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Adaptive synchronization of uncertain chaotic systems via backstepping design

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Abstract

In this paper, an approach for adaptive synchronization of uncertain chaotic systems is proposed using adaptive backstepping with tuning functions. Strong properties of global stability and asymptotic synchronization can be achieved. The proposed approach offers a systematic design procedure for adaptive synchronization of a large class of continuous-time chaotic systems in the chaos research literature. Simulation results are presented to show the effectiveness of the approach.

1. Introduction

Since the discovery of chaos synchronization [1], there has been tremendous interest in studying the synchronization of chaotic systems, see [2,3] and the references therein for a survey of the recent development. As chaotic signals could be used to transmit information in a secure and robust manner, chaos synchronization has been intensively studied in communications research [4–9] (to name just a few).

Recently, specialists from nonlinear control theory turned their attention to the study of chaos synchronization and its potential applications in communications. Fradkov and Pogromsky [10] presented a speed-gradient method for adaptive synchronization of chaotic systems. Nijmeijer and Mareels [11] casted the problem of chaos synchronization as a special case of observer design problem. Suykens et al. [13] proposed a robust nonlinear H_∞ synchronization method for chaotic Lur's systems with applications to secure communications. Pogromsky [12] considered the problem of controlled synchronization of nonlinear systems using a passivity-based design method. More recently, Fradkov et al. [14] presented an adaptive observer-based synchronization scheme, where an adaptive observer for estimating the unknown parameters of the transmitter was designed, which may correspond to the parameter modulation for message transmission. Due to these developments, chaos synchronization as well as chaos communications have attracted revived interests in the nonlinear control community.

Over the past decade, backstepping [17] has become one of the most popular design methods for adaptive nonlinear control because it can guarantee global stabilities, tracking, and transient performance for a broad class of strict-feedback systems ([19], and the references therein). In [15,16], it has been shown that many well-known chaotic systems as paradigms in the research of chaos, including Duffing oscillator, van der Pol oscillator, Rössler system, and several types of Chua's circuits, can be transformed into a class

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of nonlinear systems in the so-called nonautonomous "strict-feedback" form, and the adaptive backstepping and tuning functions control schemes have been employed and extended to control these chaotic systems with key parameters unknown. Global stability and asymptotic tracking have been achieved. In particular, the output of the controlled chaotic system has been designed to asymptotically track any smooth and bounded reference signal generated from a known reference model which may be a chaotic system.

In this paper, we consider the problem of adaptive synchronization of uncertain nonlinear systems, in particular chaotic systems, using adaptive backstepping with tuning functions [18,19]. In our approach, motivated by the backstepping design technique, the master system is any smooth, bounded, linear-in-parameter nonlinear system with key parameters unknown, while the slave system can be any smooth, nonlinear strict-feedback system. Global stability and asymptotic synchronization between the master and slave systems can be achieved. In particular, chaos synchronization can be realized when the master system is chosen as one of the chaotic systems which is in chaotic states. It should be noted that the slave system does not need to have the identical structure as the master system. In fact, the slave system can be chosen as any simple strict-feedback systems, as will be demonstrated in the simulation studies.

As indicated by Dedieu and Ogorzalek [7], the claimed advantage of chaos secure communication becomes in fact a major drawback, since the system parameters, which were taken as the cryptographic key, can be easily found using synchronization and optimization tools. In this paper, we recommend that the known nonlinear functions in the master system, rather than the system parameters, are taken as the cryptographic key for chaos secure communications. Because the master system is in a very general form, a wide class of continuous-time chaotic and hyper-chaotic systems can be designed as the transmitter. This implies that much complicated high-order chaotic systems can be employed to improve the security in chaos communications.

The rest of the paper is organized as follows: The problem formulation is presented in Section 2. Adaptive backstepping with tuning functions approach is extended to the synchronization problem in Section 3. In Section 4, the Rössler system with key constant parameters unknown is used to show the effectiveness of the proposed approach. Section 5 contain the conclusions.

