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# Neural-based direct adaptive control for a class of general nonlinear systems

T. ZHANG†, S. S. GE† and C. C. HANG†

*A direct adaptive controller based on high-order neural networks (HONNs) is presented to solve the tracking control problem for a general class of unknown nonlinear systems. The plant is assumed to be a feedback linearizable and minimum-phase system. Firstly, an ideal implicit feedback linearization control (IFLC) is established using implicit function theory. Then a HONN is applied to construct this IFLC to realize approximate linearization. The proposed controller ensures that the closed-loop system is Lyapunov stable and that the tracking error converges to a small neighbourhood of the origin. The requirements of an off-line training phase and the persistent excitation condition are eliminated. Simulation results verify the effectiveness of the proposed controller and the theoretical discussion.*

## 1. Introduction

The control problem of nonlinear systems has become a topic of considerable research importance. For affine nonlinear system control, many remarkable results have been obtained using feedback linearization methods (Isidori 1989, Behtash 1990). To relax some of the exact model-matching restrictions, several adaptive schemes have been introduced to solve the problem of parametric uncertainties (Kanellakopoulos *et al.* 1991, Teel *et al.* 1991, Marino and Tomei, 1995). However, for unknown general nonlinear systems there is no effective method to design adaptive controllers that can guarantee good tracking performance and the closed-loop stability.

Recently, active research has been carried out in neural network (NN) control for nonlinear dynamic systems. The main properties of neural networks for control applications can be summarized as follows; they can approximate any continuous mapping with any accuracy; the approximation capability is achieved through learning and parallel processing; the fault tolerance is easily accomplished. These good properties suggest that they are good candidates for implementing real-time adaptive controllers for nonlinear systems. Narendra and Parthasarthy (1990), Jin *et al.* (1993) and Chen and

Liu (1994) have shown that to guarantee stable and efficient on-line control using the backpropagation (BP) learning algorithm, the identification must be sufficiently accurate before the control action is initiated. To cope with this problem, several stable adaptive NN controller design approaches have been proposed based on Lyapunov theory to guarantee the stability of closed-loop system (Sanner and Slotine 1992, Lewis *et al.* 1996, Spooner and Passino 1996, Polycarpou 1996, Ge and Hang 1996, Ge 1996), which can guarantee the stability of systems. These methods, however, can be only applied to affine nonlinear systems.

For the control problem of general nonlinear systems, several researchers (Psaltis *et al.* 1988, Goh 1994, Levin and Narendra 1996) have suggested using neural networks as emulators of inverse systems. The main idea is that if the controller is an inverse operator of a nonlinear plant, then the reference input to the controller will produce a control input to the plant, which will in turn produce an output identical to the reference input. Based on the implicit function theory, the NN controllers have been studied to emulate the inverse controller in these references. Generally speaking, they are not adaptive controllers, because all these NN controllers need off-line training. The direct adaptive NN control problem for non-affine nonlinear systems has been studied by Goh and Lee (1994). A strong assumption is made there that the system states are required to be uniformly persistently exciting, which is difficult to be checked/guaranteed in a practical application.

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High-order networks are expansions of the first-order Hopfield (1984) and Cohen-Grossberg (1983) models that allow higher-order interactions between neurons. The strong storage capacity and approximation capability of high-order neural networks have been shown by Paretto and Niez (1986) and Kosmatopoulos *et al.* (1995). In this paper we develop a direct adaptive controller based on HONNs for a general class of unknown nonlinear systems. The paper is organized as follows: §2 describes the class of nonlinear systems under discussion and input–output linearization of the system; §3 gives the approximation of HONNs and the control structure. A robust adaptive law and the stability analysis of the closed-loop system are discussed in §4. The effectiveness of the proposed controller is illustrated via the simulation results in §5. The conclusion is given in §6.

## 2. Linearizing feedback control

### 2.1. Problem statement

Consider a SISO nonlinear system described by a general form

$$\left. \begin{aligned} \dot{x} &= f(x, u), \\ y &= h(x), \end{aligned} \right\} \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}$  and  $y \in \mathbb{R}$  are the state variables, control input and output, respectively. The mapping  $f(\cdot, \cdot) : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$  is an unknown smooth vector field and the function  $h(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}$  is an unknown smooth function.

The control objective can be described as follows: given a desired trajectory  $y_d(t)$  find a control  $u$  such that the system's output tracks the desired trajectory with an acceptable accuracy, while all states and control remain bounded. In the case of affine nonlinear systems, e.g.

$$\left. \begin{aligned} \dot{x} &= f_1(x) + g_1(x)u, \\ y &= h(x), \end{aligned} \right\} \quad (2)$$

with  $f_1(x)$  and  $g_1(x)$  smooth vector fields, it is clear that the problem can be solved if there exists a control of the form

$$u = \alpha_1(x) + \alpha_2(x)w, \quad (3)$$

that results in a linear map from  $w$  to  $y$  for the affine system (2). The existence of such a linearizing feedback is in turn guaranteed if (2) possesses a so-called 'relative degree' from  $u$  to  $y$  (Isidori 1989). Nevertheless, if (1) is not in affine form, it is not easy to find such an explicit linearizing feedback to achieve feedback linearization. We next define the 'relative degree' introduced by Tsinias and Kalouptsidis (1983) and establish an implicit feedback linearization control  $u = \alpha(x, w)$  to

obtain a linear map from  $w$  to  $y$  for the general nonlinear system (1).

Let  $L_f h$  denote the Lie derivative of the function  $h(x)$  with respect to the vector field  $f(x, u)$

$$L_f h = \frac{\partial[h(x)]}{\partial x} f(x, u).$$

Higher-order Lie derivatives can be defined recursively as  $L_f^k h = L_f(L_f^{k-1} h)$ ,  $k > 1$ . Let  $U = \Omega_x \times \Omega_u$  with  $x \in \Omega_x \subset \mathbb{R}^n$  and  $u \in \Omega_u \subset \mathbb{R}$  being compact sets. The system (1) is said to have strong relative degree  $\rho$  in  $U$  if there exists a positive integer  $1 \leq \rho < \infty$  such that

$$\frac{\partial[L_f^i h]}{\partial u} = 0, \quad i = 0, 1, \dots, \rho - 1, \quad \frac{\partial[L_f^\rho h]}{\partial u} \neq 0, \quad (4)$$

for all  $(x, u) \in U$  (Tsinias and Kalouptsidis 1983).

