u & -1 & 0 & B_\beta & 0\\ Z_u & Z_q & Z_\theta & Z_\beta & Z_\omega\end{bmatrix}\begin{bmatrix}u\\ q\\ \theta\\ \beta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 0\\ Z_\Omega\delta_\Omega\end{bmatrix}$$
$$\dot{h} = -\omega$$

where u denotes the body-frame forward velocity; ω denotes the body-frame upward velocity; θ and q denote the pitch angle and rate, respectively; β denotes the longitudinal tilt of the main rotor axis; h denotes the altitude; and δ_Ω denotes the main rotor RPM input.

The control parameters are set to be $k_1 = 2.0$, $k_2 = 8.5$, and $k_b = 1.0$, while the NN parameters are $\mu = 1$, $\Gamma_W = 5I$, $\Gamma_V = 50I$, and $\sigma_W = \sigma_V = 5$. For the high-gain observer, we choose $\epsilon = 0.01$, $\gamma_1 = 2$, and $\bar{\pi}_2 = 0.08$. The lower and upper saturation limits of the control are 393 and 1348 r/min, respectively. The initial conditions are $u(0) = q(0) = \theta(0) = \beta(0) = \omega(0) = 0, h(0) = 10$, $\hat{W} = 0$, and $\hat{V} = 0$. For convenience, and without loss of generality, we initialized the observer error as 0.

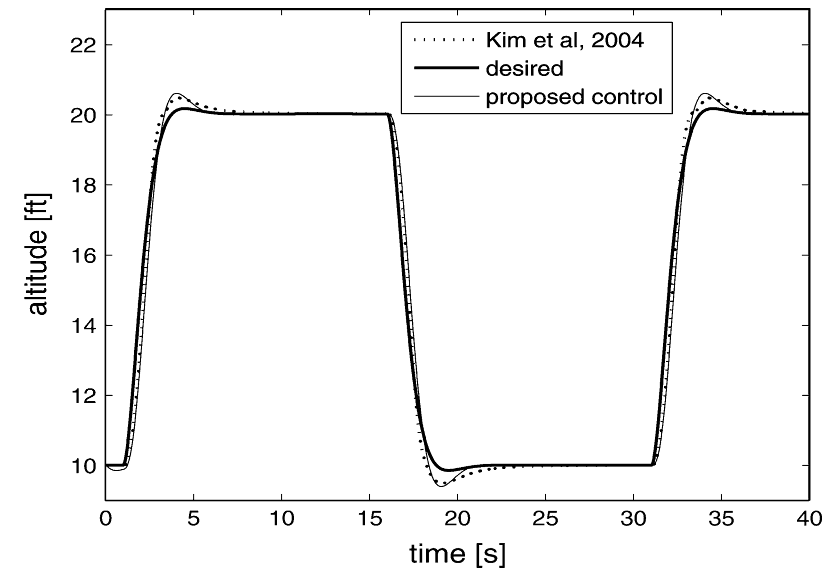

Fig. 1. Comparison of tracking performance.

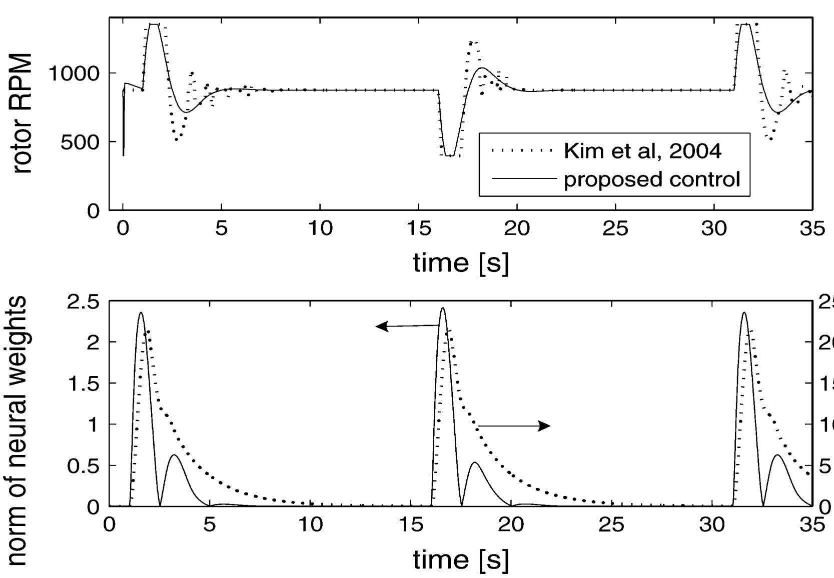

Fig. 2. Comparison of rotor RPM input and norm of neural weights.