2. Problem formulation

Consider the master system in the form of any smooth, bounded, linear-in-the-parameters nonlinear (chaotic) system as

$$\dot{x}_{di} = f_{di}(x_d, t) + \theta^{\mathrm{T}} F_{di}(x_d, t), \quad 1 \leqslant i \leqslant m, \tag{1}$$

where $x_d = [x_{d1}, x_{d2}, \ldots, x_{dm}]^{\mathrm{T}} \in R^m$ is the state vector; $\theta = [\theta_1, \theta_2, \ldots, \theta_p] \in R^p$ is the vector of unknown constant parameters; $f_{di}(\cdot), F_{di}(\cdot)$, $i = 1, 2, \ldots, m$ are known smooth nonlinear functions, with their jth derivatives ($j = 0, \ldots, m - i$) uniformly bounded in t.

The slave system is in the form of strict-feedback nonlinear (chaotic) system as

$$\begin{aligned} \dot{x}_1 &= g_1(x_1, t)x_2 + f_1(x_1, t), \\ &\vdots \\ \dot{x}_{n-1} &= g_{n-1}(x_1, \ldots, x_{n-1}, t)x_{n-1} + f_{n-1}(x_1, \ldots, x_{n-1}, t), \\ \dot{x}_n &= g_n(x, t)u + f_n(x, t), \quad n \leqslant m, \end{aligned} \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$ and $u \in R$ are the states and control action, respectively; $g_i(\cdot) \neq 0, f_i(\cdot)$, $i = 1, \ldots, n$ are known, smooth nonlinear functions, with their jth derivatives ($j = 0, \ldots, n - i$) uniformly bounded in t.

The problem is to design an adaptive synchronization algorithm

$$u = U(x, x_d, \hat{\theta}, t), \quad \dot{\hat{\theta}} = H(x, x_d, \hat{\theta}, t), \tag{3}$$

where $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_p] \in R^p$ is the vector of parameter estimates for the unknown parameter vector θ, to guarantee global stability and force the state $x_1(t)$ of the slave system (2) to asymptotically synchronize with the state $x_{d1}(t)$ of the master system (1), i.e., to achieve

$$x_1(t) - x_{d1}(t) \to 0, \quad \text{as } t \to \infty. \tag{4}$$

3. Adaptive synchronization via backstepping

In order to design an adaptive synchronization algorithm (3) to achieve the objective (4), adaptive backstepping with tuning functions is employed. For clarity and conciseness of presentation, we present the design procedure for the master system (1) and the slave system (2) with $m \geqslant 3, n = 3$, although it can be easily generalized to systems of any order with the assumption $m \geqslant n$.

Step 1. Define $z_1 = x_1 - x_{d1}$. Its derivative is given by

$$\begin{aligned}
\dot{z}_1 &= \dot{x}_1 - \dot{x}_{d1} \\
&= g_1 x_2 + f_1 - f_{d1} - \theta^{\mathrm{T}} F_{d1} \\
&= g_1 z_2 + g_1 \alpha_1 + g_1 x_{d2} - \theta^{\mathrm{T}} F_{d1} + f_1 - f_{d1} \\
&= g_1 z_2 + g_1 \alpha_1 - \hat{\theta}^{\mathrm{T}} F_{1s} + f_{1s} + (\hat{\theta} - \theta)^{\mathrm{T}} F_{1s},
\end{aligned} \tag{5}$$

where $z_2 = x_2 - x_{d2} - \alpha_1$, α_1 is an artificial control to be defined later, and

$$\begin{aligned}
F_{1s} &= F_{d1}, \\
f_{1s} &= f_1 - f_{d1} + g_1 x_{d2}.
\end{aligned}$$

For the design of α_1 to stabilize (5), consider the Lyapunov function candidate

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} (\hat{\theta} - \theta)^{\mathrm{T}} \Gamma^{-1} (\hat{\theta} - \theta). \tag{6}$$