**Assumption 1:** *The system (1) possesses a strong relative degree  $\rho \leq n$  for all  $(x, u) \in U$ .*

Define

$$\phi_j(x) = L_f^{j-1} h, \quad j = 1, 2, \dots, \rho.$$

Under Assumption 1, it was shown by Isidori (1989) that there exists other  $n - \rho$  functions  $\phi_{\rho+1}(x), \dots, \phi_n(x)$  such that the mapping

$$\Phi(x) = [\phi_1(x), \phi_2(x), \dots, \phi_n(x)]^T, \quad (5)$$

has a jacobian matrix which is non-singular in  $\Omega_x$ . Therefore  $\Phi(x)$  is a diffeomorphism on  $\Omega_x$ . By setting

$$\xi = [\phi_1(x), \dots, \phi_\rho(x)]^T,$$

$$\eta = [\phi_{\rho+1}(x), \dots, \phi_n(x)]^T,$$

(1) can be transformed into a normal form in the new coordinate,  $[\xi^T, \eta^T]^T = \Phi(x)$ , as follows:

$$\left. \begin{aligned} \dot{\xi}_i &= \xi_{i+1}, \quad i = 1, \dots, \rho - 1, \\ \dot{\xi}_\rho &= b(\xi, \eta, u), \\ \dot{\eta} &= q(\xi, \eta, u), \\ y &= \xi_1, \end{aligned} \right\} \quad (6)$$

where

$$b(\xi, \eta, u) = L_f^\rho h,$$

$$q(\xi, \eta, u) = [q_1(\xi, \eta, u), q_2(\xi, \eta, u), \dots, q_{n-\rho}(\xi, \eta, u)]^T,$$

$$q_i(\xi, \eta, u) = L_f \phi_{\rho+i}(x), \quad i = 1, 2, \dots, n - \rho,$$

$$x = \Phi^{-1}(\xi, \eta).$$

Define the domain of the normal form (6) as

$$\bar{U} = \{(\xi, \eta, u) | (\xi, \eta) \in \Phi(\Omega_x); u \in \Omega_u\}.$$

Let  $b_u = \partial[b(\xi, \eta, u)]/\partial u$  and  $\dot{b}_u = d(b_u)/dt$ . According to Assumption 1, we know that  $\partial[b(\xi, \eta, u)]/\partial u \neq 0$  for all  $(\xi, \eta, u) \in \bar{U}$ , this implies that the smooth function  $b_u$  is strictly either positive or negative for all  $(\xi, \eta, u) \in \bar{U}$ .

From now on, without losing generality, we shall assume that the sign of  $b_u$  is positive.

**Assumption 2:** *There exist positive constants  $b_0, b_1$  and  $b_2$  such that  $b_0 \leq b_u \leq b_1$  and  $|b_u| \leq b_2$  for all  $(\xi, \eta, u) \in \bar{U}$ .*

As  $b_u$  is defined as  $\partial[b(\xi, \eta, u)]/\partial u$ , it can be viewed as the control gain of the plant. The requirement for  $b_u \geq b_0$  means that the control gain of the system is larger than a positive constant. Most feedback linearization methods (Isidori 1989, Teel *et al.* 1991, Marino and Tomei 1995) need this assumption. We also require  $b_u$  and  $|b_u|$  to be bounded by positive constants. In practical applications, this does not pose a strong restriction on the class of systems. The reason is that if the controller is continuous, the situations in which a finite input causes an infinitely large effect on the system rarely happen in physical systems because of the smoothness of  $b_u$ .

If (1) is controlled by the feedback control  $u = \beta(\xi, \eta)$ , such that

$$\dot{\xi}_\rho = b(\xi, \eta, \beta(\xi, \eta)) = 0,$$

the subsystem

$$\dot{\eta} = q(0, \eta, \beta(0, \eta)),$$

is then called the zero dynamics of (1).

**Definition:** The system (1) is said to be a *hyperbolically minimum-phase system* if its corresponding zero dynamics are exponentially stable (Behtash 1990).  $\square$

**Assumption 3:** *The system (1) is hyperbolically minimum-phase. In addition, if the control input  $u$  is designed as a function of state  $x$ , the function  $q(\xi, \eta, u)$  is Lipschitz in  $\xi$  uniformly, i.e. there exists a Lipschitz constant  $L_\xi$  for  $q(\xi, \eta, u)$  such that*

$$\|q(\xi, \eta, u) - q(0, \eta, u_0)\| \leq L_\xi \|\xi\|, \quad \forall (\xi, \eta, u) \in \bar{U}, \quad (7)$$

where  $u_0 = \beta(0, \eta)$ .

Under Assumption 3 by the converse theorem of Lyapunov (Hahn 1967), there exists a Lyapunov function  $V_0(\eta)$  that satisfies

$$\sigma_2 \|\eta\|^2 \leq V_0(\eta) \leq \sigma_1 \|\eta\|^2, \quad (8)$$

$$\frac{\partial V_0}{\partial \eta} q(0, \eta, u_0) \leq -\lambda_a \|\eta\|^2, \quad (9)$$

$$\left\| \frac{\partial V_0}{\partial \eta} \right\| \leq \lambda_b \|\eta\|, \quad (10)$$

where  $\sigma_1, \sigma_2, \lambda_a$  and  $\lambda_b$  are positive constants.

Define the vectors  $\xi_d$  and  $\tilde{\xi}$  as

$$\left. \begin{aligned} \xi_d &= [y_d, \dot{y}_d, \dots, y_d^{(\rho-1)}]^T, & \xi_d &\in \mathbb{R}^\rho, \\ \tilde{\xi}_d &= [\xi_d^T, y_d^{(\rho)}]^T, & \tilde{\xi}_d &\in \mathbb{R}^{\rho+1}, \\ \tilde{\xi} &= [\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_\rho]^T = \xi - \xi_d, \end{aligned} \right\} \quad (11)$$

and a filtered tracking error as

$$e_s = [\Lambda^T \quad 1] \tilde{\xi}, \quad (12)$$

where  $\Lambda = [\lambda_1, \lambda_2, \dots, \lambda_{\rho-1}]^T$  is chosen such that the polynomial

$$s^{\rho-1} + \lambda_{\rho-1} s^{\rho-2} + \dots + \lambda_1,$$

is Hurwitz. With this choice, we have  $\tilde{\xi}(t) \rightarrow 0$  exponentially as  $e_s \rightarrow 0$ .

