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HARD DISK DRIVES CONTROL IN MOBILE APPLICATIONS*

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Abstract In this paper, control design is investigated for hard disk drives in mobile applications with unknown external arbitrarily fast time-varying disturbances. The disturbances can be estimated with exponential accuracy using the proposed disturbance observer based on a series of integral filters. The position error signal will converge to zero with the proposed control technique for systems subjected to the unknown disturbances. The effectiveness of the proposed method is demonstrated by extensive simulation studies.

Key words Hard disk drives, time-varying disturbances.

1 Introduction

Recently, hard disk drives (HDDs) have been installed in many forms of mobile electronic devices, including mobile phones, portable media players and laptops among others. In these applications, the challenge is how to improve the HDD performance in the ever presence of external shocks or disturbances as compared to conventional applications. Since these external disturbances can increase the position error signal (PES) of the read/write (R/W) head, and even damage the data stored on the disks, it is important to compensate for them.

At present, shock sensors are commonly used in many HDD products to detect external shock vibrations or disturbances so as to avoid the malfunction of writing and reading in case of severe shock[1]. In practice, accelerometers are frequently selected to measure acceleration signal to inject to the feedforward controller for disturbance rejection. Disturbance rejection control is presented based on the identification of transfer function from the accelerometer[2]. In [3], adaptive feedforward control was investigated using a finite impulse response (FIR) filter to model the dynamics between the accelerometer and the PES. Vibration rejection control was studied for hard disk dives using acceleration feedforward control where accelerometer phase delay is also considered[4]. In these works, the disturbances are assumed to be constant or slow time-varying. When the disturbances are fast time-varying, the performances of the existing control methods are unsatisfactory.

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Sliding mode control is known to be robust to parametric uncertainties and disturbances. Several representative works on the topic are available in the literature. Among them continuous-time sliding mode control was designed and applied for a special class of HDDs where accurate measurement of positions are available[5]. In [6], adaptive sliding mode control was developed for time optimal in seeking and tracking of the read/write heads. In addition, sliding mode combined with learning control was also proposed for track-following in HDDs[7]. Discrete-time sliding mode control was designed for settling and track-following so as to force the heads to slide on a hyperplane[8]. Nevertheless, sliding mode control is a discontinuous control action. Switching in the sliding mode control is well known to deliver robustness which, however, comes with control chattering that may not be desirable in HDDs.

Motivated by the work in [9], where a novel disturbance observer based on integral filters was proposed for disturbance rejection in servo mechanisms, in this paper, we investigate control design for HDDs in mobile applications with arbitrarily fast time-varying disturbances, where the difficulty lies in both the desired trajectory and the actual positional signals are not available for feedback design. The main contributions of this paper are as follows:

i) control design for HDDs in mobile applications through disturbance rejection;
ii) exponential observer design for arbitrarily fast time-varying disturbances in HDDs for mobile applications; and
iii) discussions on the differences in available signals between HDDs and the standard servo systems.

The paper is organized as follows. In Section 2, we describe the hard disk drive system and its dynamics under study. Section 3 presents the control system design for HDDs in mobile applications with arbitrarily fast time-varying disturbances, and detailed stability analysis. Finally, simulation studies are shown in Section 4 to show the effectiveness of the proposal approach, followed by the conclusion in Section 5.

2 Problem Formulation

The overview of a HDD is shown in Figure 1. The data are accessed via the electro-mechanical voice-coil-motor (VCM) actuator. The high-performance servo system is needed to place the acturator tip, holding a R/W head, over one of the circular concentric data tracks written on the rotating disks. The position error signal (PES) represents the displacement of the R/W head from the desired track location.

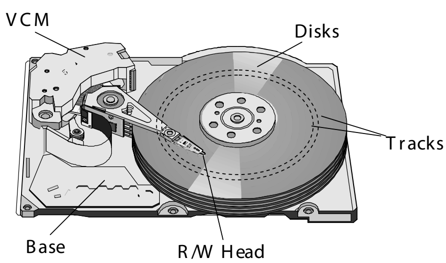

Figure 1 Overview of hard disk drives

Consider the double integrator model representing the VCM-actuator as follows[5,10]:

$$m\ddot{y} + f(y, \dot{y}) = u + d(t), \tag{1}$$

where the scalar y, $\dot{y}$ and $\ddot{y}$ are the position, velocity, and acceleration of the VCM-actuator tip respectively; m is the system inertia/mass; while the function $f(y, \dot{y})$ represents bias-forces due to static pivot, bearing friction and VCM-flex-cable; u is the control input; and $d(t)$ is the external disturbance.