To compare our controller with that of [6], we consider the tracking of the altitude h according to a desired trajectory $h_d(t)$ defined by

$$\begin{bmatrix} \dot{h}_d \\ \ddot{h}_d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2.25 & -2.4 \end{bmatrix} \begin{bmatrix} h_d \\ \dot{h}_d \end{bmatrix} + \begin{bmatrix} 0 \\ 2.25 \end{bmatrix} h_{\rm ref}$$

$$h_{\rm ref}(t) = \begin{cases} 10, & \text{if } 0 \le t < 1 \\ 20, & \text{if } 1 \le t < 16 \\ 10, & \text{if } 16 \le t < 31 \\ 20, & \text{if } t \ge 31. \end{cases} \tag{47}$$

Note that in our comparison, both controllers are simulated without engine dynamics.

It can be seen in Fig. 1 that the tracking performance under the proposed control is reasonably good, with the altitude signal tracking the desired trajectory closely. The comparison shows that the performances under the two different controls are similar. From Fig. 2, it is clear that the control signals and neural weights are well behaved. The control of [6] exhibits more fluctuations, and the neural weights evolve to significantly larger amplitudes. Although this does not set out any comparative advantages, it does demonstrate different mechanisms at work in the two control schemes. The effect of the parameter ϵ on tracking errors and observer errors are shown in Fig. 3, where it is seen that as ϵ diminishes, the tracking error under the full-state feedback control is recovered, and faster convergence of the observer is achieved.

B. Nonlinear Model of X-Cell 50

Consider nonlinear model of X-cell 50 model helicopter in vertical flight [2], [21]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_0 + a_1 x_2 + a_2 x_2^2 + (a_3 + a_4 x_4 - \sqrt{a_5 + a_6 x_4}) x_3^2 \\ \dot{x}_3 &= a_7 + a_8 x_3 + (a_9 \sin x_4 + a_{10}) x_3^2 + a_{\rm th} \\ \dot{x}_4 &= x_5 \\ \dot{x}_5 &= a_{11} + a_{12} x_4 + a_{13} x_3^2 \sin x_4 + a_{14} x_5 - K_2 u \end{aligned} \tag{48}$$

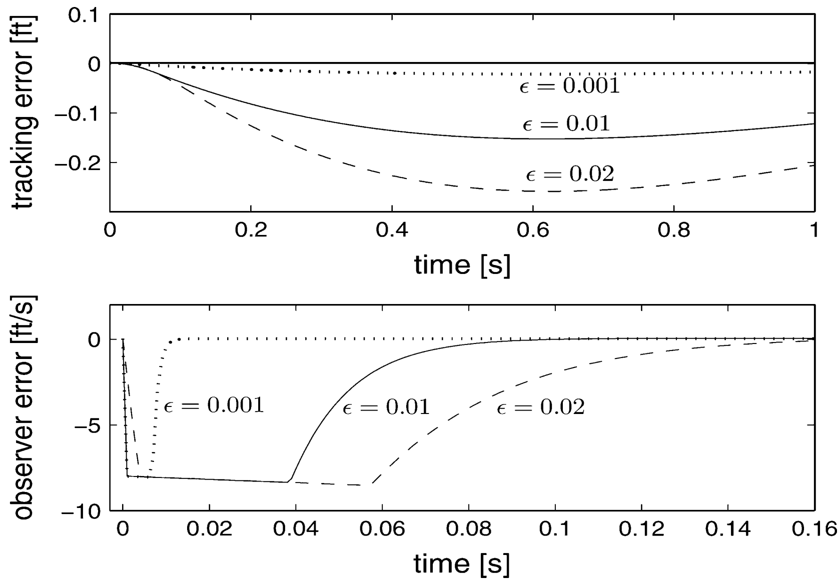

Fig. 3. Effect of ϵ on tracking and observer errors.