**Assumption 4:** *The desired trajectory vector  $\bar{\xi}_d$  is continuous and available, and  $\|\bar{\xi}_d\| \leq c$ , with  $c$  a known bound.*

**Lemma 1:** *For the zero dynamics of (1), if Assumptions 3 and 4 are satisfied, then there exist positive constants  $L_a, L_b$  and  $T_0$ , such that*

$$\|\eta\| \leq L_a |e_s| + L_b \|\xi_d\|, \quad \forall t > T_0. \quad (13)$$

**Proof:** According to Assumption 3, there exists a Lyapunov function  $V_0(\eta)$ . Differentiating  $V_0(\eta)$  along (6) yields

$$\begin{aligned} \dot{V}_0(\eta) &= \frac{\partial V_0}{\partial \eta} q(\xi, \eta, u) \\ &= \frac{\partial V_0}{\partial \eta} q(0, \eta, u_0) + \frac{\partial V_0}{\partial \eta} [q(\xi, \eta, u) - q(0, \eta, u_0)]. \end{aligned} \quad (14)$$

By using (7)–(10) in (14), we have

$$\dot{V}_0(\eta) \leq -\lambda_a \|\eta\|^2 + \lambda_b L_\xi \|\eta\| \|\xi\|. \quad (15)$$

By considering (11), (12) and Assumption 4, we can derive

$$\|\xi\| \leq d_1 \|\xi_d\| + d_2 |e_s|, \quad (16)$$

with  $d_1$  and  $d_2$  positive constants, Thus

$$\dot{V}_0(\eta) \leq -\lambda_a \|\eta\|^2 + \lambda_b L_\xi \|\eta\| (d_1 \|\xi_d\| + d_2 |e_s|).$$

Therefore,  $\dot{V}_0(\eta) \leq 0$ , whenever

$$\|\eta\| \geq \frac{\lambda_b L_\xi}{\lambda_a} (d_1 \|\xi_d\| + d_2 |e_s|). \quad (17)$$

By letting  $L_a = \lambda_b L_\xi d_2 / \lambda_a$  and  $L_b = \lambda_b L_\xi d_1 / \lambda_a$ , it can be seen that there exists a positive constant  $T_0$  such that (13) holds for all  $t > T_0$ .  $\square$

## 2.2. Ideal implicit feedback-linearization control

From (6), the time derivative of the filtered tracking error can be written as

$$\dot{e}_s = b(\xi, \eta, u) - y_d^{(\rho)} + [0 \quad \Lambda^T] \tilde{\xi}. \quad (18)$$

We have the following lemma to establish the existence of an ideal IFLC input  $u^*$ , which can linearize (1) and bring the output of the system to the desired trajectory  $y_d(t)$ .

**Lemma 2:** *Consider (1) satisfying Assumptions 1–4. For each positive constant  $k_v$ , there exist a subset  $\Phi_0 \subset \Phi(\Omega_x)$  and a unique ideal IFLC input  $u^*$  in the compact set  $\Omega_u$*

such that for all  $(\xi(0), \eta(0)) \in \Phi_0$ , the error equation (18) can be written in a linear form

$$\dot{e}_s = -k_v e_s. \quad (19)$$

Subsequently, (19) leads to  $\lim_{t \rightarrow \infty} |y(t) - y_d(t)| = 0$ .

**Proof:** By adding and subtracting  $k_v e_s$  to the right-hand side of the error equation (18), we have

$$\dot{e}_s = b(\xi, \eta, u) + \nu - k_v e_s, \quad (20)$$

where  $\nu$  is defined as

$$\nu = k_v e_s - y_d^{(\rho)} + [0 \quad \Lambda^T] \tilde{\xi}. \quad (21)$$

By considering (11) and (21), we have  $\partial \nu / \partial u = 0$ . From Assumption 1,  $\partial [b(\xi, \eta, u)] / \partial u \neq 0$  holds for all  $(\xi, \eta, u) \in \bar{U}$ . Thus

$$\frac{\partial [b(\xi, \eta, u) + \nu]}{\partial u} \neq 0, \quad \forall (\xi, \eta, u) \in \bar{U}.$$

Using the implicit function theorem (Lang 1983), for each desired trajectory  $y_d(t)$  that satisfies Assumption 4 there exist a subset  $\Phi_0 \subset \Phi(\Omega_x)$  and a unique local solution  $u = g_u(\xi, \eta, \nu) \in \Omega_u$  such that  $b(\xi, \eta, g_u(\xi, \eta, \nu)) + \nu = 0$  holds for all  $(\xi(0), \eta(0)) \in \Phi_0$ . As  $[\xi^T, \eta^T]^T = \Phi(x)$ , there exists a function  $g^*(x, \nu)$  such that  $g^*(x, \nu) = g_u(\xi, \eta, \nu)$ . Therefore, if we choose the ideal IFLC input as

$$u^*(z) = g^*(x, \nu), \quad z = [x^T, \nu]^T \in \Omega_z \subset \mathbb{R}^{n+1}, \quad (22)$$

then

$$b(\xi, \eta, u^*) + \nu = 0. \quad (23)$$

The compact set

$$\Omega_z = \{(x, \nu) \mid x \in \Omega_x; \nu = k_v e_s - y_d^{(\rho)} + [0 \quad \Lambda^T] \tilde{\xi}; \|\tilde{\xi}_d\| \leq c\}.$$

Note that the implicit function theorem is only a local result. However, if the strong relative degree is well defined on  $U$ , all the local implicit functions can be strung together to form a global (valid globally in  $U$ ) implicit function. Thus under the action of  $u^*$ , (20) and (23) imply that (19) holds. As  $k_v > 0$ , (19) is asymptotically stable, i.e.  $\lim_{t \rightarrow \infty} |e_s| = 0$ , which leads to  $\lim_{t \rightarrow \infty} |y(t) - y_d(t)| = 0$ .  $\square$

In the case of an affine nonlinear system, e.g.

$$b(\xi, \eta, u) = b_1(\xi, \eta) + b_2(\xi, \eta)u,$$

it is easy to find an explicit input

$$u = b_2^{-1}(\xi, \eta)[-b_1(\xi, \eta) + w],$$

for feedback linearization when  $b_2(\xi, \eta) \neq 0$ . However, for the general nonlinear function  $b(\xi, \eta, u)$ , the above lemma only suggests the existence of an ideal IFLC input  $u^*$ , it does not provide the methods to construct it. In the next section a high-order neural network is

applied to construct it for approximate feedback linearization.

### 3. High-order neural networks and control structure

The structure of a HONN can be expressed as

$$\left. \begin{aligned} q(W, z) &= W^T S(z), \quad W \text{ and } S(z) \in \mathbb{R}^l, \\ S(z) &= [s_1(z), s_2(z), \dots, s_l(z)]^T, \end{aligned} \right\} \quad (24)$$

$$s_i(z) = \prod_{j \in I_k} [s(z_j)]^{d_j(k)}, \quad i = 1, 2, \dots, l \quad (25)$$

where  $z \in \Omega_z \subset \mathbb{R}^{n+1}$ , the integer  $l$  is the number of NN nodes,  $\{I_1, I_2, \dots, I_l\}$  is a collection of  $l$  unordered subsets of  $\{1, 2, \dots, n+1\}$  and  $d_j(k)$  are non-negative integers,  $W$  is the adjustable synaptic weight vector. The activation function  $s(z_j)$  is a monotone increasing and differentiable sigmoidal function. It can be chosen as a logistic function, a hyperbolic tangent function, or others. In this research  $s(z_j)$  is chosen as a hyperbolic tangent function