Remark 1 In standard servo control systems, it is common practice and understanding that the positional signals including its position y, velocity $\dot{y}$, and sometimes its acceleration $\ddot{y}$, are available for feedback control design. For tracking purpose, the desired trajectory y_d, its first and second derivatives $\dot{y}_d$ and $\ddot{y}_d$, are known bounded and continuous signals as well. As such, the tracking error $e = y - y_d$ is easily computable.

However, in hard disk drives, it is not the case anymore. We do not have access to both the position of R/W head y, and the track position (desired trajectory) y_d, which is some kind of signal (servo pattern) written on disk surface. The main objective of track following servo is to maintain the head on the track. Since we do not know the exact shape of the servo track, we can only demodulate a signal using a sensor to tell us the relative distance between head and track center, which is PES. As PES can be measured quite accurately, its derivative can be estimated quite well and is usually assumed to be known for control system design.

In an effort to show the difference between HDDs and standard servo control systems mathematically, let us denote PES as follows:

$$e = y - y_d, \tag{2}$$

though neither y nor y_d is available for HDDs.

Assumption 1 For HDDs in mobile applications, both the PES signal e, and its first order derivative $\dot{e}$, are available.

Assumption 2 The external disturbance $d(t)$ is bounded with an unknown bound.

Assumption 3 The track position y_d, and its first and second derivatives $\dot{y}_d$ and $\ddot{y}_d$, are bounded and continuous signals, though they are not available for feedback control in HDDs.

The control objective is to present a control strategy to reject the external arbitrarily fast time varying disturbance and to make the PES ideally at zero in the tracking mode.

3 Control Design

From equations (1) and (2), we can obtain the following tracking error dynamics

$$m\ddot{e} = u - f(y, \dot{y}) + d(t) - m\ddot{y}_d. \tag{3}$$

Remark 2 In the standard servo setting where signals y, $\dot{y}$, and $\ddot{y}_d$ are available, we can design the following ideal certainty equivalent control

$$u = -(k_p e + k_d \dot{e}) + f(y, \dot{y}) + m\ddot{y}_d - \widehat{\omega}(t), \tag{4}$$

where k_p and k_d are positive constants and $\widehat{\omega}(t)$ is the disturbance observer for $d(t)$. For simplicity of analysis, it is very common to assume that [10]

$$f(y, \dot{y}) = c_{1f}\dot{y} + c_{2f}y, \tag{5}$$

where constant c_{1f} and c_{2f} are the viscous and elastic friction coefficients.

However, for HDDs in mobile applications, both y and $\dot{y}$ are unknown, not to mention $f(y, \dot{y})$. As mentioned earlier, $\ddot{y}_d$ is not available in the HDDs system. As such, the term $f(y, \dot{y}) + m\ddot{y}_d$ cannot appear in the control even though both the forms of $f(y, \dot{y})$ and m are assumed to be known.

In contrast to standard servo control setting, consider the following control based on available signals e and $\dot{e}$ as:

$$u = -(k_p e + k_d \dot{e}) - \widehat{\omega}(t), \tag{6}$$

where $k_p > c_{2f}$ and $k_d > c_{1f} + \frac{1}{4}$. Substituting equation (6) into equation (3) leads to the following closed-loop dynamics

$$m\ddot{e} = -(k_p e + k_d \dot{e}) - \widehat{\omega}(t) - (c_{1f}\dot{y} + c_{2f}y) + d(t) - m\ddot{y}_d. \tag{7}$$

Adding and subtracting $c_{1f}\dot{y}_d + c_{2f}y_d$ on the right hand side of equation (7), we have

$$m\ddot{e} = -[(k_p - c_{2f})e + (k_d - c_{1f})\dot{e}] + \omega(t) - \widehat{\omega}(t), \tag{8}$$

where $\omega(t) = d(t) - m\ddot{y}_d - c_{1f}\dot{y}_d - c_{2f}y_d$. From Assumptions 2 and 3, we know that $\omega(t)$ is upper-bounded.