The derivative of V_1 is

$$\begin{aligned}
\dot{V}_1 &= z_1 \dot{z}_1 + (\hat{\theta} - \theta)^{\mathrm{T}} \Gamma^{-1} \dot{\hat{\theta}} \\
&= g_1 z_1 z_2 + z_1 (g_1 \alpha_1 - \hat{\theta}^{\mathrm{T}} F_{1s} + f_{1s}) + (\hat{\theta} - \theta)^{\mathrm{T}} \Gamma^{-1} (\dot{\hat{\theta}} + \Gamma F_{1s} z_1).
\end{aligned} \tag{7}$$

Define the tuning function τ_1 for $\hat{\theta}$ as

$$\tau_1 = -\Gamma F_{1s} z_1. \tag{8}$$

To make the bracketed term multiplying z_1 in Eq. (7) be equal to $-c_1 z_1^2$, we choose

$$\alpha_1 = \frac{1}{g_1} \Big(-c_1 z_1 + \hat{\theta}^{\mathrm{T}} F_{1s} - f_{1s} \Big). \tag{9}$$

Note that the $(\hat{\theta} - \theta)$-term would have been eliminated with the choice of adaptive law $\dot{\hat{\theta}} = \tau_1$. Since this is not the last design step, we postpone the choice of adaptive law and tolerate the presence of $(\hat{\theta} - \theta)$ in $\dot{V}_1$ as follows:

$$\dot{V}_1 = -c_1 z_1^2 + g_1 z_1 z_2 + (\hat{\theta} - \theta) \Gamma^{-1} (\dot{\hat{\theta}} - \tau_1). \tag{10}$$

The second term $g_1 z_1 z_2$ will be canceled in the next step. The closed-loop form of (5) with (9) is

$$\dot{z}_1 = -c_1 z_1 + g_1 z_2 + (\hat{\theta} - \theta)^{\mathrm{T}} F_{1s}. \tag{11}$$

Step 2. The derivative of z_2 is expressed as

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{x}_{d2} - \dot{\alpha}_1 \\ &= g_2 z_3 + g_2\alpha_2 + g_2 x_{3r} + f_2 - f_{d2} - \theta^{\mathrm{T}} F_{d2} - \frac{\partial\alpha_1}{\partial x_1}(g_1 x_2 + f_1) - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - \sum_{k=1}^{m}\frac{\partial\alpha_1}{\partial x_{dk}}\left(f_{dk} + \theta^{\mathrm{T}} F_{dk}\right) - \frac{\partial\alpha_1}{\partial t} \\ &= g_2 z_3 + g_2\alpha_2 - \hat{\theta}^{\mathrm{T}} F_{2s} + f_{2s} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\left(\tau_2 - \dot{\hat{\theta}}\right) + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} F_{2s},\end{aligned} \tag{12}$$

where $z_3 = x_3 - x_{d3} - \alpha_2$, α_2 is the virtual control to be defined later, and

$$\begin{aligned}F_{2s} &= F_{d2} + \sum_{k=1}^{m}\frac{\partial\alpha_1}{\partial x_{dk}} F_{dk}, \\ f_{2s} &= f_2 - f_{d2} + g_2 x_{d3} - \frac{\partial\alpha_1}{\partial x_1}(g_1 x_2 + f_1) - \sum_{k=1}^{m}\frac{\partial\alpha_1}{\partial x_{dk}} f_{dk} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2 - \frac{\partial\alpha_1}{\partial t}.\end{aligned} \tag{13}$$

For the design of α_2 to stabilize the (z_1, z_2)-subsystem defined by (11) and (12), choose the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_2^2. \tag{14}$$

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2 = &- c_1 z_1^2 + g_2 z_2 z_3 + z_2\Big(g_1 z_1 + g_2\alpha_2 - \hat{\theta}^{\mathrm{T}} F_{2s} + f_{2s}\Big) + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}\left(\tau_2 - \dot{\hat{\theta}}\right) \\ &+ \left(\hat{\theta} - \theta\right)^{\mathrm{T}}\Gamma^{-1}\left(\dot{\hat{\theta}}_1 - \tau_1 + \Gamma F_{2s} z_2\right).\end{aligned} \tag{15}$$