$$s(z_j) = \frac{e^{z_j} - e^{-z_j}}{e^{z_j} + e^{-z_j}}. \quad (26)$$

It is shown in the literature (Paretto and Niez 1986, Kosmatopoulos *et al.* 1995) that the neural network  $W^T S(z)$  satisfies the conditions of the Stone-Weierstrass Theorem and can therefore approximate any continuous function over a compact set. Because the ideal IFLC input  $u^*(z)$  defined in (22) is a continuous function, if the number of HONN nodes  $l$  is large enough (i.e. the number of NN higher-order terms is large enough), then there exists an ideal weight  $W^*$  so that the ideal input  $u^*(z)$  can be approximated by the high-order neural networks  $W^{*T} S(z)$  to any degree of accuracy on the compact set  $\Omega_z$ :

$$u^*(z) = W^{*T} S(z) + \epsilon_u(z), \quad W^* \text{ and } S(z) \in \mathbb{R}^l, \quad (27)$$

where  $\epsilon_u(z)$  is called the NN approximation error. The ideal weight  $W^*$  is defined as

$$W^* := \arg \min_{W \in \mathbb{R}^l} \left\{ \sup_{z \in \Omega_z} |W^T S(z) - u^*(z)| \right\}, \quad \Omega_z \subset \mathbb{R}^{n+1}.$$

**Assumption 5:** On the compact set  $\Omega_z$ , the ideal neural network weight  $W^*$  and the approximation error are bounded by

$$\|W^*\| \leq w_m, \quad |\epsilon_u(z)| \leq \bar{\epsilon}_u, \quad (28)$$

with  $w_m$  and  $\bar{\epsilon}_u$  positive constants.

Let the HONN controller take the form

$$u = \hat{W}^T S(z) - k_s |e_s| e_s, \quad (29)$$

where  $\hat{W}$  is the estimate of the HONN weight  $W^*$ . The HONN emulator  $\hat{W}^T S(z)$  in the control law (29) is used

to approximate the ideal IFLC input  $u^*$  for feedback linearization, whereas  $k_s|e_s|e_s$ , with  $k_s$  a positive constant in (29), is a bounding control term, which is used to guarantee the boundedness of the system states. The proposed NN control structure is shown in Fig. 1. Define the weight estimation error as

$$\tilde{W} = \hat{W} - W^* \tag{30}$$

Because the function  $b(\xi, \eta, u)$  in (18) is implicit with respect to the input  $u$ , it is difficult to discuss it directly. The following lemma is needed for further analysis.

**Lemma 3—Mean value theorem:** Assume that  $\bar{f}(\omega) : \mathbb{R}^n \rightarrow \mathbb{R}$  is differentiable at each point of an open set  $U_\omega \subset \mathbb{R}^n$ . Let  $\omega_a$  and  $\omega_b$  be two points of  $U_\omega$  such that  $\omega_\lambda = \lambda\omega_a + (1 - \lambda)\omega_b$  belongs to  $U_\omega$  for all  $0 < \lambda < 1$ . Then there exists  $\lambda$  such that

$$\bar{f}(\omega_a) - \bar{f}(\omega_b) = \frac{\partial \bar{f}}{\partial \omega} \Big|_{\omega=\omega_\lambda} (\omega_a - \omega_b). \tag{31}$$

The proof of Lemma 3 can be found in Apostol (1957).

By using Lemma 3 there exists  $\lambda$ ,  $0 < \lambda < 1$ , such that

$$b(\xi, \eta, u) = b(\xi, \eta, u^*) + b_{u_\lambda}(u - u^*), \tag{32}$$

where  $b_{u_\lambda} = \partial[b(\xi, \eta, \bar{u})]/\partial \bar{u}|_{\bar{u}=u_\lambda}$  with  $u_\lambda = \lambda u + (1 - \lambda)u^*$ . By considering (32) and (23), we can write the error system (20) as

$$\dot{e}_s = -k_v e_s + b_{u_\lambda}(u - u^*).$$

As  $(\xi, \eta, u_\lambda) \in \bar{U}$ , we have  $b_{u_\lambda} \geq b_0 > 0$  (Assumption 2). Divided by  $b_{u_\lambda}$ , the above equation can be written as

$$b_{u_\lambda}^{-1} \dot{e}_s = -k_v b_{u_\lambda}^{-1} e_s + u - u^*. \tag{33}$$

By substituting (27) and (29) into (33), we obtain

$$\begin{aligned} b_{u_\lambda}^{-1} \dot{e}_s &= -k_v b_{u_\lambda}^{-1} e_s + \hat{W}^T S(z) - k_s |e_s| e_s \\ &\quad - W^{*T} S(z) - \epsilon_u(z) \\ &= -k_v b_{u_\lambda}^{-1} e_s + \tilde{W}^T S(z) - k_s |e_s| e_s - \epsilon_u(z). \end{aligned} \tag{34}$$

In the next section, a NN weight updating algorithm is given to realize the direct adaptive control.

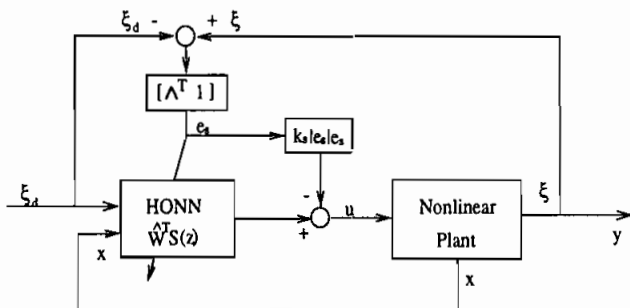


Figure 1. Direct adaptive control using high-order neural networks.

#### 4. Adaptive law and stability analysis

We present the NN weight tuning algorithm as follows:

$$\dot{\hat{W}} = -\Gamma[S(z)e_s + \delta|e_s|(\hat{W} - W^0)], \tag{35}$$

where  $\Gamma = \Gamma^T > 0$ ,  $\delta > 0$ , and  $W^0$  is a designed constant vector to be specified later. The first term on the right-hand side of (35) is the modified backpropagation algorithm, and the second term corresponds to the  $e$ -modification term in the adaptive control (Narendra and Annaswamy 1987), which can improve the robustness of the controller in the presence of the HONN approximation error. The following theorem shows the main results of this paper.

**Theorem:** For (1), with Assumptions 1–5 satisfied, let the controller be given by (29), and the neural network weights are updated by (35). Then, there exist compact sets  $\Phi_0$  and  $\Omega_0$ , and positive constants  $c^*$ ,  $\delta^*$ ,  $k_s^*$  and  $l^*$  such that if: all initial states  $(\xi(0), \eta(0)) \in \Phi_0$  and  $\hat{W}(0) \in \Omega_0$ ; and  $c \leq c^*$ ,  $\delta \geq \delta^*$ ,  $k_s \geq k_s^*$ ,  $l \geq l^*$ , then the trajectories of the system remain in the compact set  $\bar{U}$  and the tracking error converges to a small neighbourhood of the origin. In addition, the tracking error can be made arbitrarily small by increasing the gains  $k_v$  and/or  $k_s$  and the NN nodes  $l$ .