On examining equation (8), if we can design a disturbance observer such that $|\widehat{\omega}(t) - \omega(t)| \le \delta e^{-\sigma t}$, where δ and σ are positive constants, then the stability of the closed loop system (8) will be obtained easily. Consider the following differential equation:

$$\dot{\widehat{\omega}}(t) - \dot{\omega}(t) + \gamma(\widehat{\omega}(t) - \omega(t)) + e^{-\gamma t}\dot{\omega}(t) = 0, \tag{9}$$

where γ is a positive constant, its solution is $\widehat{\omega}(t) = (1 - e^{-\gamma t})\omega(t) + e^{-\gamma t}\omega(0)$. It follows that $\widehat{\omega}(t)$ converges to its true value $\omega(t)$ exponentially. However, since $\omega(t)$ and $\dot{\omega}(t)$ are not available, we cannot obtain $\widehat{\omega}(t)$ from (9) directly.

Lemma 1 *Considering the following integral filters,*

$$\dot{\eta}(t) = -\alpha\eta(t) + (\alpha - \beta)e^{-\beta t}\eta(0) + \int_0^t e^{-\alpha(t-s)}\omega(s)ds, \tag{10}$$

$$\dot{\widehat{\eta}}(t) = -\alpha\widehat{\eta}(t) + (\alpha - \beta)e^{-\beta t}\widehat{\eta}(0) + \int_0^t e^{-\alpha(t-s)}\widehat{\omega}(s)ds, \tag{11}$$

where α and β are positive constants, $\eta(0)$ and $\widehat{\eta}(0)$ are initial values, we have the following conclusions:

i) *The signal $\widehat{w}(t)$ can converge to it true value $w(t)$ exponentially, i.e.,*

$$|\tilde{\omega}(t)| \le \delta e^{-\sigma t}, \tag{12}$$

where $\widetilde{\omega}(t) = \widehat{\omega}(t) - \omega(t)$, δ is a positive constant and σ is a positive design parameter;

ii) *The signal $\widehat{w}(t)$ can be obtained from the following integral equation*

$$\int_0^t e^{\alpha s}\widehat{\omega}(s)ds = \alpha e^{\alpha s}\int_0^t e^{-\alpha s}\psi(s)ds + \psi(t), \tag{13}$$

where

$$\psi(t) = (\beta - \gamma)e^{(\alpha+\gamma-\beta)t}(\widehat{\eta}(0) - \eta(0)) + \beta e^{(\alpha-\beta)t}\eta(0) + (e^{\gamma t} - 1)\int_0^t e^{\alpha s}v(s)ds$$
$$+[\alpha + (\gamma - \alpha)e^{\gamma t}]\int_0^t \left[\theta(s) + \int_0^s e^{\alpha r}v(r)dr\right]ds + (e^{\gamma t} - 1)\theta(t) \tag{14}$$

are computable signals, with $v = (k_p - c_{2f})e + (k_d - c_{1f})\dot{e}$, $\theta(t) = me^{\alpha t}\dot{e}(t) - m\dot{e}(0) - m\alpha\int_0^t e^{\alpha s}\dot{e}(s)ds$, and constant $\gamma > 0$ is a design parameter.

See Appendix A for the proof of Lemma 1.

The stability of the closed-loop system is given in the following theorem.