Define tuning function τ_2 for $\hat{\theta}$ as

$$\tau_2 = \tau_1 - \Gamma F_{2s} z_2. \tag{16}$$

To make the bracketed term multiplying z_2 in Eq. (15) be equal to $-c_2 z_2^2$, we choose

$$\alpha_2 = \frac{1}{g_2}\Big(- c_2 z_2 - g_1 z_1 + \hat{\theta}^{\mathrm{T}} F_{2s} - f_{2s}\Big). \tag{17}$$

Again, we postpone the choice of adaptive laws $\dot{\hat{\theta}}$ and tolerate the presence of $(\hat{\theta} - \theta)$ in $\dot{V}_2$ as

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 + g_2 z_2 z_3 + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}\left(\tau_2 - \dot{\hat{\theta}}\right) + \left(\hat{\theta} - \theta\right)\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_2\right). \tag{18}$$

The term $g_2 z_2 z_3$ will be canceled in the next step. The closed-loop form of (12) with (17) is

$$\dot{z}_2 = -g_1 z_1 - c_2 z_2 + g_2 z_3 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\left(\tau_2 - \dot{\hat{\theta}}\right) + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} F_{2s}. \tag{19}$$

Step 3. Since this is our last step, the derivative of z_3 is expressed as

$$\begin{aligned}\dot{z}_3 &= \dot{x}_3 - \dot{x}_{d3} - \dot{\alpha}_2 \\ &= g_3 u + f_3 - \theta^{\mathrm{T}} F_{d3} - f_{d3} - \frac{\partial\alpha_2}{\partial x_1}(g_1 x_2 + f_1) - \frac{\partial\alpha_2}{\partial x_2}(g_2 x_3 + f_2) - \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} - \sum_{k=1}^{m}\frac{\partial\alpha_2}{\partial x_{dk}}\left(f_{dk} + \theta^{\mathrm{T}} F_{dk}\right) - \frac{\partial\alpha_2}{\partial t} \\ &= g_3 u - \hat{\theta}^{\mathrm{T}} F_{3s} + f_{3s} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\left(\tau_3 - \dot{\hat{\theta}}\right) + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} F_{3s},\end{aligned} \tag{20}$$

where

$$F_{3s} = F_{d3} + \sum_{k=1}^{m} \frac{\partial \alpha_2}{\partial x_{dk}} F_{dk},$$
$$f_{3s} = f_3 - f_{d3} - \frac{\partial \alpha_2}{\partial x_1}(g_1 x_2 + f_1) - \frac{\partial \alpha_2}{\partial x_2}(g_2 x_3 + f_2) - \frac{\partial \alpha_2}{\partial \hat{\theta}} \tau_3 - \sum_{k=1}^{m} \frac{\partial \alpha_2}{\partial x_{dk}} f_{dk} - \frac{\partial \alpha_2}{\partial t}. \tag{21}$$

For the design of control u to stabilize the (z_1, z_2, z_3) error system defined by (11), (19) and (20), choose the following Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2} z_3^2. \tag{22}$$

The derivative of V_3 is

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \left(\tau_2 - \dot{\hat{\theta}}\right) + z_3 \left(g_2 z_2 + g_3 u - \hat{\theta}^{\mathrm{T}} F_{3s} + f_{3s}\right) + z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}} \left(\tau_3 - \dot{\hat{\theta}}\right)$$
$$+ \left(\hat{\theta} - \theta\right)^{\mathrm{T}} \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_2 + \Gamma F_{3s} z_3\right). \tag{23}$$