**Proof:** The proof has two steps. We first assume that the system trajectories remain in the compact set  $\bar{U}$  so that the transformation from (1) to the normal form (6) and the NN approximation in Assumption 5 are valid. With this assumption, we show that the tracking error converges to a small neighbourhood of the origin. Then we show that for a proper choice of the reference signal  $y_d(t)$  and control parameters, the trajectories do remain in the compact set  $\bar{U}$ .

*Step 1:* Consider the Lyapunov function candidate as

$$V = \frac{1}{2}[b_{u_\lambda}^{-1} e_s^2 + \tilde{W}^T \Gamma^{-1} \tilde{W}]. \tag{36}$$

By differentiating (36) along (34) and (35), we have

$$\begin{aligned} \dot{V} &= e_s[-k_v b_{u_\lambda}^{-1} e_s - k_s |e_s| e_s + \tilde{W}^T S(z) - \epsilon_u(z)] \\ &\quad + \frac{1}{2} \frac{d(b_{u_\lambda}^{-1})}{dt} e_s^2 + \tilde{W}^T \Gamma^{-1} \dot{\tilde{W}} \\ &= -k_s |e_s| e_s^2 - \frac{k_v}{b_{u_\lambda}} e_s^2 - \frac{\dot{b}_{u_\lambda}}{2b_{u_\lambda}^2} e_s^2 - \epsilon_u(z) e_s \\ &\quad - \delta |e_s| \tilde{W}^T (\hat{W} - W^0). \end{aligned} \tag{37}$$

By completing the square it can be shown that

$$2\tilde{W}^T (\hat{W} - W^0) = \|\tilde{W}\|^2 + \|\hat{W} - W^0\|^2 - \|W^* - W^0\|^2. \tag{38}$$

Therefore

$$\begin{aligned} \dot{V} &\leq |e_s| \left[ -k_s e_s^2 - \frac{k_v}{b_{u_\lambda}} |e_s| - \frac{\dot{b}_{u_\lambda}}{2b_{u_\lambda}^2} |e_s| - \frac{\delta}{2} \|\tilde{W}\|^2 \right. \\ &\quad \left. + \frac{\delta}{2} \|W^* - W^0\|^2 + |\epsilon_u(z)| \right] \\ &\leq |e_s| \left[ -k_s \left( |e_s| - \frac{\beta_0}{k_s} \right)^2 - \frac{k_v}{b_{u_\lambda}} |e_s| - \frac{\delta}{2} \|\tilde{W}\|^2 + \beta_1 \right], \end{aligned} \quad (39)$$

where

$$\begin{aligned} \beta_0 &= \frac{|\dot{b}_{u_\lambda}|}{4b_{u_\lambda}^2}, \\ \beta_1 &= \frac{\delta}{2} \|W^* - W^0\|^2 + |\epsilon_u(z)| + \frac{b_{u_\lambda}^2}{16k_s b_{u_\lambda}^4}. \end{aligned}$$

According to Assumptions 2 and 5, we know that  $b_0 \leq b_{u_\lambda} \leq b_1$ ,  $|\dot{b}_{u_\lambda}| \leq b_2$  and  $|\epsilon_u(z)| \leq \bar{\epsilon}_u$ . As  $k_v$ ,  $k_s$  and  $\delta$  are chosen as positive constants, it can be seen that  $\beta_0$  and  $\beta_1$  are bounded:

$$\left. \begin{aligned} \beta_0 &\leq \frac{b_2}{4b_0^2}, \\ \beta_1 &\leq \frac{\delta}{2} \|W^* - W^0\|^2 + \bar{\epsilon}_u + \frac{b_2^2}{16k_s b_0^4}. \end{aligned} \right\} \quad (40)$$

Now define

$$\Theta_e := \left\{ e_s \mid |e_s| \leq \min \left\{ \frac{\beta_0}{k_s} + \sqrt{\frac{\beta_1}{k_s}, \frac{\beta_1 b_1}{k_v}} \right\} \right\}, \quad (41)$$

$$\Theta_w := \left\{ \tilde{W} \mid \|\tilde{W}\| \leq \max \left\{ \sqrt{\frac{2\beta_1}{\delta}}, \|\tilde{W}(0)\| \right\} \right\}, \quad (42)$$

$$\begin{aligned} \Theta_{ew} := &\left\{ (e_s, \tilde{W}) \mid k_s \left( |e_s| - \frac{\beta_0}{k_s} \right)^2 + \frac{k_v}{b_1} |e_s| \right. \\ &\left. + \frac{\delta}{2} \|\tilde{W}\|^2 \leq \beta_1 \right\}. \end{aligned} \quad (43)$$

It is obvious that  $\Theta_e$ ,  $\Theta_w$  and  $\Theta_{ew}$  are compact sets. Expressions (39) and (43) imply that  $\dot{V} \leq 0$  so long as  $V$  is outside the compact set  $\Theta_{ew}$ . According to a standard Lyapunov theorem (Narendra and Annaswamy 1989), we conclude that  $e_s$  and  $\tilde{W}$  are bounded. Furthermore, from (39) and (41), it can be seen that  $\dot{V}$  is strictly negative so long as  $e_s$  is outside the set  $\Theta_e$ . Therefore there exists a constant  $T$  such that, for  $t > T$ ,  $e_s$  will converge to  $\Theta_e$ . Using Lemma 1, we know that the internal dynamic  $\eta$  will converge to

$$\Theta_\eta := \left\{ \eta(t) \mid \|\eta\| \leq L_u \min \left\{ \frac{\beta_0}{k_s} + \sqrt{\frac{\beta_1}{k_s}, \frac{\beta_1 b_1}{k_v}} \right\} + L_b c \right\}, \quad \forall t > T + T_0.$$

Let the vector  $\zeta = [\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_{\rho-1}]^T$ ; then a state representation for the mapping (12) is

$$\dot{\zeta} = A\zeta + be_s, \quad (44)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -\lambda_1 & -\lambda_2 & \dots & -\lambda_{\rho-1} \end{bmatrix} \in \mathbb{R}^{(\rho-1) \times (\rho-1)}, \\ b &= \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^{\rho-1}. \end{aligned} \quad (45)$$