Theorem 1 *Consider the closed-loop system consisting of system* (1) *satisfying Assumptions* 1–3*, controller* (6) *and observer* (13)*. The external disturbance can be rejected exponentially and the PES will converge to zero, i.e., the estimate,* $\widehat{\omega}(t)$*, globally exponentially converges to its true value,* $\omega(t)$*, and the tracking error* $e \to 0$ *and* $\dot{e} \to 0$*, as* $t \to \infty$*.*

Proof According to Lemma 1, we know that $\widetilde{w}(t)^2 \le \delta^2 e^{-2\sigma t}$. As $t \to \infty$, we have $\widetilde{\omega}(t)^2 \to 0$, i.e., the estimate $\widehat{\omega}(t)$ given in (13) globally exponentially converges to its true value $\omega(t)$. Consider the Lyapunov function candidate

$$V = \frac{1}{2}(k_p - c_{2f})e^2 + \frac{1}{2}m\dot{e}^2, \tag{15}$$

where $k_p > c_{2f}$. The time derivative of V along (8) is given

$$\begin{aligned}\dot{V} &= (k_p - c_{2f})e\dot{e} + m\dot{e}\ddot{e} = -(k_d - c_{1f})\dot{e}^2 + \dot{e}(\omega(t) - \widehat{\omega}(t)) \\ &\le -\left(k_d - c_{1f} - \frac{1}{4}\right)\dot{e}^2 + \widetilde{\omega}(t)^2 \le -k_0\dot{e}^2 - k_1\dot{e}^2 + \widetilde{w}(t)^2.\end{aligned} \tag{16}$$

where $k_d - c_{1f} - \frac{1}{4} = k_0 + k_1 > 0$ with $k_0 > 0$ and $k_1 > 0$. When $|\dot{e}| > \frac{|\widetilde{\omega}(t)|}{\sqrt{k_1}}$, we have $\dot{V} \le 0$. Therefore, we know $|\dot{e}| \le \frac{|\widetilde{\omega}(t)|}{\sqrt{k_1}}$. Noticing that $|\widetilde{\omega}(t)| \to 0$, as $t \to \infty$, we have $|\dot{e}| \to 0$, as $t \to \infty$, which, subsequently, leads to $\ddot{e} \to 0$, as $t \to \infty$, then from the closed-loop error equation (8), the conclusion that $e \to 0$, as $t \to \infty$, is achieved. ∎

4 Simulation Study

The hard disk drive under consideration is a Hitachi GST 1.8-inch hard disk drive with model number HTC426020G7AT00 and spindle motor rotational speed 4200 RPM as shown in Figure 2[11].

Figure 3 shows the frequency response of VCM positioner of the 1.8-inch disk drive[11]. By curve-fitting to the measured frequency response in Figure 3, we obtain the model plant G as follows

$$\begin{aligned}G &= G_P G_{R1} G_{R2}, \\ G_P &= \frac{5.527 \times 10^7}{s^2 + 502.7s + 3.948 \times 10^5},\end{aligned} \tag{17}$$

$$G_{R1} = \frac{0.954s^2 + 1031s + 6.964 \times 10^8}{s^2 + 1056s + 6.964 \times 10^8}, \tag{18}$$

$$G_{R2} = \frac{2.527 \times 10^9}{s^2 + 6032s + 2.527 \times 10^9}, \tag{19}$$

where G_p is the nominal plant without resonance modes, while G_{R1} and G_{R2} are the resonance modes at 4.2kHz and 8kHz respectively. In implementation, to deal with the above resonance modes, we use two corresponding notch filters at 4.2kHz and 8kHz respectively as follows

$$\begin{aligned}C_{N1} &= \frac{s^2 + 2639s + 6.964 \times 10^8}{s^2 + 5278s + 6.964 \times 10^8}, \\ C_{N2} &= \frac{s^2 + 1.005 \times 10^4 s + 2.527 \times 10^9}{s^2 + 5.027 \times 10^4 s + 2.527 \times 10^9}.\end{aligned} \tag{20}$$

The nominal plant (17) can be transformed to the form of model plant (1) with $m = 1.809\times10^{-8}$, $f(y,\dot{y}) = 9.095\times10^{-6}\dot{y} + 7.143\times10^{-3}y$.