To eliminate the $(\hat{\theta} - \theta)$-term in $\dot{V}_3$ from Eq. (23), we choose the parameter adaptive law for $\hat{\theta}$ as

$$\dot{\hat{\theta}} = \tau_3 = \tau_2 - \Gamma F_{3s} z_3. \tag{24}$$

Noting that

$$\tau_2 - \dot{\hat{\theta}} = \tau_2 - \tau_3 = \Gamma F_{3s} z_3. \tag{25}$$

Eq. (23) can be written as

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + \left(z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} + z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}}\right)\left(\tau_3 - \dot{\hat{\theta}}\right) + z_3 \left(g_2 z_2 + g_3 u - \hat{\theta}^{\mathrm{T}} F_{3s} + f_{3s} + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma F_{3s}\right)$$
$$= -c_1 z_1^2 - c_2 z_2^2 + z_3 \left(g_2 z_2 + g_3 u - \hat{\theta}^{\mathrm{T}} F_{3s} + f_{3s} + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma F_{3s}\right). \tag{26}$$

Finally, we choose the control u such that the bracketed term multiplying z_3 in Eq. (26) be equal to $-c_3 z_3^2$

$$u = \frac{1}{g_3} \left(- c_3 z_3 - g_2 z_2 + \hat{\theta}^{\mathrm{T}} F_{3s} - f_{3s} - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma F_{3s}\right). \tag{27}$$

Thus, we have

$$\dot{V}_3 = -\sum_{k=1}^{3} c_k z_k^2. \tag{28}$$

The closed-loop form of (20) with (27) is

$$\dot{z}_3 = -g_2 z_2 - c_3 z_3 + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} F_{3s} - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma F_{3s}. \tag{29}$$

The global stability and asymptotic single state synchronization of the closed-loop system is summarized in Theorem 1.

Theorem 1. *The closed-loop adaptive system consisting of the master system* (1), *the slave system* (2), *the controller* (27) *and the adaptive law* (24) *has a globally uniformly stable equilibrium at* $(z, \hat{\theta}) = (0, \theta)$. *This guarantees the global boundedness of all the signals in the closed-loop system, including the states of the slave system* $x = [x_1, x_2, x_3]^{\mathrm{T}}$, *the control* u *and parameter estimates* $\hat{\theta}$, *and* $\lim_{t\to\infty} z(t) = 0$, *which means that asymptotic single state identical synchronization is achieved*

$$\lim_{t\to\infty}[x_1(t) - x_{d1}(t)] = 0. \tag{30}$$

Proof. The $(z_1, \ldots, z_n)$-system corresponds to the closed-loop adaptive system consisting of the master system (1), the slave system (2), the controller (27) and the adaptive law (24). The derivative of the Lyapunov function (20) along (z_1, z_2, z_n)-system is (28), which proves that equilibrium $z = 0$ is globally uniformly stable.

Combining (24) with (28) we conclude that $\hat{\theta}$ are bounded. Since $z_1 = x_1 - x_{d1}$ and x_{d1} is bounded, we see that x_1 is also bounded. The boundedness of x_i, $i = 2, 3$ follows from the boundedness of α_{i-1} and x_{di}, and the fact that $x_i = z_i + x_{di} + \alpha_{i-1}$, $i = 2, 3$. Using (27), we conclude that the control u is also bounded.

From the LaSalle–Yoshizawa theorem [19], it further follows that, all the solutions of the $(z_1, \ldots, z_n)$-system converge to the manifold $z = 0$ as $t \to \infty$. From the definition $z_1 = x_1 - x_{d1}$, we conclude that $x_1(t) - x_{d1}(t) \to 0$ as $t \to \infty$, which means that global asymptotic single state identical synchronization is achieved. □

Remark 1. For chaos secure communications, we recommend that the known nonlinear functions F_{di} in the master system (1), rather than the system parameters θ, are taken as the cryptographic key. Because the master system is in a very general form (1), a wide class of continuous-time chaotic and hyper-chaotic systems can be designed as the transmitter. This implies that much complicated high-order chaotic systems can be employed to improve the security in chaos communications.

Remark 2. In the proposed approach, the master system (1) only has parametric uncertainties that appear linearly with respect to the known nonlinear functions. For the case when both parametric uncertainty and unknown nonlinear functions are present in the master system, where these unknown nonlinear functions could be due to modeling errors, external disturbances, time variations in the system, robust adaptive control design can be used to guarantee robustness with respect to bounded uncertainties and exogenous disturbances (see, e.g., [20] and the references therein). Ultimately uniform boundedness and generalized synchronization can be achieved.