As  $s^{\rho-1} + \lambda_{\rho-1}s^{\rho-2} + \dots + \lambda_1$  is Hurwitz, the matrix  $A$  is stable. Therefore, there exist constants  $k_0 > 0$  and  $\lambda_0 > 0$  such that  $\|\exp(At)\| \leq k_0 \exp(-\lambda_0 t)$ . The solution for  $\zeta$  in (44) can be written as

$$\zeta(t) = \exp(At)\zeta(0) + \int_0^t \exp[A(t-\tau)]be_s d\tau,$$

and it follows that

$$\begin{aligned} \|\zeta(t)\| &\leq k_0 \exp(-\lambda_0 t) \|\zeta(0)\| + \frac{1 - \exp(-\lambda_0 t)}{\lambda_0} k_0 e_{\max} \\ &\leq k_0 \exp(-\lambda_0 t) \|\zeta(0)\| + \frac{k_0 e_{\max}}{\lambda_0}, \quad \forall t > T, \end{aligned} \quad (46)$$

where

$$e_{\max} = \min \left\{ \frac{\beta_0}{k_s} + \sqrt{\frac{\beta_1}{k_s}, \frac{\beta_1 b_1}{k_v}} \right\},$$

and  $k_0 \exp(-\lambda_0 t) \|\zeta(0)\|$  decays exponentially. Noting that  $e_{\max}$  can be reduced by increasing  $k_s$  and  $k_v$ , therefore the tracking error  $y - y_d = \tilde{\xi}_1$  will converge to a small neighbourhood of the origin.

In summary, in the case that  $(\xi, \eta, u) \in \bar{U}$ , if  $k_v$ ,  $k_s$  and  $\delta$  are chosen as positive constants, there exists a constant  $T$  such that: the tracking error converges to a small neighbourhood of the origin for all  $t > T$ ; the internal dynamics  $\eta$  converge to  $\Theta_\eta$  for all  $t > T + T_0$ ; and the parameter estimate error  $\tilde{W}$  is bounded by  $\Theta_w$  for all time. In addition, by increasing the gains  $k_v$  and/or  $k_s$  and the number of neural nodes  $l$ , the origin  $\Theta_e$  can be made as small as desired; therefore an arbitrarily small tracking error can be achieved.

*Step 2:* In this part we show that for a proper choice of the tracking signal  $y_d(t)$  and control parameters the trajectories do remain in the compact set  $\bar{U}$ . From (12) and  $\tilde{\xi} = [\zeta^T \tilde{\xi}_\rho]^T$ , it is shown that  $\tilde{\xi}_\rho = e_s - \Lambda^T \zeta$ . By considering (41) and (46), there exist positive constants  $\bar{k}_0$  and  $\bar{k}_1$  such that

$$\begin{aligned} \|\tilde{\xi}(t)\| &\leq \|\zeta(t)\| + |\tilde{\xi}_\rho(t)| \leq \bar{k}_0 \exp(-\lambda_0 t) \|\zeta(0)\| \\ &\quad + \bar{k}_1 e_{\max}, \quad \forall t > T. \end{aligned} \quad (47)$$

From the previous analysis, it can be seen that  $\dot{V} \leq 0$  and  $\dot{V}_0(\eta) \leq 0$  so long as  $(e_s, \tilde{W})$  and  $\eta$  are outside the compact sets  $\Theta_{ew}$  and  $\Theta_\eta$ , respectively. Let

$$\left. \begin{aligned} \Omega_D &:= \{(e_s, \tilde{W}, \eta) | V \leq D; \eta \in \Theta_\eta\}, \\ \Theta &:= \{(e_s, \tilde{W}, \eta) | (e_s, \tilde{W}) \in \Theta_{ew}; \eta \in \Theta_\eta\}, \\ \bar{D} &:= \inf \{D | \Omega_D \supset \Theta\}. \end{aligned} \right\} \quad (48)$$

Then clearly all the trajectories that start in  $\Omega_{\bar{D}}$  will remain in  $\Omega_{\bar{D}}$  for all time. From (48) we can see that  $\bar{D}$  is a function of  $c, \delta, k_v, k_s$  and  $\bar{\epsilon}_u$ . Noting that  $\bar{\epsilon}_u$  can be made arbitrarily small by increasing the NN nodes  $l$ ; and (47) implies that

$$\|\xi\| \leq \bar{k}_0 \exp(-\lambda_0 t) \|\zeta(0)\| + \bar{k}_1 e_{\max} + \|\xi_d\|,$$

then there exist positive constants  $c^*, \delta^*, k_s^*$  and  $l^*$  such that for all  $c \leq c^*, \delta \geq \delta^*, k_v > 0, k_s \geq k_s^*$  and  $l \geq l^*$  we have

$$(e_s, \tilde{W}, \eta) \in \Omega_{\bar{D}} \Rightarrow (\xi, \eta, u) \in \bar{U}.$$

In summary, there exist compact sets  $\Phi_0$  and  $\Omega_0$ , such that for all  $\|\xi_d\| \leq c^*, \delta \geq \delta^*, k_s \geq k_s^*, l \geq l^*$  and all initial conditions  $(\xi(0), \eta(0)) \in \Phi_0$  and  $\dot{W}(0) \in \Omega_0$ , we obtain  $(\xi, \eta, u) \in \bar{U}$ , for all time. This completes the proof.  $\square$

**Remark 1:** There is a design trade-off between the NN complexity and the magnitude of the tracking error. The greater the number of NN nodes, the smaller  $\bar{\epsilon}_u$  will be, which leads to smaller output tracking error. From (41) it can be seen that increasing  $k_s$  and  $k_v$  will reduce the tracking error. It should be noticed that if  $k_s$  and/or  $k_v$  are chosen to be too large, the controller may become a high-gain controller, which is not only expensive but may also excite the unmodelled dynamics of the practical systems.

**Remark 2:** The parameter  $W^0$  in the weight update law (35) is designed as the prior estimate of the ideal weight  $W^*$ . From the definition of  $\beta_1$  in (39), it can be seen that the smaller  $W^0 - W^*$ , the smaller  $\Theta_e$  will be. If no such estimate is available, the parameter  $W^0$  can simply be set to zero. The parameter  $\delta$  in adaptive law (35) needs to be designed carefully. From (40) and (41) it is shown that smaller tracking error will be achieved by choosing smaller  $\delta$ . Nevertheless, (42) implies that a smaller  $\delta$  may cause larger NN weight errors. In this case, the control signal  $u$  may be very large and out of the region  $U$  in which Assumptions 1–3 and 5 hold. On the other hand, if  $\delta$  is designed as a very large constant, a large tracking error will occur. Therefore, the parameter  $\delta$  should be chosen to be neither too small nor too large.  $\square$