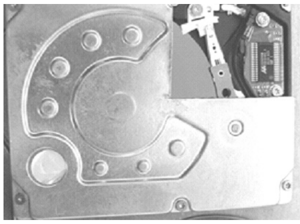

Figure 2 Hitachi GST 1.8" disk drive

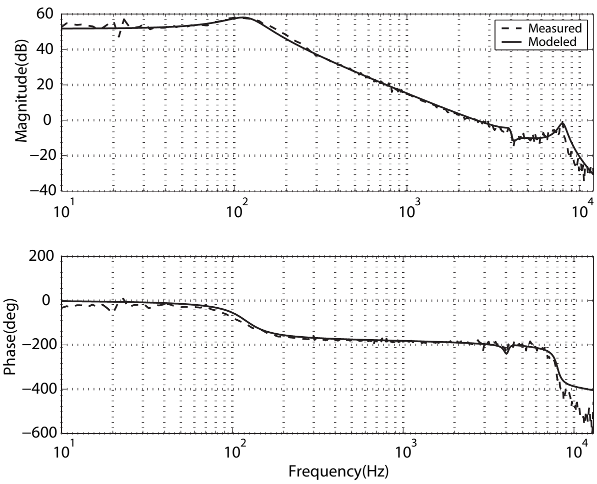

Figure 3 Frequency response of the VCM actuator

For the purposes of simulation, we assume that the external disturbances are acting vertically to the disk surface, which include repeatable runout (RRO) and nonrepeatable runout (NRRO), i.e., $d(t) = d_{\text{RRO}} + d_{\text{NRRO}}$. For RRO, we assume that it consists of a sum of sinusoids of known frequencies, i.e.,

$$d_{RRO} = [\sin(2\pi ft) + \sin(4\pi ft) + \sin(6\pi ft)] \times 10^{-7},$$

where $f = \frac{4200}{60} = 70$Hz. For NRRO, we assume it is a zero mean random noise, and dominates in low frequency range. Thus, the following low pass filter

$$D(s) = \frac{3.948\times10^7}{s^2 + 8886s + 3.948\times10^7} \tag{21}$$

is used to represent the transfer function from external NRRO to the actuator. For the controller u given in equation (6) in Section 3, the design parameters are chosen as $k_p = 20$ and $k_d = 0.005$. For the disturbance observer $\widehat{w}$, its parameters are chosen as $\alpha = 600$, $\beta = 500$, $\gamma = 1000$, and

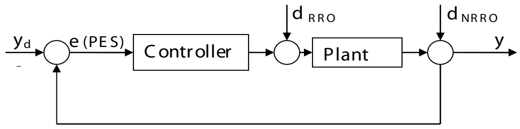

Figure 4 Overall servo loop of hard disk drives

$\lambda = 1.0$. The overall servo loop of hard disk drives is shown in Figure 4, where the control part includes $C = C_{N1}C_{N2}u$.

The simulation results are plotted in Figures 5–7. From Figure 5, we can see that, without disturbance observer $\widehat{w}$, the PES signal can converge to the neighborhood of zero, but still oscillates around zero. With disturbance observer, the PES signal can converge to zero in $0.0015s$. From Figure 7, we see that the estimate of the disturbance also exponentially converges to its true value in $0.0015s$.

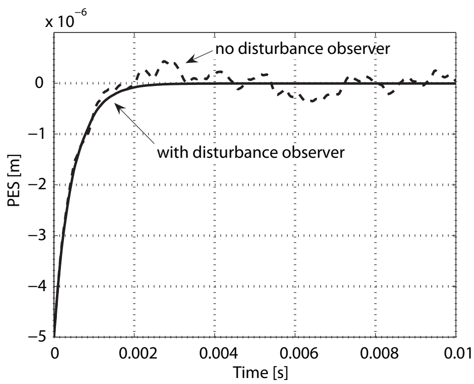

Figure 5 PES signal

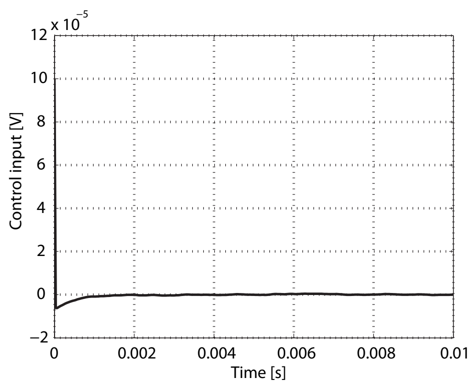

Figure 6 Control input signal with disturbance observer

5 Conclusion

In this paper, control design has been presented for hard disk drives in mobile application with external arbitrarily fast time-varying disturbances. The disturbances can be estimated with exponential accuracy using the proposed disturbance observer based on the series of integral filters. The position error signal converges to zero with the proposed control for systems subjected to the unknown disturbances. The effectiveness of the proposed method has been demonstrated by representative simulation cases.