4. Simulation studies

In the simulation studies, the master system is chosen as the chaotic Rössler system [21] with all the key parameters unknown, and the slave system is designed as: (i) the simplest version of first order strict-feedback systems; (ii) the same Rössler system as the master system except that the system parameters are different. Other chaotic systems such as the Duffing oscillator, the van der Pol oscillator and the Chua's circuits can all be taken as the master and the slave systems and can be designed readily by the same design procedure.

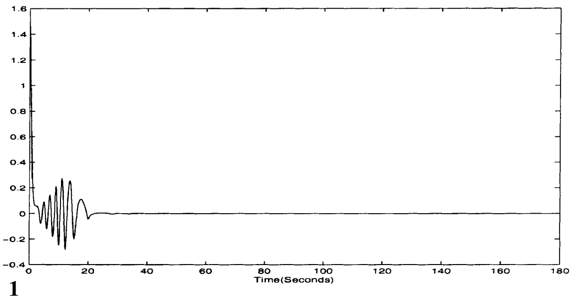
Consider the Rössler system [21] as the master system described as (after some simple state transformations)

$$\begin{aligned}
\dot{x}_{d1} &= x_{d2} + \theta_1 x_{d1}, \\
\dot{x}_{d2} &= -x_{d3} - x_{d1}, \\
\dot{x}_{d3} &= \theta_2 + x_{d3}(x_{d2} - \theta_3),
\end{aligned} \tag{31}$$

where θ_1, θ_2 and θ_3 are constant system parameters.

Assuming that the Rössler system (31) (master system) is in chaotic state with parameters $\theta_1 = 0.15$, $\theta_2 = 0.20$ and $\theta_3 = 10$. Our objective is to force state $x_1(t)$ of the slave system to asymptotically synchronize with state $x_{d1}(t)$ of the master system (31).

For case (i), the slave system is designed as a simple first order system $\dot{x}_1 = u$. Since the Rössler system (31) is in the strict-feedback form, while the slave system is a first order system, i.e., $n = 1$, two master system state signals, x_{d1} and x_{d2}, are employed by the slave system. At the same time, only one parameter estimate $\hat{\theta}_1$ can be obtained.



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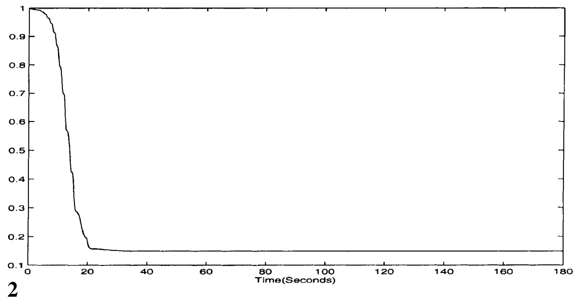

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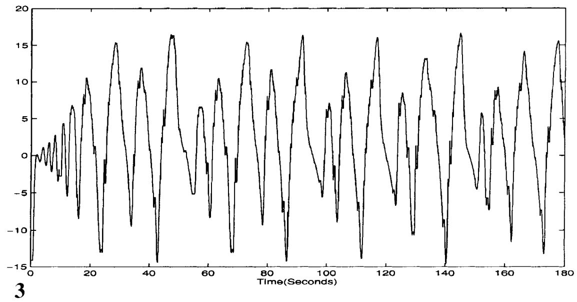
Fig. 1. Tracking error $x_1(t) - x_{d1}(t)$.

Fig. 2. Boundedness of the parameter estimate: $\hat{\theta}_1$.