**Remark 3:** Compared with the traditional exact linearization techniques and the neural network methods, the

proposed adaptive NN controller clearly has some intrinsic advantages. There is no need to search for an explicit controller to cancel the nonlinearities of the system exactly. In fact, even though the implicit function  $f(x, u)$  in (1) is known exactly, there does not always exist an explicit controller for feedback linearization. Instead of solving the implicit function for the explicit controller, a high-order neural networks is applied to approximate the ideal IFLC input  $u^*$  to achieve feedback linearization. Secondly, the requirements of an off-line training phase and the persistent excitation condition are not needed for the convergence of the tracking error.  $\square$

**Remark 4:** The adaptive law (35) is derived from the Lyapunov method and the  $e$ -modification term is introduced to achieve some robustness in the presence of the HONN approximation error. The bounding control term  $k_s |e_s| e_s$  in the controller (29) is used to limit the upper bounds of the system states; it therefore guarantees the validity of the strong relative degree and the NN approximation in Assumptions 1 and 5, respectively.  $\square$

## 5. Simulation

In this section the proposed adaptive control method is illustrated using a third-order SISO nonlinear plant

$$\left. \begin{aligned} \dot{x}_1 &= 2x_2 + x_1^2 + x_3, \\ \dot{x}_2 &= x_1 \exp x_2 + (1 + x_3^2)u + u^3 + \frac{[1 + \sin(x_1 + u)]u}{1 + x_2^2 + x_3^2}, \\ \dot{x}_3 &= -2x_3 + 0.2x_1^2 x_2, \\ y &= x_1. \end{aligned} \right\} \quad (49)$$

The nonlinearity is an implicit function with respect to  $u$ . Obviously, it is impossible to obtain an explicit controller for exact feedback linearization. As

$$\begin{aligned} \dot{y} &= L_f h = 2x_2 + x_1^2 + x_3, \quad \frac{\partial \dot{y}}{\partial u} = 0, \\ \ddot{y} &= L_f^2 h = 2\dot{x}_2 + 2x_1(2x_2 + x_1^2 + x_3) \\ &\quad - 2x_3 + 0.2x_1^2 x_2, \\ \frac{\partial \ddot{y}}{\partial u} &= 2 + 2x_3^2 + 6u^2 \\ &\quad + \frac{2 + 2\sin(x_1 + u) + 2[\cos(x_1 + u)]u}{1 + x_2^2 + x_3^2}. \end{aligned}$$

As  $\partial \ddot{y} / \partial u > 2, \forall (x, u) \in \mathbb{R}^4$ , the plant is of relative degree 2. Now choose the transformation

$$\Phi(x) = [\xi_1, \xi_2, \eta]^T = [x_1, L_f h, x_3]^T. \quad (50)$$



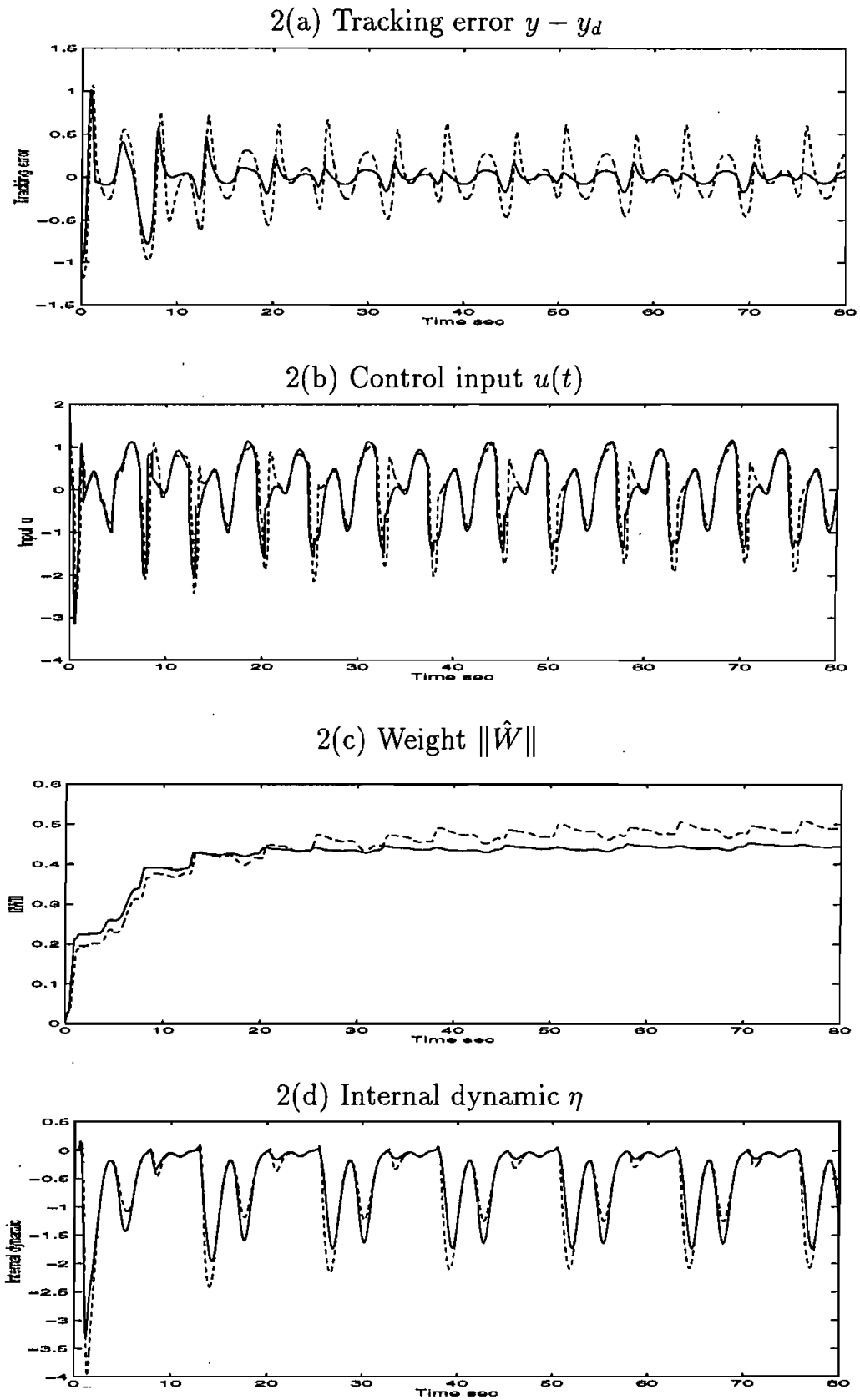


Figure 2. Direct adaptive control using high-order neural networks: case A (dashed line)  $k_s = 3 \cdot 0$ ,  $k_f = 0 \cdot 1$ ,  $l = 12$ ; case B (solid line)  $k_s = 12 \cdot 0$ ,  $k_f = 0 \cdot 1$ ,  $l = 24$ .