Appendix A: Proof of Lemma 1

The lemma can be proved easily by following the materials in [9]. It is given here as a lemma for clarity and completeness.

i) Consider the following differential equation

$$\dot{\widehat{\eta}}(t) - \dot{\eta}(t) + \gamma(\widehat{\eta}(t) - \eta(t)) + e^{-\gamma t}\dot{\eta}(t) = 0. \tag{22}$$

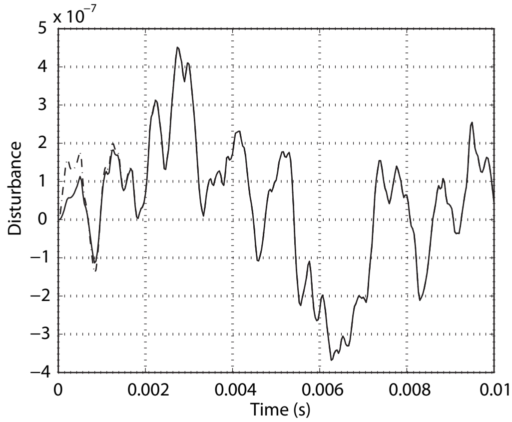

Figure 7 Disturbance signal (w: dash-dotted; $\widehat{w}$: solid)

Its solution is

$$\widehat{\eta}(t) = (1 - e^{-\gamma t})\eta(t) + e^{-\gamma t}\eta(0).$$

It means that $\widehat{\eta}(t)$ converges to $\eta(t)$ exponentially. From (10) and (11), we can obtain that

$$\eta(t) = e^{-\alpha t}\eta(0) + e^{-\alpha t}\int_0^t e^{\alpha s}\left[(\alpha-\beta)e^{-\beta s}\eta(0) + \int_0^s e^{-\alpha(s-r)}\omega(r)dr\right]ds, \tag{23}$$

$$\widehat{\eta}(t) = e^{-\alpha t \hat{\eta}(0)} + e^{-\alpha t}\int_0^t e^{\alpha s}\left[(\alpha-\beta)e^{-\beta s}\widehat{\eta}(0) + \int_0^s e^{-\alpha(s-r)}\widehat{\omega}(r)dr\right]ds, \tag{24}$$

Substituting (23) and (24) into (22) results in

$$\int_0^t e^{\alpha s}\widetilde{\omega}(s)ds + (\gamma-\alpha)\int_0^t\int_0^s e^{\alpha r}\widetilde{\omega}(r)drds = \phi(t), \tag{25}$$

where

$$\begin{aligned}\phi(t) = {} & (\beta-\gamma)e^{(\alpha-\beta)t}[\widehat{\eta}(0) - \eta(0)] + \beta e^{(\alpha-\beta-\gamma)t}\eta(0) - e^{-\gamma t}\int_0^t e^{\alpha s}\omega(s)ds \\ & + \alpha e^{-\gamma t}\int_0^t\int_0^s e^{\alpha r}\omega(r)drds. \end{aligned} \tag{26}$$

Define the following variable

$$\chi(t) = \int_0^t\int_0^s e^{\alpha r}\widetilde{\omega}(r)drds. \tag{27}$$

Differentiating both sides of (27) and combining (25), we obtain $\dot{\chi}(t) = -(\gamma-\alpha)\chi(t) + \phi(t)$. Its solution is

$$\chi(t) = e^{-(\gamma-\alpha)t}\int_0^t e^{(\gamma-\alpha)s}\phi(s)ds. \tag{28}$$