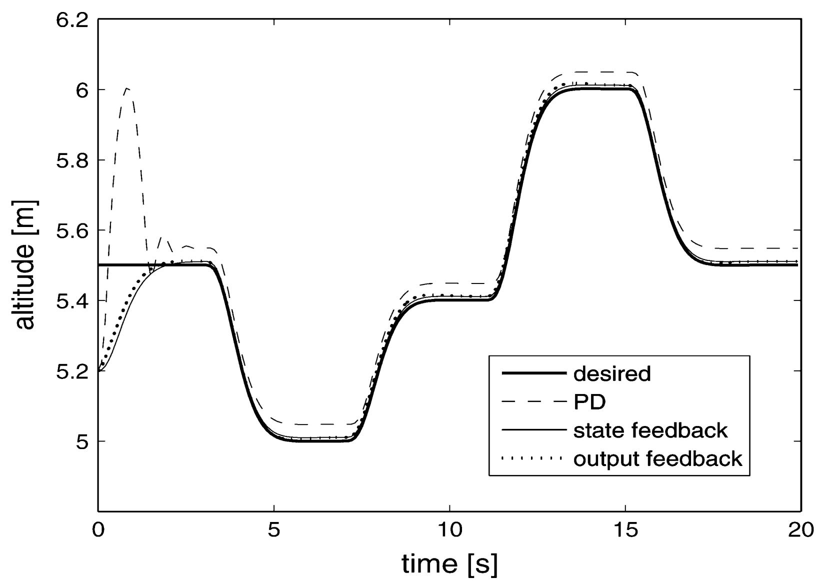

Fig. 4. Comparison of tracking performance.

where x_1 denotes altitude, x_2 denotes altitude rate, x_3 denotes rotor speed, x_4 denotes the collective pitch angle, x_5 denotes the collective pitch rate, $a_{\mathrm{th}} = 111.69s^{-2}$ is a constant input to the throttle, and u is the input to the collective servomechanisms.

Let output y be the altitude x_1. By restricting the throttle input to be constant, we obtain a SISO system in which u is the only input variable forcing the output y to track a desired trajectory, $y_d(t)$, generated by the dynamic system

$$y_d^{(4)} + 150.06y_d^{(3)} + 154.35y_d^{(2)} + 64.19y_d^{(1)} + 12.6y_d = 150.06y_{\mathrm{ref}} \quad (49)$$

where

$$y_{\mathrm{ref}}(t) = \begin{cases} 5.5, & \text{if } 0 \leq t < 3 \\ 5.0, & \text{if } 3 \leq t < 7 \\ 5.4, & \text{if } 7 \leq t < 11 \\ 6.0, & \text{if } 11 \leq t < 15 \\ 5.5, & \text{if } t \geq 15. \end{cases} \quad (50)$$

The control parameters are chosen as $k_1 = 2.0$, $k_2 = 3.0$, $k_2 = 4.5$, $k_4 = 5.5$, and $k_b = 0.6$, while the NN parameters are $\mu = 0.01$, $\Gamma_W = 50I$, $\Gamma_V = 20.4I$, $\sigma_V = 0.055$, and $\sigma_W = 0.05$. For the high-gain observer, we choose $\epsilon = 5 \times 10^{-4}$, $\gamma_1 = 4$, $\gamma_2 = 6$, $\gamma_3 = 4$, $\bar{\pi}_2 = 5 \times 10^{-4}$, $\bar{\pi}_3 = 5 \times 10^{-8}$, and $\bar{\pi}_4 = 1 \times 10^{-11}$. The saturation limits of the control are ± 450 m·rad. The initial conditions are $x(0) = [5.2, 0, 95.36, 0, 0]^T$, $\hat{W} = 0$, and $\hat{V} = 0$.

As shown in Fig. 4, good tracking performance is achieved by the proposed adaptive NN control for both full-state and output feedback schemes. The initial error is efficiently reduced and the altitude trajectory lies in close proximity of the desired trajectory. We compare the performance of the NN controller with a linear proportional-derivative (PD) controller $u_{pd} = K_p(y - y_d) + K_d(\dot{y} - \dot{y}_d)$, where $K_p = 5000$ rad and $K_d = 500$ rad·s are chosen so that the tracking errors are reasonably small and the control magnitude is constrained to $|u_{pd}| \leq 450$ m·rad. Although steady-state errors are comparable between PD and NN