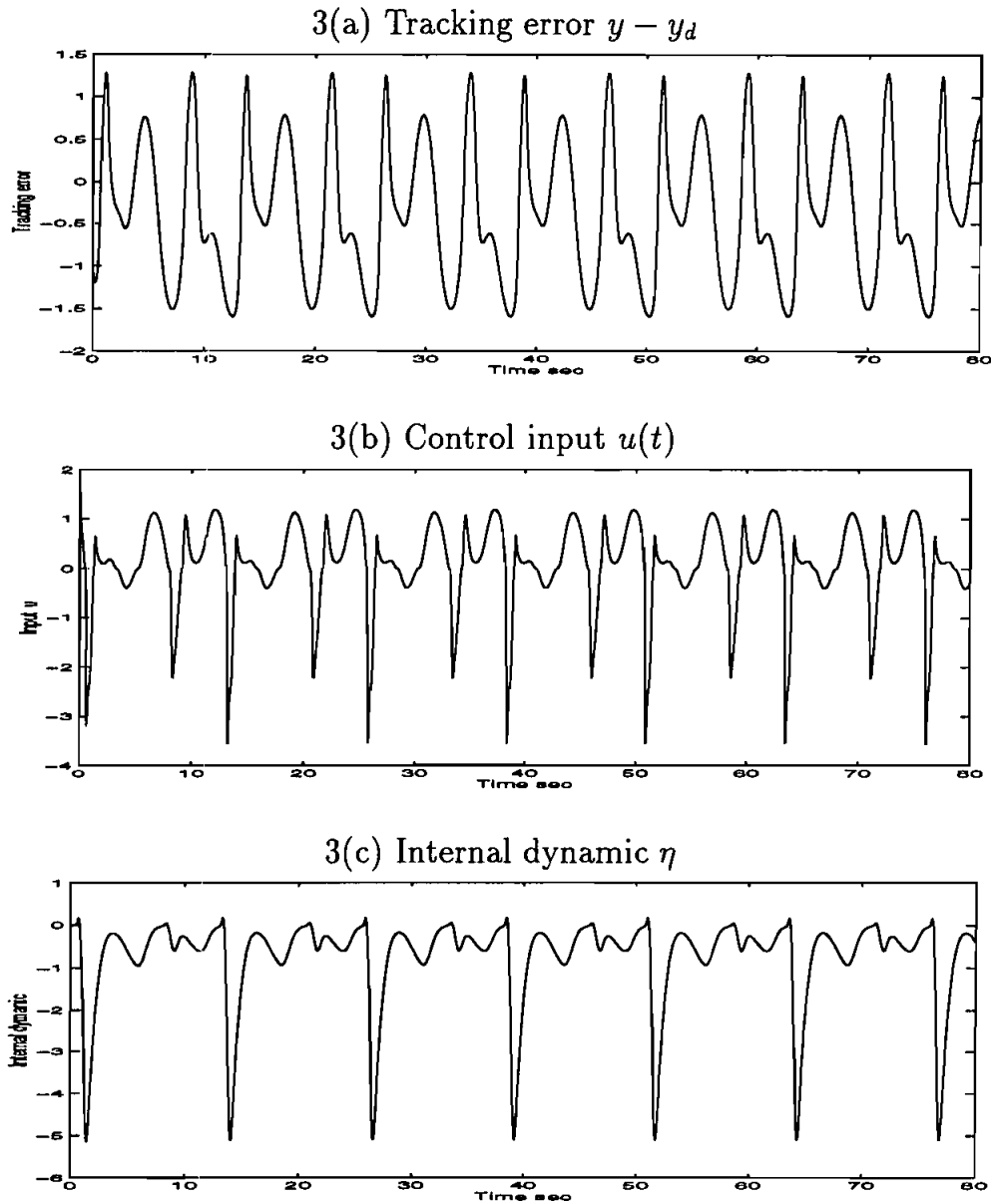


Figure 3. Response of controller  $u = -k_s|e_s|e_s$  with  $k_s = 0.1$ .

Equations (49) can be transformed into the normal form

$$\begin{aligned} \dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= L_f^2 h, \\ \dot{\eta} &= -(2 + 0.1\xi_1^2)\eta + 0.1\xi_1^2(\xi_2 - \xi_1^2). \end{aligned}$$

Clearly, the internal dynamic  $\eta$  is exponentially stable if

$$|\eta| > \frac{0.1\xi_1^2|\xi_2 - \xi_1^2|}{2 + 0.1\xi_1^2},$$

therefore Assumption 3 is satisfied. In the following simulation study, we suppose that no exact model of

(49) is available. The adaptive control method in this paper is used for tracking control.

The reference signal is chosen as

$$y_d(t) = 2 \sin(t) + \cos(0.5t).$$

A neural network controller

$$u(t) = \hat{W}^T S(z) - k_s|e_s|e_s, \quad \text{with } k_s = 0.1,$$

has been chosen. The input vector is  $z = [x_1, x_2, x_3, v]^T$  with  $\Lambda = 2.0$ . The parameters in the weight update law (35) are chosen as  $\Gamma = \text{diag}\{0.02\}$ ,  $\delta = 3.0$  and  $W^0 = 0.0$ . The initial conditions are  $x(0) = [0.0, 0.0, 0.0]^T$  and  $\hat{W}(0) = 0.0$ .

Figure 2 shows the simulation results for two cases with different parameters  $k_v$  and  $l$ . Because of the large initial weight errors ( $\hat{W}(0) = 0 \cdot 0$ ), large tracking error  $y - y_d$  is shown in Fig. 2(a) during the first 20 s. Through the NN learning process, the tracking error converges to a small set after 20 s. The boundedness of the NN weight estimates and internal dynamic are shown in Fig. 2(c) and Fig. 2(d). By comparing the tracking performance of case A ( $k_v = 3 \cdot 0$  and  $l = 12$ ) with that of case B ( $k_v = 12 \cdot 0$  and  $l = 24$ ), we can see that the tracking error decreases and better transient performance are obtained in case B by increasing the gain  $k_v$  and NN nodes  $l$ . This verifies the results of Theorem 1.

To study the contribution of the neural networks, we let  $u = -k_s |e_s| e_s$  with  $k_s = 0 \cdot 1$ ; that is, with no neural networks. The simulation results in Fig. 3 show that the output cannot track the reference signal effectively. However, it can be seen that the states of the system are bounded using this controller. Therefore, by adding this bounding control term, the states and control input are guaranteed to be in the compact set  $U$ .

## 6. Conclusion

This paper has addressed the adaptive control problem for nonlinear systems using high-order neural networks. The newly proposed adaptive controller applies to a general class of unknown nonlinear systems and guarantees the stability of the system. Robustness results are achieved by employing a modified back-propagation updating law with the  $e$ -modification term. No requirements of an off-line training phase and the persistent excitation condition are needed. The theoretical analysis and the simulation results show that the proposed method is very effective in controlling nonlinear dynamic systems.

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