Differentiating both sides of (28) twice, we have

$$\ddot{\chi}(t) = (\gamma-\alpha)^2 e^{-(\gamma-\alpha)t}\int_0^t e^{(\gamma-\alpha)s}\phi(s)ds - (\gamma-\alpha)\phi(t) + \dot{\phi}(t), \tag{29}$$

where

$$\begin{aligned}\dot{\phi}(t) &= (\alpha-\beta)(\beta-\gamma)e^{(\alpha-\beta)t}[\widehat{\eta}(0)-\eta(0)] + \beta(\alpha-\beta-\gamma)e^{(\alpha-\beta-\gamma)t}\eta(0) - e^{(\alpha-\gamma)t}\omega(t)\\ &+(\alpha+\gamma)e^{-\gamma t}\int_0^t e^{\alpha s}\omega(s)ds - \gamma\alpha e^{-\gamma t}\int_0^t\int_0^s e^{\alpha r}\omega(r)drds. \end{aligned}\tag{30}$$

Differentiating both sides of (27) twice, we have

$$\ddot{\chi}(t) = e^{\alpha t}\widetilde{\omega}(t). \tag{31}$$

Combining (29) and (31), we have

$$\widetilde{\omega}(t) = (\gamma-\alpha)^2 e^{-\gamma t}\int_0^t e^{(\gamma-\alpha)s}\phi(s)ds - e^{-\alpha t}(\gamma-\alpha)\phi(t) + e^{-\alpha t}\dot{\phi}(t). \tag{32}$$

Substituting (26) and (30) into (32), and since $w(t)$ is upper-bounded, we have

$$\widetilde{w}(t) = c_1 e^{-\beta t} + c_2 e^{-\gamma t} + c_3 e^{-(\beta+\gamma)t},$$

where c_1, c_2 and c_3 are constants. Obviously, we can see that there exist positive constants δ and σ such that $|\widetilde{w}(t)| \le \delta e^{\sigma t}$ with $\sigma = \min(\beta,\gamma)$.

ii) From (8), we have

$$\omega(t) = m\ddot{e} + v + \widehat{\omega}(t). \tag{33}$$

Define

$$\int_0^t e^{\alpha s} m\ddot{e}(s)ds = \theta(t).$$

Then, we have

$$\theta(t) = me^{\alpha t}\dot{e}(t) - m\dot{e}(0) - m\alpha\int_0^t e^{\alpha s}\dot{e}(s)ds.$$

Therefore, we do not need $\ddot{e}(t)$ signal. Substituting (33) into (25), we have

$$\int_0^t e^{\alpha s}\widehat{\omega}(s)ds - \alpha\int_0^t\int_0^s e^{\alpha s}\widehat{\omega}(r)drds = \psi(t), \tag{34}$$

where

$$\begin{aligned}\psi(t) &= (\beta-\gamma)e^{(\alpha+\gamma-\beta)t}(\widehat{\eta}(0)-\eta(0)) + \beta e^{(\alpha-\beta)t}\eta(0) + (e^{\gamma t}-1)\int_0^t e^{\alpha s}v(s)ds\\ &+[\alpha+(\gamma-\alpha)e^{\gamma t}]\int_0^t\left[\theta(s)+\int_0^s e^{\alpha r}v(r)dr\right]ds + (e^{\gamma t}-1)\theta(t).\end{aligned}$$

To simplify (34), we define the following signal

$$\zeta = \int_0^t\int_0^s e^{\alpha s}\widehat{\omega}(r)drds. \tag{35}$$

Its first derivative is

$$\dot{\zeta} = \int_0^t e^{\alpha s}\widehat{\omega}(s)ds. \tag{36}$$

Substituting (35) and (36) into (34), we have

$$\dot{\zeta} - \alpha\zeta = \psi. \tag{37}$$

Its solution is

$$\zeta = e^{\alpha s}\int_0^t e^{-\alpha s}\psi ds. \tag{38}$$

Substituting (38) to (37), we have

$$\int_0^t e^{\alpha s}\widehat{\omega}(s)ds = \alpha e^{\alpha t}\int_0^t e^{-\alpha s}\psi(s)ds + \psi(t). \tag{39}$$

As such, we can numerically solve $\widehat{w}$ from (39) using existing integral equation solving methods in [12, 13]. The proof is completed.

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