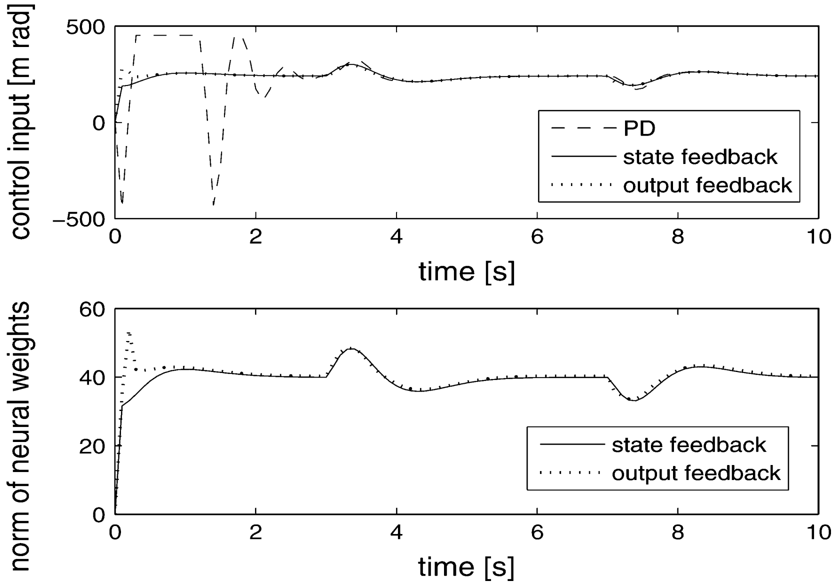

Fig. 5. Control signals and norm of neural weights.

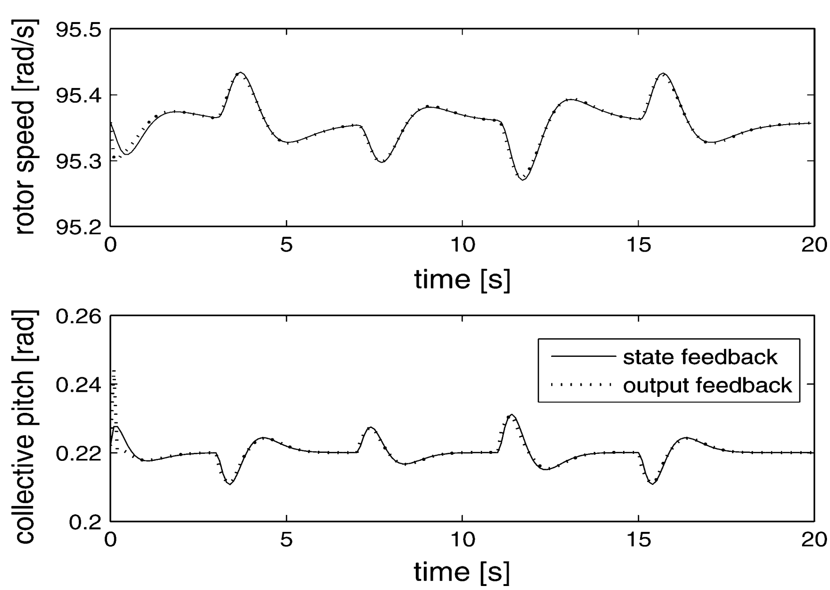

Fig. 6. Rotor speed and collective pitch.

control, the PD control gives poorer transient performance as it attempts to compensate for the initial error, due to the inability of the linear PD control to adequately compensate for the effects of nonlinearity and coupling.

The boundedness of the control input and the neural weights, for full-state and output feedback NN control, as well as the PD control, is shown in Fig. 5. The size of input signal under PD control is much larger than that under NN control, as seen by the fact that the PD control signal initially fluctuates between the saturation limits. This can be explained by the fact that a large PD control gain is required to compensate for nonlinearities, thus amplifying the control effort greatly when the initial error is large. In Fig. 6, it is shown that the rotor speed and collective pitch angle, for both full-state and output feedback NN control, are bounded. In particular, it is confirmed that the collective pitch angle remains in the region $[0, 0.44]$ rad as restricted in practical operations.

V. Conclusion

In this brief, we have proposed adaptive NN control, based on the implicit function theorem and the mean value theorem, for helicopters in vertical flight under a general SISO nonlinear nonaffine system representation. Our results show that the robust adaptive NN controller drives the output tracking error to a small neighborhood of the origin, while keeping all closed-loop trajectories bounded within a compact set. The simulation study demonstrated the effectiveness of the proposed control on linear and nonlinear helicopter models for vertical flight, and comparison with the control schemes in [6] showed similar level of performance.

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