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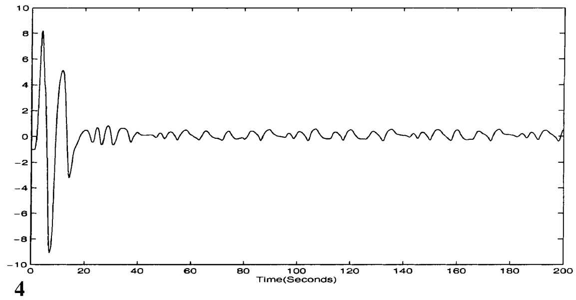

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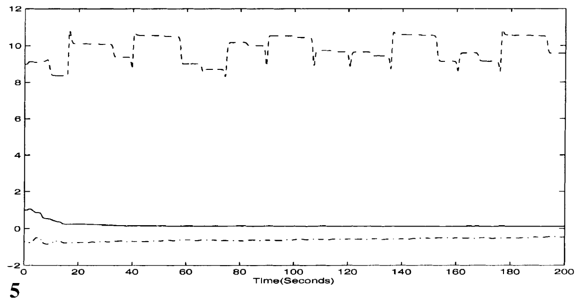
Fig. 3. Boundedness of the control u.

Fig. 4. Tracking error $x_1(t) - x_{d1}(t)$.

In the following simulation, the parameters of controller (27) and adaptive law (24) are chosen as $c_1 = 10$ and $\Gamma = \mathrm{diag}\{0.05\}$. The initial conditions are chosen that $x_1(0) = 2$, $x_{d1}(0) = 0.5$, $x_{d2}(0) = 0.4$ and $x_{d3}(0) = 0.1$.

Numerical simulation results are shown in Figs. 1–3. As shown in Fig. 1, state $x_1(t)$ of the slave system asymptotically synchronize with state $x_{d1}(t)$ of the master system (1), while at the same time the parameter estimate $\hat{\theta}_1$ and the control u remain bounded as shown in Figs. 2 and 3, respectively.

For case (ii), the slave system is chosen as the same as the master system except that the system parameters are $\theta_1 = 0.20$, $\theta_2 = 0.20$ and $\theta_3 = 10$. In the following simulation, the parameters of controller (27) and adaptive law (24) are chosen as $c_1 = 2, c_2 = 2, c_3 = 2$ and $\Gamma = \mathrm{diag}\{0.001, 0.1, 0.1\}$. The initial conditions are chosen as $x_1(0) = 7$, $x_2(0) = 3$, $x_3(0) = 0.4$, $x_{d1}(0) = 8$, $x_{d2}(0) = 3$ and $x_{d3}(0) = 0.4$.



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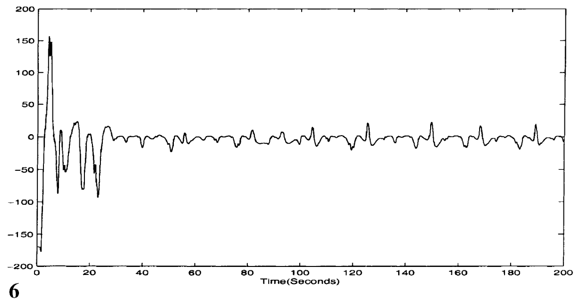

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Fig. 5. Boundedness of the parameter estimates: $\hat{\theta}_1$ (solid line), $\hat{\theta}_2$ (dash–dot line) and $\hat{\theta}_3$ (dashed line).

Fig. 6. Boundedness of the control u.

Numerical simulation results are shown in Figs. 4–6. As shown in Fig. 4, state $x_1(t)$ of the slave system asymptotically synchronize with state $x_{d1}(t)$ of the master system (1). It can be shown that, at the same time, states $x_2(t)$ and $x_3(t)$ of the slave system, the parameter estimates $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$ and the control u remain bounded. The boundedness of parameter estimates and control signal u are shown in Figs. 5 and 6, respectively.

5. Conclusion

An approach for adaptive synchronization of uncertain chaotic systems using backstepping with tuning functions design method has been presented in this paper. These results demonstrate the fruitfulness of modern nonlinear and adaptive control theory for applications to chaos synchronization and communications. The proposed approach offers a systematic design procedure for the adaptive synchronization of most of chaotic systems in the chaos research literature, which implies that much complicated high-order chaotic systems can be employed to improve the security in chaos communications.

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